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RESEARCH ARTICLE

CLASSIFICATIONS IN FUZZY PSEODOMETRIC SPACES

SERVET KUTUKCU[1*], ELIF AYDIN[2]

[1,2]Department of Mathematics, OndokuzMayis University 55139 Kurupelit, Samsun, Turkey

*E-mail: skutukcu@omu.edu.tr

ABSTRACT

In this paper, we give strong and stationary structures in fuzzy pseudometric spaces and present relations between them illustrating examples.

Keywords: strong, stationary, fuzzy pseudometric.

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1. INTRODUCTION

A triangular norm, shortly t-norm, is a kind binary operation on the unit interval [0,1] used in multivalued logic especially fuzzy logic generalizes intersection in a lattice, conjunction in logic and triangle inequality in ordinary metric spaces. In [6], a t-norm $\ast : [0,1] \times [0,1] \to [0,1]$ defined as $a\ast1=$ for all $a\in[0,1]$, $\ast$ is symmetric, $\ast$ is nondecreasing in each variable and $\ast$ is associative. We will make use of three basic t-norms, namely the minimum operator, the algebraic product and the Lukasiewicz t-norm TL defined by $a\ast b = \min\{a,b\} = TM$, $a\ast b = a.b = TP$ and $a\ast b = \max\{0, a+b-1\} = TL$, respectively. These t-norms are ranked as $TL \le TP \le TM$, in fact, TM is the strongest t-norm. Using t-norms to generalize triangle inequality, George and Veeramani introduced fuzzy metric space,

Definition 1.1 [1]. A 3-tuble $(X,M,\ast)$ is called a fuzzy metric space, shortly FM-space, if X is an arbitrary set, $\ast$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$ satisfying following conditions; for all $x,y,z\in X$ and $t,s>0$

FM1) $M(x,y,t) > 0$,

FM2) $M(x,y,t) = 1$ iff $x = y$,

FM3) $M(x,y,t) = M(y,x,t)$,

FM4) $M(x,y,t+s) \ge M(x,z,t) \ast M(z,y,s)$,

FM5) $M(x,y,.) : (0,\infty) \to [0,1]$ is continuous.

Also M is called a fuzzy metric on X.

If we take FM2*) x = y then M(x,y,t) = 1 instead of FM2, then (X,M,$*$) is called fuzzy pseudometric space, shortly FPM-space.

If we take minimum operator $*$, then (X,M,$*$) is called fuzzy ultrametric space.

Throughout the paper, we will denote $(0,\infty)$ with IR^+.

2.STRONG FUZZY PSEODOMETRIC SPACES

Definition 2.1. Let (X,M,$*$) be a FPM-space. M is said to be a strong fuzzy pseodometric on X, or (X,M,$*$) is said to be a strong FPM-space if $M(x,y,t) \geq M(x,z,t) * M(z,y,t)$ for all $x,y,z\in X$ and t>0.

Remark 2.1. From Definition 1.1, every strong FPM-space is a FPM-space, but the converse is not true.

Example 2.1. Let X = {x,y,z} and $a*b = a.b$ for all $a,b\in[0,1]$. Define $M:X\times X\times(0,\infty) \to [0,1]$ by M(x,x,t) = M(y,y,t) = M(z,z,t) = 1, M(x,z,t) = M(z,x,t) = M(y,z,t) = M(z,y,t) = t/t+1 and M(x,y,t) = M(y,x,t) = $t^2/(t+2)^2$. Then M is a fuzzy pseodometric metric but not a strong fuzzy pseodometric on X.

Example 2.2. Let X = {x,y,z} and $a*b = \max\{0,a+b-1\}$ for all $a,b\in[0,1]$. Define $M:X\times X\times(0,\infty) \to [0,1]$ by M(x,x,t) = M(y,y,t) = M(z,z,t) = 1, M(x,z,t) = M(z,x,t) = M(y,z,t) = M(z,y,t) = 2t+1/2t+2 and M(x,y,t) = M(y,x,t) = t/t+2. Then M is a fuzzy pseodometric metric but not a strong fuzzy pseodometric on X.

Example 2.3. Let X = IR^+ and $a*b = a.b$ for all $a,b\in[0,1]$. Define $M:X\times X\times(0,\infty) \to [0,1]$ by $M(x,y,t) = \min\{x,y\}/\max\{x,y\}$. Then M is a fuzzy pseodometric metric and also a strong fuzzy pseodometric on IR^+.

Theorem 2.1. Let (X,d) be a pseodometric space and $a*b = a.b$ for all $a,b\in[0,1]$. Define $M_d:X\times X\times(0,\infty) \to [0,1]$ by $M_d(x,y,t) = t/t+d(x,y)$. Then M_d is a fuzzy pseodometric metric and also a strong fuzzy pseodometric on X. (X,M_d,$*$) is called standard strong FPM-space.

Proof. In [8], we know that (X,M_d,$*$) is a FPM-space. We only show that M_d is strong. Since (X,d) is a pseodometric space, we have $d(x,z) \leq d(x,y) + d(y,z)$ for $x,y,z\in X$. Then

$$1+\frac{d(x,z)}{t} \leq 1+\frac{d(x,y)+d(y,z)}{t}$$

$$\leq \frac{t^2+td(x,y)+td(y,z)+d(x,y).d(y,z)}{t^2}$$

$$\leq \frac{[t+d(x,y)][t+d(y,z)]}{t^2}$$

and so,

$$\frac{t}{t+d(x,z)} \geq \frac{t}{t+d(x,y)}.\frac{t}{t+d(y,z)}$$

hence,

$$M_d(x,z,t) \geq M(x,y,t) * M(y,z,t).$$

This completes the proof.

Definition 2.2. Let (X,M,$*$) be a strong FPM-space. If $*$ is minimum operator then (X,M,$*$) is said to be a strong fuzzy ultra-pseodometric space.

The proof of following theorem is easily omitted.

Theorem 2.2. Standard strong FPM-space (X,M_d,$*$) is a strong fuzzy ultra-pseodometric space if and only if d is ultra-pseodometric on X.

3. STATIONARY FUZZY PSEODOMETRIC SPACES

Definition 3.1. Let (X,M,$*$) be a FPM-space. M is said to be a stationary fuzzy pseudometric on X or (X,M,$*$) is said to be a stationary FPM-space if M does not depend on t, i.e. M(x,y,t) = M(x,y) is constant for all x,y$\in$ X.

Remark 3.1. From Definition 1.1, every stationary FPM-space is a FPM-space, but the converse is not true.

Example 3.1. From [8], we know that (X,M_d,$*$) is a FPM-space. But (X,M_d,$*$) is not a stationary FPM-space since the function M_d depends on t. If we take t = 1, then (X,M_d,$*$) is a stationary FPM-space. For t = 1, we call (X,M_d,$*$) is a standard stationary FPM-space.

Example 3.2. A FPM-space (X,M,$*$) give in Example 2.3 is a stationary FPM-space.

Example 3.3. Let X = (0,1/2) and a$*$b = max{0,a+b-1} for all a,b$\in$[0,1]. Define M:X×X×(0,∞) $\to$ [0,1] by $M(x,y,t)=\begin{cases} 1, & x=y \\ x+y, & x\neq y \end{cases}$. Then M is a stationary fuzzy pseodometric on X.

Theorem 3.1. Every stationary FPM-space is a strong FPM-space.

Proof. Let (X,M,$*$) be a stationary FPM-space. Then, from (FM4), M(x,y) $\geq$ M(x,z) $*$ M(z,y) for all x,y$\in$ X. Since M is not depend on t, we have M(x,y,t) $\geq$ M(x,z,t) $*$ M(z,y,t). From Definition 2.1, (X,M,$*$) is a strong FPM-space.

This completes the proof.

Remark 3.2. The converse of Theorem 3.1 is not true.

Example 3.4. Let X = IR^+ and a$*$b = min{a,b} for all a,b$\in$[0,1]. Define M: IR^+× IR^+×(0,∞) $\to$ [0,1] by $M(x,y,t)=\begin{cases} 1, & x=y \\ \dfrac{t}{t+1}, & x\neq y \end{cases}$. Then M is a strong fuzzy pseodometric but not a stationary fuzzy pseodometric on X.

Theorem 3.2. Let (X,M,$*$) be a FPM-space and define a function family {M_t : t$\in$$IR^+$} as M_t: X× X $\to$ (0,1] by M_t(x,y) = M(x,y,t). Then (X,M_t,$*$) is a stationary FPM-space for all t$\in$$IR^+$ if and only if (X,M,$*$) is a strong FPM-space.

Proof. Let (X,M_t,$*$) be a stationary FPM-space for all t$\in$$IR^+$. Then, from Theorem 3.1, (X,M_t,$*$) is a strong FPM-space for all t$\in$$IR^+$. Since M_t(x,y) = M(x,y,t) for all t$\in$$IR^+$, then (X,M,$*$) is a strong FPM-space.

Let (X,M,$*$) be a strong FPM-space. Then, from Remark 3.1, (X,M,$*$) is a FPM-space. Define a function φ : $IR^+$$\to$ (0,1] by φ(t) = k (a constant), then φ is increasing, continuous and $M(x,y,t)=M(x,y,k)=M_k(x,y)=M_t(x,y)$. So the distance M(x,y,t) is not depend on t for all t>0. Hence, (X,M,$*$) i.e. (X,M_t,$*$) be a stationary FPM-space for all t$\in$$IR^+$.

This completes the proof.

Definition 3.2. Let (X,M,$*$) be a stationary FPM-space. If $*$ is minimum operator then (X,M,$*$) is said to be a stationary fuzzy ultra-pseodometric space.

The proof of following theorem is easily omitted.

Theorem 3.3. Standard stationary FPM-space (X,M_d,$*$) is a stationary fuzzy ultra-pseodometric space if and only if d is ultra-pseodometric on X.

3. OPEN PROBLEMS

Problem 3.1. In [3], Kutukcuet.all defined intuitionistic fuzzification of Menger spaces. What are intuitionistic classifications of the notions strong and stationary?

Problem 3.2. In [4], Park defined intuitionistic fuzzification of metric spaces. What are intuitionistic classifications of the notions strong and stationary?

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