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RESEARCH ARTICLE

SOME RESULTS ON THE REDUCED COLOR ENERGY OF GRAPHS

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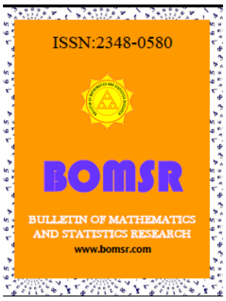

ABSTRACT

Recently we have defined three types of reduced color energy of graphs and also discussed an admissible and forbidden class of colored graphs based on their reduced color energy with an arbitrary fixed coloring with minimum colors. In this article we define fourth type of reduced color energy, i.e., the sum of absolute negative color eigenvalues and investigate an admissible and forbidden class of colored graphs based on their fourth type reduced color energy with an arbitrary fixed coloring with minimum colors. We also find fourth type of reduced color energy of graphs of order six whose reduced energy does not exceed 5.

AMS Subject Classifications: (05C50).

Keywords: Color Eigenvalues, Color energy and Reduced Color Energy,

1.0 INTRODUCTION

In this paper we shall consider only simple, finite connected colored graphs. The vertex set of colored graph G is denoted by $V(G)$ and its order by $|G| = n$. A coloring of graph G is a coloring of its vertices such that no two adjacent vertices receive the same color. The minimum number of colors needed for coloring of a graph G is called chromatic number and is denoted by $\chi(G)$. Recently C. Adiga and et.al., in [1] have introduced color matrix, color energy and investigated many properties, results of color energy and color eigenvalues of a graph.

The color spectrum of such colored graph is the set $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$ of colored eigenvalues of its $(-1, 0, 1)$ color matrix whose entries are $A_c(G) = (a_{ij})$, where

$$a_{ij} = \begin{cases} 1, & if\ v_i\ and\ v_j\ are\ adjacent\ with\ c(v_i) \neq c(v_j) \\ -1, & if\ v_i\ and\ v_j\ are\ not\ adjacent\ with\ c(v_i) = c(v_j) \\ 0, & otherwise \end{cases}$$

The characteristic polynomial of $A_c(G)$ is denoted by $P_c(G,\lambda) = det(A_c(G) - \lambda I)$. If the colors are used minimum then the color matrix is denoted by $A_\chi(G)$. The eigenvalues of G are eigenvalues of $A_\chi(G)$ they are called chromatic eigenvalues. Since $A_\chi(G)$ is real and symmetric its color eigenvalues are also real numbers. The color energy is defined as $E_\chi(G) = \sum_{i=1}^{n}|\lambda_i|$. If the graph G is colored with $\chi(G)$ colors, then the color energy of a graph with respect $\chi(G)$ is denoted by $E_\chi(G)$. This is called chromatic energy of G. If the distinct color eigenvalues of $A_\chi(G)$ are $\lambda_1 > \lambda_2 > \cdots > \lambda_r,\ r \le n$ with their multiplicities are $m_{1,} m_2, \ldots. m_r,$ then we have $Spec_c(G) = \begin{pmatrix} \lambda_{1,} & \lambda_2, \ldots\ldots, & \lambda_r \\ m_{1,} & m_2, \ldots\ldots, & m_r \end{pmatrix}$.

Reduced Color Energy of Graphs

The different types of reduced (ordinary) energy of connected simple graphs are defined and studies in [5, 6]. Motivated by papers [1-4] and that fact that, color matrix is generalization of adjacency matrix of a graph G (see [1]), so we were defined and discussed reduced color energy of graphs of type-I, Type-II and Type-III in [1, 2]. Since the color matrix is generalization of the adjacency matrix and also the fact that absolute sum of negative color eigenvalues have close resemblance with sum of positive color eigenvalues for arbitrary fixed coloring. Therefore we define reduced energy of Type-IV and discuss the class of colored graphs which are admissible and forbidden, whose reduced color energy does not exceed 10 for some special class of graphs and also investigate an admissible and forbidden class of graphs of order six, whose reduced color energy does not exceed 5 with arbitrary fixed coloring, see in Table 2.

Preliminary Results

Definition 1. (Reduced Color Energy of Type-I [5]**):** The reduced color energy of type-I, i.e., $R_1(G)$ is sum of all color eigenvalues of G including their multiplicities and excluding the largest color eigenvalue λ_1 of G.

Definition 2.(Reduced Color Energy of Type-II [6]**):** The reduced color energy of type-II is the sum of all positive color eigenvalues of G including their multiplicities and it is denoted by $R_2(G)$.

Definition 3. (Reduced Color Energy of Type-III [6]**):** The reduced color energy of graph G of type-III, i.e., $R_3(G)$ is the sum of absolute color eigenvalues excluding smallest eigenvalues $\lambda_n(G)$ of colored graph G.

Note 1. We have found an admissible and forbidden class of graphs whose $R_1(G)$, $R_2(G)$ and $R_3(G)$ does not exceed 10 for some special class of graphs with arbitrary fixed coloring in [5,6].

Note 2. An admissible and forbidden class of graphs whose $R_1(G)$, $R_2(G)$ and $R_3(G)$ does not exceed 5 for up-to six vertices with an arbitrary fixed coloring are listed in Table 2 and in Table 1 (their arbitrary fixed color code) at the end.

Main Results

Now we define reduced color energy of Type-IV. The proof technique involved here is similar to the proof techniques which are discussed in [5, 6].

Definition 4. (Reduced Color Energy of Type-IV**):** The reduced color energy of type-IV i.e., $R_4(G)$ is sum of all negative color eigenvalues of G including their multiplicities.

Now we shall prove an important property of general class

Theorem 1. The class $C_4(a)$ is finite for any real number $a \ge 1$.

Proof: Let G be an arbitrary colored graph from the class of $C_4(a)$, then we have

$$a \ge \sum_{i=1}^{k}|-\lambda_i| \ge \sum_{i<0}^{k}|-\lambda_i| \ge \sum_{i>0}^{k}|-\lambda_i|$$

Where $-\lambda_i\ (i = 2, 3, \ldots, k)$ are negative color eigenvalues.

Thus we have $G \in S(a)$, where $S(a) = \{G/ \sum_{i>0}^{k} |-\lambda_i| \leq a\}$.
Hence $C_4(a) \subseteq S(a)$. The class $S(a)$ is finite for every $a \geq 1$. Therefore $C_4(a)$ is finite for every $a \geq 1$.

In this article we completely describe $C_4(10)$ for special class of colored graphs and describe $C_4(5)$ for colored graphs of order six with fixed arbitrary coloring with $\chi(G)$ colors.
If H is any connected subgraph of a colored graph G, we denote it by $H \subseteq G$. Heredity property is universal concept in mathematics as we well other than mathematics subjects. So by heredity property of color energy, we have $E_\chi(H) \subseteq E_\chi(G)$ whence we have that any connected subgraph of an admissible colored graph is also admissible. This implies that the method of forbidden subgraphs can be consistently applied.
Next $K_{n_1,n_2,\dots,n_m}$, P_n and C_n be the complete $m-$partite colored graph, colored path graph and colored cycle graph with n vertices, respectively. Since complete bipartite colored graph $K_{n,\ m}$ has just one negative color eigenvalue $r(G)$ (including their multiplicity), it will belong to the class $C_4(a)$ if and only if $r(G) \leq a$.
We shall firstly determine the exact values of parameter n, m for which the colored graph $K_{n,\ m}$ $(n \leq m)$ is admissible.
Proposition 1. The colored graph $K_{n,m}$ $(n \leq m)$ is admissible exactly for the following values of parameter n, m

1. $n = 1,\ m = 1, 2, 3,4,5,6,7,8,9,10$
2. $n = 2,\ m =2, 3,4,5,6,7,8,9$
3. $n = 3,\ m = 3,4,5,6,7,8.$
4. $n = 4,\ m = 4,5,6,7.$
5. $n = 5,\ m = 5,6.$

Proof: Since $K_{n,\ m}$ is complete bipartite colored graph and color spectrum of $K_{n,\ m}$ is

$$Spec_\chi(K_{n,\ m}) = \begin{pmatrix} -(n+m-1) & 1 \\ 1 & n+m-1 \end{pmatrix}$$

Therefore it will belong to the class $C_4(10)$ if and only if $|-(n+m-1)| \leq 10$, and $K_{n,\ m}$ when $n = m = 6$ is forbidden graph and hence the statement.
Now we look at $K_{n_1,n_2,\dots,n_m}$ complete $m-$partite colored graphs. Since $K_{n_1,n_2,\dots,n_m}$ colored graphs have just two positive color eigenvalues $r(G)$ (including their multiplicity), it will belong to the class $C_4(10)$ if and only if $r(G) \leq 10$.
We shall determine the exact values of parameter $n_1, n_2, \dots, n_m$ for which the colored graph $K_{n_1 n_2,\dots n_m}$, $(n_1 \leq n_2 \leq \dots \leq n_m)$ is admissible.
Proposition 2. The colored graph $K_{n,m,k}$ $(n \leq m \leq k)$ is admissible exactly for the following values of parameter n, m, k

1. $n = 1,\ m = 1,\ k = 1, 2, 3, 4, 5,6,7,8$
2. $n = 2,\ m = 2, k = 2,3,4,5.$
3. $n = 3,\ m = 3, k = 3,$

Proof. It is easy to check that all the above colored graphs are admissible. Besides, since the colored graph $K_{n,m,k}$ is forbidden for the values of parameters $(n, m, k) = (1, 1, 9)$, $(2, 2, 6,)$ and $(3, 3, 4,)$. By heredity property of color energy proves the statement.
Proposition 3. The colored graph $K_{n,m,k,l}$ $(n \leq m \leq k \leq l)$ is admissible exactly for the following values of parameter n, m, k, l,

1. $n = 1,\ m = 1,\ k = 1,\ l = 1\ 2, 3, 4, 5, 6, 7.$
2. $n = 2,\ m = 2,\ k = 2,\ l = 2.$

Proof: It is easy to check that all the above colored graphs are admissible. Besides, since the colored graph $K_{n,m,k,l}$ is forbidden for the values of parameters $(n, m, k, l) = (1, 1, 1, 8),\ (2, 2, 2, 3)$. By heredity property of color energy proves the statement.

Proposition 4. The colored graph $K_{n,m,k,l,p}(n \leq m \leq k \leq l \leq p)$ is admissible exactly for the following values of parameter n, m, k, l, p

1. $(n, m, k, l, p) = (1, 1, 1, 1, 1),\ (1, 1, 1, 1,\ 2)\ (1, 1, 1, 1, 3), (1, 1, 1, 1, 4), (1, 1, 1, 1, 5)$.

Proof: It is easy to check that all the above colored graphs are admissible. Besides, since the colored graph $K_{n,m,k,l}$ is forbidden for the values of parameters $(n, m, k, l, p) = (1, 1, 1, 1\ 6),\ (2, 2, 2, 2, 2)$. By heredity property of color energy proves the statement.

Proposition 5. The colored graph $G = K_{n,m,k,l,p,r}$ is admissible exactly for the following values of parameter n, m, k, l, p, r

$$(n, m, k, l, p, r) = (1, 1, 1, 1, 1\ 1), (1, 1, 1, 1, 1, 2)\ (1,1, 1, 1, 1, 3), (1, 1, 1, 1, 1, 4).$$

Proof. It is easy to check that all the above colored graphs are admissible. Besides, since the colored graph $K_{n,m,k,l,p,r}$ is forbidden for the values of parameters $(n, m, k, l, p\ r) = (1, 1, 1, 1\ 1\ 5)$, $(2, 2, 2, 2\ 2\ ,2)$. By heredity property of color energy proves the statement.

Proposition 6. The colored graph $G = K_{n,m,k,l,p,r,s}$, is admissible exactly for the following values of parameter n, m, k, l, p, r, s

$$(n, m, k, l, p, r, s) = (1, 1, 1, 1, 1, 1\ 1), (1, 1, 1, 1, 1, 1, 2)\ (1,1, 1,1, 1, 1, 3), (1, 1, 1, 1, 1, 1, 4).$$

Proof. It is easy to check that all the above colored graphs are admissible. Besides, since the colored graph $K_{n,m,k,l,p,r}$ is forbidden for the values of parameters $(n, m, k, l, p\ r, s) = (1, 1, 1, 1, 1, 1,5)$, $(2, 2, 2, 2\ 2, 2, 2)$. By heredity property of color energy proves the statement.

Proposition 7. The colored graph $G = K_{n,m,k,l,p,r,s,t}$, is admissible exactly for the following values of parameter n, m, k, l, p, r, s, t

$$(n, m, k, l, p, r, s, t) = (1, 1, 1, 1, 1,1, 1\ 1), (1, 1, 1, 1, 1, 1, 1, 2).$$

Proof. It is easy to check that all the above colored graphs are admissible. Besides, since the colored graph $K_{n,m,k,l,p,r}$ is forbidden for the values of parameters $(n, m, k, l, p\ r, s, t) = (1, 1, 1, 1, 1,1, 1, 3)$, $(2, 2, 2, 2\ 2, 2, 2,2)$. By heredity property of color energy proves the statement.

Proposition 8. The null colored graph G of order $n \leq 11$ is admissible.

Proof. Let G be null colored graph of order n. Since $\chi(G) = 1$, the color spectrum of null graph is $\begin{pmatrix} -(n-1) & 1 \\ 1 & (n-1) \end{pmatrix}$. Thus G is admissible if $R_4(G) = |-(n-1)| \leq 10$, for $n \leq 11$. otherwise forbidden if $n = 12$.

Note: The $E_\chi(K_n) = E_\chi(\overline{K_n})$, thus G is admissible if $n \leq 11$.

Proposition 9. Let $\overline{(K_{1,\ n-1})_c}$ be the complement of the colored star graph of order n, which is colored with χ colors, is admissible if $n \leq 12$.

Proof. We know that $Spec\overline{(K_{1,\ n-1})_c}$ is $\begin{pmatrix} 0 & -(n-2) & 1 \\ 1 & 1 & n-2 \end{pmatrix}$

and $R_4\left(\overline{(K_{1,\ n-1})_c}\right) = |-(n-2)| \leq 10$, therefore is admissible if $n \leq 12$, and by heredity property of color energy provides the statement. On the other hand if for $n \leq 13$ then $R_4\left(\overline{(K_{1,\ n-1})_c}\right) = 11 \nleq 10$ are forbidden colored graphs.

Proposition 10. Let $\overline{(K_{m,\ n})_c}$ be the complement of the colored complete bipartite graph of order n, which is colored with χ colors, is admissible exactly for the following values of parameter $m, n,\ (m \leq n)$.

1. $m = 1, n = 1,2,3, . . ,11$.

2. $m = 2, n = 2, 3, \dots, 9, 10.$
3. $m = 3, n = 3,4, ..7,8,9$
4. $m = 4, n = 4\ ,5,6,7,8.$
5. $m = 5, n = 5,6,7.$
6. $m = 6, n = 5,6.$

Proof. We know that $Spec\overline{(K_{m,\ n})_c} = \begin{pmatrix} -(n-1) & -(m-1) & 1 \\ 1 & 1 & m+n-2 \end{pmatrix}$ and $R_4\overline{(K_{m,\ n})_c} = |-(m+n-2)| \leq 10$ and is admissible for above values of parameters (m, n). Besides, since the graph $\overline{K_{n,\ m}}$ is forbidden colored graph for the values of parameters $(n, m) = (4, 6), (5, 5)$, and above;

Proposition 11. If S_n^0 is the colored crown graph of order $2n$ is admissible if $n \leq 6$.

Proof. We know that $Spec_\chi(S_n^0) = \begin{pmatrix} -2(n-1) & 0 & 2 \\ 1 & n & n-1 \end{pmatrix}$ and $R_4(S_n^0) = |-2(n-1)| \leq 10$ and hence is admissible if $n \leq 6$. Further by heredity property of color energy proves the statement. On the other hand whence $n \geq 7$ are forbidden colored graphs.

Proposition 12. If $\overline{(S_n^0)}$ is the complement of the colored crown graph of order $2n$ is admissible if $n \leq 6$.

Proof. We know that $Spec_\chi\overline{(S_n^0)} = \begin{pmatrix} -(n-2) & -n & 2 & 0 \\ 1 & 1 & n-1 & n-1 \end{pmatrix}$ and $|R_4(G)| = |-(n-2)| + |-n| = 2(n-1)$ So $\overline{(S_n^0)}$ is admissible if $R_4\overline{(S_n^0)} = 2(n-1) \leq 10$ whence $n \leq 6$. Further By heredity property of color energy proves the statement. On the other hand if $n \geq 7$, $R_4\overline{(S_n^0)} = 2(n-1) = 12 \nleq 10$ thus $\overline{(S_n^0)}$ are forbidden colored graphs.

Proposition 13. If C_n is a colored cycle of order n is admissible if $n \leq 11$,

Proof. It is easy to check that, a colored cycle graphs of order up to 11 are admissible, i.e., $R_4(C_{11}) = 9.9720 \leq 10$, and By heredity property of color energy proves the statement. Besides, since the colored graph C_{12} and above graphs whose $R_4(C_{12}) \geq 10$ thus C_{12} and above are forbidden colored graphs.

The following Table-1 is the list of an upper triangular color matrix of graphs with an arbitrary fixed color and Table 2, contains the reduced color energy of graphs of the type-I- to type-IV of colored graphs up-to order six, with an arbitrary fixed coloring with $\chi(G)$ colors.

Table 1. Upper Triangular Color Matrices of Graphs

1.	1				
2.	11	1			
3.	1-1	1			
4.	111	11	1		
5.	111	1-1	1		
6.	11-1	11	0		
7.	1-11	1-1	1		
8.	111	-1-1	-1		
9.	1-10	1-1	1		
10.	1111	111	11	1	
11.	1111	111	1-1	1	
12.	1111	1-10	11	1	
13.	1111	1-11	1-1	1	
14.	111-1	111	10	0	
15.	1111	1-1-1	11	-1	
16.	1111	1-10	1-1	1	
17.	1-101	11-1	11	1	
18.	1-11-1	111	1-1	0	
19.	111-1	1-11	10	0	
20.	11-10	10-1	11	1	
21.	11-11	10-1	10	1	
22.	1-11-1	1-11	1-1	1	
23.	11-1-1	100	11	-1	
24.	1-10-1	110	1-1	1	
25.	11-10	100	1-1	1	
26.	1-11-1	1-10	1-1	1	
27.	1-101	1-10	10	1	
28.	1111	-1-1-1	-1-1	-1	
29.	1-1-10	11-1	-10	1	
30.	1-10-1	1-10	1-1	1	
31.	11111	1111	111	11	1

32.	11111	11-11	111	11	1
33.	11-101	1111	111	11	1
34.	11-111	11-11	111	11	1
35.	11111	1-100	111	11	1
36.	11111	1-11-1	111	1-1	1
37.	11111	1001	111	1-1	1
38.	11111	11-11	11-1	10	1
39.	11-111	11-11	11-1	11	1
40.	1111-1	1110	110	10	1
41.	1-11-11	1111	1-10	11	1
42.	11011	1-10 0	111	11	1
43.	11111	11-10	10-1	11	1
44.	1-1111	1-100	111	11	1
45.	11111	11-11	10 -1	10	1
46.	111-11	1-11-1	111	1-1	1
47.	11101	11-11	11-1	10	1
48.	11-101	11-11	11-1	11	1
49.	1111-1	1-110	111	10	0
50.	1111-1	1-111	110	10	0
51.	11111	-11-10	1-1 0	11	1
52.	11111	1-10-1	110	1-1	1
53.	1-1111	111-1	100	10	1
54.	1-1111	11-10	100	11	1
55.	11111	1-10-1	1-11	1-1	1
56.	111-11	111-1	100	10	1
57.	111-11	1-100	11-1	11	1
58.	11111	1-101	1-10	10	1
59.	11011	1-11-1	1-11	1-1	1
60.	111-11	1-110	10-1	11	1
61.	110-11	1-101	11-1	11	1
62.	1-11-11	1-110	1-11	11	1
63.	1-111-1	1111	10-1	10	0
64.	1111-1	11-11	100	10	0
65.	1111-1	1-101	110	10	0
66.	111-10	1100	10-1	11	1
67.	11111	-11-1-1	1-1-1	11	-1
68.	11-11-1	1111	1-10	1-1	0
69.	1-111-1	11-11	11-1	10	1
70.	11111	-11-10	1-10	1-1	1
71.	111-11	110 -1	100	10	1
72.	1-10-11	1101	1-10	11	1
73.	11111	1-10 -1	1-10	1-1	1
74.	1-1011	111-1	100	10	1
75.	111-11	1-10-1	101	1-1	1
76.	1-1111	1-101	100	10	1
77.	1-1111	1-101	100	10	1
78.	1-1011	11-10	100	11	1

79.	11-111	-11-10	1-10	11	1
80.	1-1111	1-101	101	1-1	1
81.	11-101	101-1	1-10	11	1
82.	1-11-11	1-11-1	1-1-1	1-1	1
83.	111-1-1	1100	100	11	-1
84.	111-1-1	1100	100	10	-1
85.	111-10	1100	100	1-1	1
86.	1-1-11-1	1110	-11-1	1-1	1
87.	1111-1	1-1-10	110	-10	1
88.	1-101-1	1-110	11-1	10	1
89.	1111-1	10-10	100	11	0
90.	1111-1	1-100	1-10	10	1
91.	11-100	10 -1-1	111	11	-1
92.	111-10	1-100	10-1	11	1
93.	1-111-1	10-10	11-1	10	1
94.	10-11-1	11-10	110	1-1	1
95.	11-11-1	11-10	1-10	1-1	1
96.	11011	1-10-1	1-11	1-1	0
97.	1-1111	1-10-1	100	1-1	1
98.	11011	1-10-1	1-10	1-1	1
99.	100-11	1-100	11-1	11	1
100.	1-1011	11-10	100	1-1	1
101.	11-111	1000	1-1-1	11	-1
102.	101-11	1-10-1	110	1-1	1
103.	11-111	-11-1-1	1-1-1	11	-1
104.	1-11-11	1-11-1	1-10	1-1	1
105.	1-11-1-1	1100	1-1-1	11	-1
106.	1-1-10-1	1110	-11-1	1-1	1
107.	1-10-10	111-1	1-10	10	1
108.	111-1-1	1-100	100	11	-1
109.	11-1-10	100-1	111	-10	1
110.	1-100-1	11-10	11-1	10	1
111.	1-11-10	1100	1-10	1-1	1
112.	111-10	1-10-1	100	1-1	1
113.	11-10-1	10-10	110	1-1	1
114.	11-100	10-10	1-10	11	1
115.	10-11-1	1010	1-10	1-1	1
116.	1-111-1	1-100	10-1	10	1
117.	11-11-1	1000	1-10	1-1	1
118.	1-11-10	1-100	1-10	11	1
119.	110-11	1-10-1	100	1-1	1
120.	1-1-11-1	11-10	-11-1	1-1	1
121.	1-111-1	1-1-10	11-1	-10	1
122.	1-11-11	1-10-1	1-10	1-1	1
123.	1-110-1	1-101	10-1	10	1
124.	11-1-1-1	1000	111	-1-1	-1
125.	1-1-10-1	1110	-11-1	0-1	1
126. .	1-100-1	1-110	11-1	00	1

127.	11-1-10	1000	11-1	-10	1
128.	1-100-1	10-10	11-1	10	1
129.	1-1-100	11-10	-100	11	1
130.	1-10-10	1-100	1-10	11	1
131.	1-11-1-1	1-100	1-1-1	11	-1
132.	1-10-10	1-11-1	1-10	1-1	1
133.	1-1-10-1	11-10	-11-1	1-1	1
134.	1-11-10	1-10-1	1-10	1-1	1
135.	10-11-1	1000	1-10	1-1	1
136.	1-10-10	1-10-1	1-10	1-1	1
137.	11111	-1-1-1-1	1-1-1	-1-1	-1
138.	1111-1	-1-1-10	-1-10	-10	1
139.	1-1-100	11-1-1	-100	11	-1
140.	1-1 0-10	1-11-1	1-10	0-1	1
141.	1-1 0-1-1	1-100	1-1-1	11	-1
142.	1-10-10	1-10-1	1-10	-11	1
143.	1				

Table 2: Contains color eigenvalues, color energy and four different types of reduced color energy of graphs with given fixed arbitrary coloring of graphs up-to order six vertices.

			Color Eigenvalues						**Color Enrgy and Four Types of Color Energy**				
S. No.	V	E	λ_n					λ_1	$E_c(G)$	$R_1(G)$	$R_2(G)$	$R_3(G)$	$R_4(G)$
1	2	1	-1					1	2	**1**	**1**	**1**	**-1**
2	3	3	-1	-1				2	4	**-1**	**2**	**3**	**-1**
3		2	-1	-1				2	4	**2**	**2**	**3**	**-2**
4	4	6	-1	-1	-1			3	6	**3**	**3**	**5**	**-3**
5		5	-2.2361	-1	1			2.2361	6.4722	**3.2361**	**4.2361**	**4.2361**	**-3.2361**
6		4	-2	-1	1			2	6	**3**	**4**	**4**	**-3**
7		4	-3	1	1			1	6	**2**	**5**	**3**	**-3**
8		3	-3	1	1			1	6	**2**	**5**	**3**	**-3**
9		3	-2.5616	0	1			1.5616	5.1232	**2.5616**	**3.5616**	**2.5616**	**-2.5616**
10	5	10	-1	-1	-1	-1		4	8	**4**	**4**	**7**	**-4**
11		9	-2.3723	-1	-1	1		3.3723	8.7446	**4.3723**	5.3723	6.3723	**-4.3723**
12		8	-2.2491	-1	-1	1.1464		3.1028	8.4983	**4.2492**	5.3955	6.2492	**-4.2491**
13		8	-3	-1.5616	1	1		2.5616	9.1232	**4.5616**	6.5616	6.1232	**-4.5616**
14		7	-2	-1	-1	1		3	8	**4**	**5**	6	**-4**
15		7	-3.3723	-1	1	1		2.3723	8.7446	**4.3723**	6.3723	5.3723	**-4.3723**
16		7	-2.5616	-1.8136	0.4707	1.5616		2.3429	8.7504	**4.3752**	6.4075	6.1888	**-4.3752**
17		7	-3.1028	-1.1464	1	1		2.2491	8.4983	**4.2491**	6.2492	5.3955	**-4.2492**
18		6	-3.1028	-1.1464	1	1		2.2491	8.4983	**4.2491**	6.2492	5.3955	**-4.2492**
19		6	-2.6636	-1.2527	0.1802	1.4652		2.2709	7.8326	**3.9163**	5.5617	5.169	**-3.9163**
20		6	-2	-2	0	2		2	8	**4**	6	6	**-4**
21		6	-2.7321	-1.4142	0.7321	1.4142		2	8.2926	**4.1463**	6.2926	5.5605	**-4.1463**
22		6	-4	1	1	1		1	8	**4**	7	**4**	**-4**
23		5	-3	-1	1	1		2	8	**4**	6	**5**	**-4**
24		5	-2.7321	-1.4142	0.7321	1.4142		2	8.2926	**4.1463**	6.2926	5.5605	**-4.1463**
25		5	-2.569	-1.0719	0.2854	1.158		2.1974	7.2817	**3.6408**	5.0843	**4.7127**	**-3.6409**
26		6	-3.6458	0	1	1		1.6458	7.2916	**3.6458**	5.6458	**3.6458**	**-3.6458**
27		5	-2.5616	-1.4142	1	1.4142		1.5616	7.9516	**3.9758**	6.39	5.39	**-3.9758**
28		4	-4	1	1	1		1	8	**4**	7	**4**	**-4**
29		4	-3.3234	-0.3579	1	1		1.6813	7.3626	**3.6813**	5.6813	**4.0392**	**-3.6813**
30		4	-3.2361	0	0	1.2361		2	6.4722	**3.2361**	**4.4722**	**3.2361**	**-3.2361**
31	6	15	-1	-1	-1	-1	-1	5	10	**5**	**5**	**9**	**-5**
32		14	-2.4641	-1	-1	-1	1	4.4641	10.9282	5.4641	6.4641	8.4641	-5.4641
33		13	-2.404	-1	-1	0	1.1824	4.2217	9.8081	5.0108	5.5864	7.4041	**-4.404**
34		13	-3	-1.8284	-1	1	1	3.8284	11.6568	5.8284	7.8284	8.6568	-5.8284
35		12	-2.2579	-1	-1	-1	1.2008	4.0571	10.5158	5.2579	6.4587	8.2579	-5.2579
36		12	-3.6056	-1	-1	1	1	3.6056	11.2112	5.6056	7.6056	7.6056	-5.6056

37		12	-2.3742	-1.6232	-1	-0.1577	1.2707	3.8844	10.3102	5.1551	6.4258	7.936	-5.1551
38		12	-3.1042	-1.5846	-1	1	1.1484	3.5404	11.3776	5.6888	7.8372	8.2734	-5.6888
39		12	-3	-3	1	1	1	3	12	6	9	9	-6
40		11	-2	-1	-1	-1	1	4	10	**5**	6	8	**-5**
41		11	-3.4093	-1.1733	-1	1	1.1733	3.4093	11.1652	5.5826	7.7559	7.7559	-5.5826
42		11	-2	-1.5442	-1	-0.6902	1.5127	3.7217	10.4688	5.2344	6.7471	8.4688	-5.2344
43		11	-2.3723	-2	-1	0	2	3.3723	10.7446	5.3723	7.3723	8.3723	-5.3723
44		11	-3.175	-1.2097	-1	1	1	3.3847	10.7694	5.3847	7.3847	7.5944	-5.3847
45		11	-2.7321	-1.8951	-1	0.7321	1.6027	3.2924	11.2544	5.6272	7.962	8.5223	-5.6272
46		11	-4.0642	-1.6946	1	1	1	2.7588	11.5176	5.7588	8.7588	7.4534	-5.7588
47		11	-3.2361	-1.2361	-1	1	1.2361	3.2361	10.9444	5.4722	7.7083	7.7083	-5.4722
48		11	-3.2015	-2.5616	0.4549	1	1.5616	2.7466	11.5262	5.7631	8.7796	8.3247	-5.7631
49		10	-2.3723	-2	-1	1	1	3.3723	10.7446	5.3723	7.3723	8.3723	-5.3723
50		10	-2.7371	-1.3476	-1	0.2155	1.4882	3.381	10.1694	5.0847	6.7884	7.4323	-5.0847
51		10	-3.3847	-1	-1	1	1.2097	3.175	10.7694	5.3847	7.5944	7.3847	-5.3847
52		10	-3.1623	-1.4142	-1	1	1.4142	3.1623	11.153	5.5765	7.9907	7.9907	-5.5765
53		10	-2.7913	-1.3914	-1	0.2271	1.7913	3.1642	10.3653	5.1826	7.2011	7.574	-5.1827
54		10	-2.5694	-1.8181	-1	0.5066	1.7602	3.1206	10.7749	5.3874	7.6543	8.2055	-5.3875
55		10	-3.7336	-1.8846	0.4062	1	1.6196	2.5924	11.2364	5.6182	8.644	7.5028	-5.6182
56		10	-3.107	-1.1605	-1	1	1.1605	3.107	10.535	5.2675	7.428	7.428	-5.2675
57		10	-3	-2.4679	0.0872	1	1.7989	2.5818	10.9358	5.4679	8.354	7.9358	-5.4679
58		10	-2.5616	-1.8208	-1.2859	1.1593	1.5616	2.9474	11.3366	5.6683	8.3892	8.775	-5.6683
59		10	-4	-1.5616	1	1	1	2.5616	11.1232	5.5616	8.5616	7.1232	-5.5616
60		10	-3.1249	-2.4495	0.3633	1	1.7616	2.4495	11.1488	5.5744	8.6993	8.0239	-5.5744
61		10	-3.3723	-2	0	1	2	2.3723	10.7446	5.3723	8.3723	7.3723	-5.3723
62		10	-4.2217	-1.1824	1	1	1	2.404	10.8081	5.404	8.4041	6.5864	-5.4041
63		9	-3.107	-1.1605	-1	1	1.1605	3.107	10.535	5.2675	7.428	7.428	-5.2675
64		9	-2.6688	-1.2724	-1	0.2374	1.5973	3.1065	9.8824	**4.9412**	6.7759	7.2136	**-4.9412**
65		9	-2.6853	-1.2625	-1	0.3149	1.4903	3.1425	9.8955	**4.9477**	6.753	7.2102	**-4.9478**
66		9	-2	-2	-1	0	2	3	10	**5**	7	8	**-5**
67		9	-4.4641	-1	1	1	1	2.4641	10.9282	5.4641	8.4641	6.4641	-5.4641
68		9	-3.4236	-2.1535	1	1	1	2.5771	11.1542	5.5771	8.5771	7.7306	-5.5771
69		9	-4	-1.5616	1	1	1	2.5616	11.1232	5.5616	8.5616	7.1232	-5.5616
70		9	-3.4737	-2.0175	0.3856	1	1.6738	2.4317	10.9823	5.4911	8.5506	7.5086	-5.4912
71		9	-2.7321	-1.4142	-1	0.7321	1.4142	3	10.2926	5.1463	7.2926	7.5605	-5.1463
72		9	-2.8284	-1.4142	-1.4142	1.4142	1.4142	2.8284	11.3136	5.6568	8.4852	8.4852	-5.6568
73		9	-3.3723	-2	0	1	2	2.3723	10.7446	5.3723	8.3723	7.3723	-5.3723
74		9	-2.6642	-1.4663	-1	0.3711	2	2.7594	10.261	5.1305	7.5016	7.5968	-5.1305
75		9	-3.8844	-1.2707	0.1577	1	1.6232	2.3742	10.3102	5.1551	7.936	6.4258	-5.1551
76		9	-3	-1.4142	-0.7321	1	1.4142	2.7321	10.2926	5.1463	7.5605	7.2926	-5.1463
77		9	-3	-1.4142	-0.7321	1	1.4142	2.7321	10.2926	5.1463	7.5605	7.2926	-5.1463
78		9	-2.7722	-1.4395	-1.0697	0.7497	1.8996	2.6321	10.5628	5.2814	7.9307	7.7906	-5.2814
79		9	-4.0571	-1.2008	1	1	1	2.2579	10.5158	5.2579	8.2579	6.4587	-5.2579
80		9	-4.0571	-1.2008	1	1	1	2.2579	10.5158	5.2579	8.2579	6.4587	-5.2579
81		9	-2.7321	-2.7321	0.7321	0.7321	2	2	10.9284	5.4642	8.9284	8.1963	-5.4642
82		9	-4.6458	0.6458	1	1	1	1	9.2916	**4.6458**	8.2916	**4.6458**	**-4.6458**
83		8	-3	-1	-1	1	1	3	10	**5**	7	7	**-5**

84		8	-2.5738	-1.1022	-1	0.3784	1.2046	3.0929	9.3519	**4.6759**	6.259	6.7781	**-4.676**
85		8	-2.5738	-1.1022	-1	0.3784	1.2046	3.0929	9.3519	**4.6759**	6.259	6.7781	**-4.676**
86		8	-4.2217	-1.1824	1	1	1	2.404	10.8081	5.404	8.4041	6.5864	-5.4041
87		8	-3.595	-1.3567	0	1	1.5137	2.438	9.9034	**4.9517**	7.4654	6.3084	**-4.9517**
88		8	-3.1502	-2.2071	0.4257	1	1.5814	2.3502	10.7146	5.3573	8.3644	7.5644	-5.3573
89		8	-2.3097	-1.6917	-1.1252	0.8808	1.5109	2.7348	10.2531	5.1265	7.5183	7.9434	-5.1266
90		8	-2.7224	-2.2754	0.1793	0.6322	1.7644	2.422	9.9957	**4.9979**	7.5737	7.2733	**-4.9978**
91		8	-3.1326	-2	-0.1404	1	2	2.2731	10.5461	5.2731	8.273	7.4135	-5.273
92		8	-2.7785	-2	0	0.2892	2	2.4893	9.557	**4.7785**	7.0677	6.7785	**-4.7785**
93		8	-3.719	-1.1718	0	1	1.6089	2.2819	9.7816	**4.8908**	7.4997	6.0626	**-4.8908**
94		8	-3.6964	-1.3028	0.1782	1	1.5182	2.3028	9.9984	**4.9992**	7.6956	6.302	**-4.9992**
95		8	-3.9651	-1	0.282	1	1	2.6831	9.9302	**4.9651**	7.2471	5.9651	**-4.9651**
96		8	-3.3252	-2.0761	0.7202	1	1.429	2.2522	10.8027	5.4014	8.5505	7.4775	-5.4013
97		8	-3.6964	-1.3028	0.1782	1	1.5182	2.3028	9.9984	**4.9992**	7.6956	6.302	**-4.9992**
98		8	-3.2361	-2	0	1.2361	2	2	10.4722	5.2361	8.4722	7.2361	-5.2361
99		8	-2.7448	-2.1701	-0.3111	1.3959	1.4812	2.3489	10.452	5.226	8.1031	7.7072	-5.226
100		8	-2.7913	-2.1642	0	0.7729	1.7913	2.3914	9.9111	**4.9556**	7.5197	7.1198	**-4.9555**
101		8	-4	-1	1	1	1	2	10	**5**	8	6	**-5**
102		8	-3.7217	-1.5127	0.6902	1	1.5442	2	10.4688	5.2344	8.4688	6.7471	-5.2344
103		8	-5	1	1	1	1	1	10	**5**	9	**5**	**-5**
104		8	-4.7016	0	1	1	1	1.7016	9.4032	**4.7016**	7.7016	**4.7016**	**-4.7016**
105		7	-4.0571	-1.2008	1	1	1	2.2579	10.5158	5.2579	8.2579	6.4587	-5.2579
106		7	-3.9316	-1.4142	0.6768	1	1.4142	2.2548	10.6916	5.3458	8.4368	6.76	-5.3458
107		7	-3.3938	-1.2594	-0.2523	1	1.6314	2.274	9.8109	**4.9054**	7.5369	6.4171	**-4.9055**
108		7	-3.3591	-1.4285	-0.0585	1	1.5514	2.2947	9.6922	**4.8461**	7.3975	6.3331	**-4.8461**
109		7	-3	-2	0	1	2	2	10	**5**	8	7	**-5**
110		7	-3.2774	-1.3865	0	0.3865	2	2.2774	9.3278	**4.6639**	7.0504	6.0504	**-4.6639**
111		7	-3.3743	-1.464	0.2122	1	1.1607	2.4654	9.6766	**4.8383**	7.2112	6.3023	**-4.8383**
112		7	-3.3387	-1.255	0	0.489	1.4979	2.6068	9.1874	**4.5937**	6.5806	5.8487	**-4.5937**
113		7	-2.8758	-2	-0.1883	0.8248	2	2.2393	10.1282	5.0641	7.8889	7.2524	-5.0641
114		7	-2.6165	-1.3051	-0.8458	0.2964	1.6647	2.8064	9.5349	**4.7675**	6.7285	6.9184	**-4.7674**
115		7	-3.3795	-1.6708	0.4399	1	1.6105	2	10.1007	5.0504	8.1007	6.7212	-5.0503
116		7	-3.3795	-1.6708	0.4399	1	1.6105	2	10.1007	5.0504	8.1007	6.7212	-5.0503
117		7	-3.648	-1.0822	0.2775	1	1.2348	2.2179	9.4604	**4.7302**	7.2425	5.8124	**-4.7302**
118		7	-3.6352	-1.1225	0.2524	1	1.3543	2.1511	9.5155	**4.7578**	7.3644	5.8803	**-4.7577**
119		7	-3.3087	-1.574	0	1.1474	1.4919	2.2433	9.7653	**4.8826**	7.522	6.4566	**-4.8827**
120		7	-4.7016	0	1	1	1	1.7016	9.4032	**4.7016**	7.7016	**4.7016**	**-4.7016**
121		7	-4.4279	-0.3757	1	1	1	1.8035	9.6071	**4.8035**	7.8036	5.1792	**-4.8036**
122		7	-4.3723	0	0	1	1.3723	2	8.7446	**4.3723**	6.7446	**4.3723**	**-4.3723**
123		7	-3.6458	-1.4142	1	1	1.4142	1.6458	10.12	5.06	8.4742	6.4742	-5.06
124		6	-4	-1	1	1	1	2	10	**5**	8	6	**-5**
125		6	-3.5616	-1.4142	0.5616	1	1.4142	2	9.9516	**4.9758**	7.9516	6.39	**-4.9758**
126		6	-3.0266	-1.6308	-0.201	1.2648	1.5937	2	9.7169	**4.8585**	7.7169	6.6903	**-4.8584**
127		6	-3.3252	-1.1264	-0.0908	1	1.3285	2.2139	9.0848	**4.5424**	6.8709	5.7596	**-4.5424**
128		6	-3.2397	-1.0769	0	0.2877	1.7449	2.284	8.6332	**4.3166**	6.3492	5.3935	**-4.3166**
129		6	-3.3377	-1.1199	0	1	1.1687	2.2889	8.9152	**4.4576**	6.6263	5.5775	**-4.4576**
130		6	-3.2361	-1.1701	0	0.6889	1.2361	2.4812	8.8124	**4.4062**	6.3312	5.5763	**-4.4062**

131		6	-4.4279	-0.3757	1	1	1	1.8035	9.6071	**4.8035**	7.8036	5.1792	**-4.8036**
132		6	-4.119	-0.618	0.4316	1	1.618	1.6874	9.474	**4.737**	7.7866	5.355	**-4.737**
133		6	-4.3723	0	0	1	1.3723	2	8.7446	**4.3723**	6.7446	**4.3723**	**-4.3723**
134		6	-4.0678	-0.3616	0	1	1.2446	2.1848	8.8588	**4.4294**	6.674	**4.791**	**-4.4294**
135		6	-3.3298	-1.4838	0.5288	1	1.5081	1.7768	9.6273	**4.8137**	7.8505	6.2975	**-4.8136**
136		6	-3.7136	-0.618	0	0.4829	1.618	2.2307	8.6632	**4.3316**	6.4325	**4.9496**	**-4.3316**
137		5	-5	1	1	1	1	1	10	**5**	9	**5**	**-5**
138		5	-4.2015	-0.5451	1	1	1	1.7466	9.4932	**4.7466**	7.7466	5.2917	**-4.7466**
139		5	-3.8284	-1	1	1	1	1.8284	9.6568	**4.8284**	7.8284	5.8284	**-4.8284**
140		5	-3.7785	-0.7108	0	1	1.4893	2	8.9786	**4.4893**	6.9786	5.2001	**-4.4893**
141		5	-4.0678	-0.3616	0	1	1.2446	2.1848	8.8588	**4.4294**	6.674	**4.791**	**-4.4294**
142		5	-3.7136	-0.618	0	0.4829	1.618	2.2307	8.6632	**4.3316**	6.4325	**4.9496**	**-4.3316**
										83	12	**29**	**83**

COCLUSION: In this article we have defined reduced color energy of type –IV of colored graphs. Discussed an admissible and forbidden class of colored graphs for the class $C_4(10)$ with $\chi(G)$ colors and found that, there are exactly 83 admissible connected colored graphs of order six. whose reduced color energy of Type IV does not exceed 5.

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