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RESEARCH ARTICLE

DYNAMICAL SYSTEMS ARISING FROM VECTOR FIELDS ON SMOOTH MANIFOLDS

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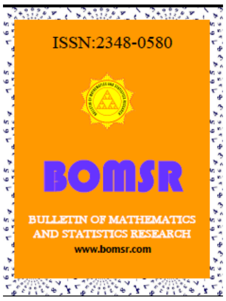

ABSTRACT

In this paper we have considered a problem that looks into a possible lift for functions defined on manifolds and flows associated with the dynamically systems on them. Initially we observe this problem on one dimensional manifolds. Later ,it was generalized to M where n is a dimensional smooth manifold $\mathbb{R}^n$,n> 1.

Keywords: Smooth manifolds ,vector fields, orientable manifolds.

1. INTRODUCTION

In this section we give account of smooth differentiable manifolds which are finite dimensional. Non trivial examples of manifolds which admits smooth differentiable structures are S^1 *the* circle group,also denoted by T^1 the one dimensional torus.They generally come from quotient topology induced by topology of R^2.The Cartesian product of the circle group taken n times gives rise to n torus which is also regarded as quotient space with induced topology of R^{n+1}.Our manifolds are orientable manifolds and the maps are orientation preserving maps ,quite naturally such Orientation preserving homeomorphisms give rise to dynamical systems and if the maps are smooth this dynamical systems are also smooth.. The theory of dynamical systems is in fact vast area of study initiated by Henri Poincare around1900. He was interested studying problems in celestial mechanics. Later Kolomogrov ,,Birkoff ,Arnold, Kac and many others have continued to investigate the problems arising in this area of research.In this paper We have noticed that a vector field arising from an integral curve giving rise to. a dynamical system which is a system of first order ordinary differential equations .

1.1 Vector fields as Flows

Let M be a smooth n-dimensional manifolds then it admits a smooth differentiable structure.

Let xεM lying in some coordinate patch of (U i) of M where U is an open subset of M and i, U:U→$\mathbb{R}^n$ is a local diffeomorphism.

Let T_xM denote the tangent space of M at x then we know that $T_xM \sim \mathbb{R}^n$
i.e there is an isomorphism with local basis with vector fields
$\{\partial|\partial xi: i = 1,2, \ldots\ldots\ldots n\}$ of TxM. *To this end consider a smooth curve* $\gamma: I \to M$,
I is the interval containing zero such that $\gamma(0)$=x
t→ $\gamma(t)$ t∈ I
then $\gamma^1(t) = \frac{d\,\gamma(t)}{dt}$ is the velocity vector
In fact a tangent vector to the curve at $\gamma(t)$ in M.
Since $\gamma^1(t)$ is a tangent vector to M at $\gamma(t)$ i.e $\gamma^1(t) \in T_{\gamma(t)}M$
We have proved the following proposition .
Proposition 1:If $\gamma: I \to M$ is a smooth curve passing through x in M,I containing zero then $\gamma^1(t) \in T_{\gamma(t)}M$, where $T_{\gamma(t)}M$ is the tangent space of M at $\gamma(t)$.
In particular $\gamma^1(0)$ is the tangent vector to M at x of $T_{\gamma(0)}M$ .This been a local description vector field on M provided a global picture because vector field in M is a map.
X:M→ TM where TM is the tangent bundle of M(this bundle is smooth because M is smooth) which attaches to each point P of M then the tangent vector X(P).
P→X(P)
Here TM $=U_{\gamma(t)}M\ T_{\gamma(t)}M$
In otherwords this is more so of defining a vector in an invariant fashion as a map
X:M→ TM
Such that πoX=IM where π:TM→M is a projection maps.
Observation:
The map that assigns to each point of M at tangent vector gives rise to a system of first order ordinary differential equation.
X(P): $\frac{d\,X(P)}{dP}$
1.2 Dynamical Systems:
We shall set up a simple dynamical system
$X^1(t)$ =rX(t), where X is a smooth vector field in M , where X is a smooth vector field in M.
where r>0 is some constant.
$f: \mathbb{R} \to \mathbb{R}$ **x^1=ax ,x(0)=a**
$x(t) = ae^{at}$
$\boldsymbol{x^1(t)}$= ae^{at}
$\boldsymbol{i.e = ax(t)}$
x(0) = e^{a0}=1
$x^1(t)$=x(0)x(t)
$\frac{x^1(t)}{x(t)}$=x(0)
Some digression
Let x:[a,b] → $\mathbb{R}$ **be continuous and differentiable on (a,b) then x^1=kx**
Is a simple system of ordinary differential equation in one variable.
Let x(0)=k then its solution gives rise to $x(t) = x(0)e^{kt}$
M → $T_rM \approx \mathbb{R}^n$→ {ei: e1=(1.......0) en=(0......0,1)}
$\{\partial|\partial xi: i = 1,2, \ldots\ldots\ldots n\}$ local vector fields.
X∈ $\mathbb{R}^n$ then X = (x1,x2,........xn) where each Xi ∈ $\mathbb{R}$
$\boldsymbol{x^1(t) = (x^1 1(t) \ldots\ldots. x^1 n(t))}$ **where each $x^1 i(t)$=** $\frac{d\,Xi(t)}{dt}$

Reflecting this in a system of n variables and treating $\frac{d}{dt}$ as a operator which is infact linear operator then X(t) = AX(t)

Where A is the nxn matrix associated with the derivative operator.

The linear isomorphism in vertation can be generalised to M.

1.3 Application :

In this section to give one application form over discussion

Let the manifold be M= $\mathbb{R}^2$

For a simply connected and bounded subset D of $\mathbb{R}^2$ with ∂D simple closed curve then for any γ**:I→**$\mathbb{R}^2$ as earlier such that γ(I)≈D

Then ∂D is a simple curve in $\mathbb{R}^2$

If X is Vector field generated by γ in $\mathbb{R}^2$ then t→ $\gamma(t)$ and X($\gamma(t)$) smoothly give rise to $X^1(\gamma(t))$=AX($\gamma(t)$)

Where A is a 2X2 matrix non singular.

Which is an system of ordinary differential equation of first order setting $\gamma(t)$=u for some point u in $\mathbb{R}^2$ then this parametric family { $\gamma(t)$:t ∈ I} gives rise to {u:u∈ $\mathbb{R}^2$}

Consequently the system of ordinary differential equation, reduces to $\boldsymbol{x^1(u) = AX(u)}$

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