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RESEARCH ARTICLE

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A NOTE ON COLOR ENERGY OF GRAPHS

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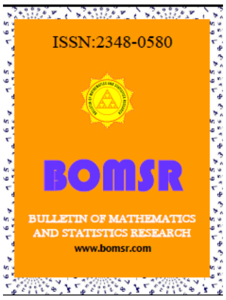

ABSTRACT

The concept of color energy and its relations with topological and color spectral properties are discussed in [1]. Recently we have defined different types of reduced color energy in [2-4]. In this article, we will define and classify the class of an admissible or forbidden colored graphs based on their color energy and investigate all admissible colored graphs of order six whose color energy does not exceed 10 with an arbitrary fixed coloring. We noticed that, there are exactly 83 admissible connected colored graphs of order six whose color energy does not exceed 10 with an arbitrary fixed coloring.

Keywords: Color Eigenvalues and Color Energy.

AMS Subject Classifications: 05C50.

INTRODUCTION

A coloring of graph G is a coloring of its vertices such that no two adjacent vertices receive the same color. The minimum number of colors needed for coloring of a graph G is called chromatic number and is denoted by $\chi(G)$. Recently C. Adiga and et.al., have introduced color matrix, color energy of graphs and obtained its characterization in [1], also they have investigated many properties and results on color energy and color eigenvalues of graphs. Throughout the paper, we shall consider only simple, finite connected colored graphs. The vertex set of colored graph G is denoted by $V(G)$ and its order by $|V(G)| = n$ and edge set of G is denoted by $|E(G)| = m$. Throughout the paper graph mean colored graphs.

The color matrix of a graph is $A_\chi(G) = (a_{ij})$, whose entries of a_{ij} are as follows

$$\begin{cases} 1, & if\ v_i\ and\ v_j\ are\ adjacent\ with\ c(v_i) \neq c(v_j) \\ -1, & if\ v_i\ and\ v_j\ are\ not\ adjacent\ with\ c(v_i) = c(v_j) \\ 0, & otherwise \end{cases}$$

The characteristic polynomial of $A_\chi(G)$ is denoted by $P_\chi(G,\lambda) = det(A_\chi(G) - \lambda I)$. The roots of the characteristic polynomial are $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$ are the eigenvalues of $A_\chi(G)$. They are called chromatic eigenvalues of G. Since $A_\chi(G)$ is real and symmetric so its color eigenvalues are also real. The color energy is defined as

$$E_\chi(G) = \sum_{i=1}^{n} |\lambda_i|.$$

This is called chromatic energy of G. If the distinct color eigenvalues of $A_\chi(G)$ are $\lambda_1 > \lambda_2 > \cdots > \lambda_r$, $r \leq n$ with their multiplicities are $m_1, m_2, \ldots m_r$, then we have

$$Spec_c(G) = \begin{pmatrix} \lambda_1, & \lambda_2, \ldots, & \lambda_r \\ m_1, m_2, \ldots & \ldots, m_r \end{pmatrix}.$$

ADMISSIBLE COLORED GRAPHS

Clustering and characterizing the graphs through an eigenvalues and graph energy is formal trend in spectral graph theory and related fields. One can see many monographs on this direction. For more details see [5-7] and references therein. Now here we will define and classify the class of admissible colored graphs whose color energy does not exceed 10 and found that, there are exactly 83 admissible connected colored graphs of order six, whose color energy does not exceed 10 with an arbitrary fixed coloring with $\chi(G)$ colors. Upper triangular matrices and corresponding color energy is shown in Table 1 and Table 2 of our early paper [4].

Now we shall state an important property of general class.

Theorem 1. The class $C_4(a)$ is finite for any real number $a \geq 1$.

Proof of the theorem similar to Theorem 1 of [4].

In this paper we investigate completely (up to six vertices) the class of $C_1(10)$ is called admissible and if other graph impossible (or forbidden) for the class.

If H is any connected colored subgraph of a colored graph G, then by heredity property we have $E_\chi(H) \subseteq E_\chi(G)$ and for any connected colored subgraph of an admissible colored graph is also admissible. This implies that the method of forbidden subgraphs can be consistently applied.

Next $K_{n_1 n_2, \ldots n_m}$, be the complete m −partite graph. Now will we find admissible class of graphs initial with $K_{n,\ m}$. We shall firstly determine the exact values of parameter n, m for which the colored graph $K_{n,\ m}$ $(n \leq m)$ is admissible.

Proposition 1: The graph $K_{n,m}$ $(n \leq m)$ is admissible exactly for the following values of parameter n, m

1. $n = 1,\ m = 1, 2, 3,4,5,$
2. $n = 2,\ m = 2, 3,4,$ and
3. $n = 3,\ m = 3.$

Proof: Since $K_{n,\ m}$ is complete bipartite graph and has spectrum

$$\begin{pmatrix} -(n+m-1) & 1 \\ 1 & (n+m-1) \end{pmatrix}.$$

Therefore it will belong to the class $C_1(10)$ if and only if $2(n+m-1) \leq 10$, which easily gives the statement.

Now we look at $K_{n_1 n_2, \ldots n_m}$ complete m −partite graphs. We shall determine the exact values of parameter $n_1, n_2, \ldots, n_m$ for which the graph $K_{n_1 n_2, \ldots n_m}$, $(n_1 \leq n_2 \leq \ldots \leq n_m)$ is admissible.

Proposition 2: The graph $K_{n,m,k}$ $(n \leq m \leq k)$ is admissible exactly for the following values of parameter n, m, k

1. $n = 1,\ m = 1,\ k = 1, 2, 3,$

Proof: It is easy to check that all the above graphs are admissible. Besides, since the graph $K_{n,m,k}$ is forbidden for the values of parameters $(n, m, k) = (1, 1, 4),\ (2, 2, 2)$.

Proposition 3: The graph $K_{n,m,k,l}$ $(n \leq m \leq k \leq l)$ is admissible exactly for the following values of parameter n, m, k, l,

1. $(n, m, l, k) = (1,1,1,1),\ (1,1,1,2)$.

Proof: It is easy to check that all the above graphs are admissible. Besides, since the graph $K_{n,m,k,l}$ is forbidden for the values of parameters $(n, m, l, k) = (1,1,1,3),\ (2,2,2,2)$.By heredity property of color energy proves the statement.

Proposition 4: The graph $K_{n,m,k,l,p}(n \leq m \leq k \leq l \leq p)$ is admissible exactly for the following values of parameter n, m, k, l, p

1. $(n, m, l, k) = (1,1,1,1,1)$.

Proposition 5: The graph $G = K_{n_1 n_2,\ldots,n_m}, (m \geq 6)$ is admissible if and only if $m = 6$ and G is the graph K_6.

Proof: The graph K_6 is obviously an admissible graph. Besides, the graphs $K_{1,1,1,1\,,2}$ and K_7 are all forbidden graphs. The statement is immediate.

Proposition6: The null graph G of order $n \leq 6$ is admissible.

Proof: Let Gbe null graph of order n. Since $\chi(G) = 1$ and $E_\chi(K_n) = E_\chi(\overline{K_n}) = 2(n-1)$.

We discuss little more admissible class of graphs.

Proposition7. The null graph G of order $n \leq 6$ is admissible.

Proof: Let Gbe null graph of order n. Since $\chi(G) = 1$ and $E_\chi(K_n) = E_\chi(\overline{K_n}) = 2(n-1)$.

Proposition 8. Let $\overline{(K_{1,\ n-1})_c}$ be the complement of the colored star graph of order n, which is colored with χ colors, is admissible if $n \leq 7$.

Proof. We know that $Spec\overline{(K_{1,\ n-1})_c}$ is $\begin{pmatrix} 0 - (n-2) & 1 \\ 1 \quad 1 & n-2 \end{pmatrix}$and $E\overline{(K_{1,\ n-1})_c} = 2n-4$, therefore is admissible if $n \leq 7$, on the other hand if for $n = 8$, then $E\overline{(K_{1,\ n-1})_c} = 12 \nleq 10$ is forbidden.

Proposition 9. Let $\overline{(K_{m.n})_c}$ be the complement of the colored complete bipartite graph of order n, which is colored with χ colors, is admissible exactly for the following values of parameter m, n, $(m \leq n)$

1. $m = 1, n = 1,2,3,4,5,6$.
2. $m = 2, n = 1,2,3,4,5$.
3. $m = 3, n = 1,2,3,4$.

Proof. We know that $Spec\overline{(K_{m,\ n})_c} = \begin{pmatrix} -(n-1) & -(m-1) & 1 \\ 1 & 1 & m+n-2 \end{pmatrix}$

and $E\overline{(K_{m,\ n})_c} = 2(n+m-2) \leq 10$ and is admissible for above values of parameters m, n. Besides, since the graph $K_{n,\ m}$ is forbidden for the values of parameters $(n, m) = (4,4)$, and above; i.e., $E\overline{(K_{m,\ n})_c} = 2(n+m-2) = 2(4+4-2) = 12 \nleq 10$.

Proposition 10. If ${S_n}^0$ is the colored crown graph of order $2n$ is admissible if $n \leq 6$.

Proof. We know that $Spec_\chi({S_n}^0) = \begin{pmatrix} -2(n-1) & 2 & 0 \\ 1 & n-1 & n \end{pmatrix}$ and $E_\chi({S_n}^0) = 4(n-1) \leq 10$ and hence is admissible if $n \leq 6$. Further by the heredity property of color energy provides the statement. On the other hand whence $n \geq 7$ are forbidden graphs.

Next result immediately follows, because $E_\chi({S_n}^0) = E\overline{\overline{({S_n}^0)}_c}$.

Proposition 11. If $\overline{({S_n}^0)}$ is the complement of the colored crown graph of order $2n$ is admissible if $n \leq 6$.

Proposition 12. If $K_{n\times 2}$ is a cocktail party colored graph of order $2n$ with χ colorsis admissible if $n \leq 2$.

Proof. We know that $Spec_\chi(K_{n\times 2}) = \begin{pmatrix} 2n-3 & -3 & 1 \\ 1 & n-1 & n \end{pmatrix}$ and $E_\chi(K_{n\times 2}) = 6(n-1) \le 10$ and hence is admissible if $n = 1,$ and 2. On the other hand $n \ge 3$ are forbidden graphs.

Proposition 13. If $(\overline{K_{n\times 2}})_c$ is the complement of cocktail party colored graph of order $2n$ with $\chi(K_{n\times 2})$ colorsis admissible if $n \le 5$.

Proof. The eigenvalues of $(\overline{K_{n\times 2}})_c$ are $1, \omega, \omega^2, \ldots. \omega^{n-1}$, where $\omega = e^{\frac{2\pi i}{n}}$ and α, α^3, $\alpha^5, \ldots \alpha^{2n-1}$, where $\alpha = e^{\frac{\pi i}{n}}$ and $E(\overline{K_{n\times 2}})_c = 2n \le 10$. Therefore $E(\overline{K_{n\times 2}})_c$ admissible whence $n \le 5$ and further by the heredity property of color energy provides the statement.. For $n \ge 6$ are forbidden graphs.

Proposition 14. If C_{2n} is a cycle of order $2n$is admissible if $n \le 4$,

Proof. We know that $E_\chi(C_{2n}) = |3-n| + 1 + n + \sum_{\substack{m=1 \\ m\neq n}}^{2n-1} \left|1 + 2\cos\left(\frac{\pi m}{n}\right)\right|$ and is easy to see that C_{2n} is admissible if $n \le 4$.

Conclusion

In this article we have discussed admissible and forbidden class of colored graphs whose energy does not exceed 10 and noticed that they are exactly 83 admissible connected graphs of order six. See Table 2 of [4].

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