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RESEARCH ARTICLE

WAVE PROPAGATION AT AN INTERFACE BETWEEN MICROPOLAR ELASTIC SOLID AND MICROPOLAR VISCOELASTIC SOLID WITH STRETCH

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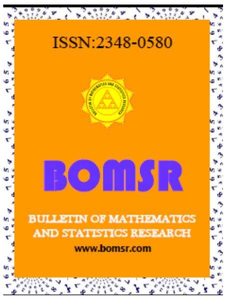

ABSTRACT

The present investigation deals with wave propagation at an interface separating micropolar elastic solid half space and micropolar viscoelastic solid half space with stretch. A longitudinal wave is considered to be incident on the plane interface through micropolar elastic solid half space. A longitudinal wave bumps into obliquely at the interface. Amplitude ratios of various reflected and refracted waves have been computed numerically for a specific model and thus the results are represented graphically with an angle of incidence of incident wave. Effects of axial stretch and viscosity on the amplitude ratios are discussed. A particular case of reflection at free surface of micropolar elastic solid has been deduced and discussed with the help of graphs.

Keywords: Amplitude ratio, longitudinal wave, reflection, refraction, axial stretch, micropolar viscoelastic solid

1. Introduction

The micropolar theory of elasticity constructed by Eringen and his co-workers intended to be applied on such materials and for problems where the ordinary theory of elasticity fails because of microstructure in the materials. Micropolar elastic materials, roughly speaking, are the classical elastic materials with extra independent degree of freedom for the local rotations. These materials respond to spin inertia, body and surface couples and as a consequence they exhibit certain new static and dynamic effects, e.g. new types of waves and couples stresses.

A micropolar elastic solid is distinguished from an elastic solid by the fact that it can support body and surface couples. These solids can undergo local deformations and micro-rotations such materials may be imagined as bodies which are made of rigid short cylinders or dumbbell type molecules.

From a continuum mechanical point of view, micropolar elastic solids may be characterized by a set of constitutive equations which define the elastic properties of such materials. A linear theory as a special case of the nonlinear theory of micro-elastic solids was first constructed by Eringen and Suhubi (1964a, b). Later, Eringen (1965) and (1966) recognized and extended this theory.

Eringen (1966a, 1990) developed the theories of 'micropolar continua' and 'microstructures continua' which are special cases of the theory of 'micromorphic continua' earlier developed by Eringen and his coworkers (1964). Thus,the Eringen's '3M' theories (Micromorphic, Microstretch, Micropolar) are the generalization the classical theory of elasticity. In classical continuum, each particle of a continuum is represented by a geometrical point and can have three degree of freedom of translation during the process of deformations.

Eringen's theory of micropolar elasticity keeps importance because of its applications in many physical substance for example material particles having rigid directors, chopped fibres composites, platelet composites, aluminium epory, liquid crystal with side chains, a large class of substance like liquid crystal with rigid molecules, rigid suspensions, animal blood with rigid cells, foams, porous materials, bones, magnetic fields, clouds with dust, concrete with sand and muddy fluids are example of micropolar materials.

The linear theory of micropolar viscoelasticity was developed by Eringen (1967). Mc Carthy & Eringen (1969) discussed wave propagation conditions and growth equations. Kumar *et al* (1990) discussed a plane problem in a micropolar viscoelastic solid half space with stretch. Singh (2000) discussed on reflection and transmission of plane harmonic waves at an interface between liquid and micropolar viscoelastic solid with stretch. Singh and Kumar have studied on wave reflection at viscoelastic-micropolar elastic interface. Many problems related to waves and vibrations have been discussed in micropolar viscoelastic solid with stretch by several researchers. Recently, Kumari (2013), Gade and Raghukanth (2015), Zhang at el. (2016) and Merkel & Luding (2016) discussed such waves and vibrations. The present paper is concerned with longitudinal wave propagation in micropolar elastic solid, reflection and refraction of longitudinal waves at the interface between micropolar elastic solid half space and micropolar viscoelastic solid half space with stretch. A particular case of reflection at free surface of micropolar elastic solid has been deduced and discussed with the help of graphs.

2. Basic equations and constitutive relations:

For medium $\mathbf{M_1}$ (Micropolar elastic solid)

The equation of motion in micropolar elastic medium are given by Eringen (1968) as

$$({c_1}^2+{c_3}^2)\nabla^2\varphi=\frac{\partial^2\varphi}{\partial t^2}, \tag{1}$$

$$({c_2}^2+{c_3}^2)\nabla^2 U+{c_3}^2\nabla\times\Phi=\frac{\partial^2 U}{\partial t^2}, \tag{2}$$

$$({c_4}^2\nabla^2-2{\omega_0}^2)\Phi+{\omega_0}^2\nabla\times U=\frac{\partial^2\Phi}{\partial t^2}, \tag{3}$$

where

$$c_1^2=\frac{\lambda+2\mu}{\rho}\,,\qquad c_2^2=\frac{\mu}{\rho}\,,\qquad c_3^2=\frac{\kappa}{\rho}\,,\qquad c_4^2=\frac{\gamma}{\rho j}\,,\qquad \omega_0^2=\frac{\kappa}{\rho j}\,, \tag{4}$$

Parfitt and Eringen (1969) have shown that eq. (1) corresponds to longitudinal wave propagating with velocity V_1, given by ${V_1}^2={c_1}^2+{c_3}^2$ and equations. (2) - (3) are coupled equations in vector potentials U and Φ and these correspond to coupled transverse and micro-rotation waves. If $\frac{\omega^2}{{\omega_0}^2}>20$, there exist two sets of coupled-wave propagating with velocities $1/\lambda_1$ and $1/\lambda_2$.

where

$$\lambda_1{}^2 = \frac{1}{2}\left[B - \sqrt{B^2 - 4C}\right], \quad \lambda_2{}^2 = \frac{1}{2}\left[B + \sqrt{B^2 - 4C}\right], \tag{5}$$

and

$$B = \frac{q(p-2)}{\omega^2} + \frac{1}{(c_2{}^2 + c_3{}^2)} + \frac{1}{c_4{}^2}, \qquad C = \left(\frac{1}{c_4{}^2} - \frac{2q}{\omega^2}\right)\frac{1}{(c_2{}^2 + c_3{}^2)},$$

$$p = \frac{\kappa}{\mu + \kappa}, \quad q = \frac{\kappa}{\gamma}. \tag{6}$$

We consider a two dimensional problem by taking the following components of displacement and micro- rotation as

$$U = (u, 0, w), \qquad \Phi = (0, \Phi_2, 0), \tag{7}$$

where

$$u = \frac{\partial\varphi}{\partial x} - \frac{\partial\psi}{\partial z}, \qquad w = \frac{\partial\varphi}{\partial z} + \frac{\partial\psi}{\partial x}, \tag{8}$$

and components of stresses as

$$t_{zz} = (\lambda + 2\mu + \kappa)\frac{\partial^2\varphi}{\partial z^2} + \lambda\frac{\partial^2\varphi}{\partial x^2} + (2\mu + \kappa)\frac{\partial^2\psi}{\partial x\,\partial z}, \tag{9}$$

$$t_{zx} = (2\mu + \kappa)\frac{\partial^2\varphi}{\partial x\,\partial z} - (\mu + \kappa)\frac{\partial^2\psi}{\partial z^2} + \mu\frac{\partial^2\psi}{\partial x^2} - \kappa\Phi_2, \tag{10}$$

$$m_{zy} = \gamma\frac{\partial\Phi_2}{\partial z}, \tag{11}$$

For medium M_2 (Micropolar viscoelastic solid with stretch)

Following Eringen (1967, 1971), the constitutive and field equations of micropolar viscoelastic solid with stretch, the absence of body forces and body couples, can be written as

$$\bar{t}_{kl} = \bar{\lambda}\bar{u}_{r,r}\delta_{kl} + \bar{\mu}\left(\bar{u}_{k,l} + \bar{u}_{l,k}\right) + \bar{\kappa}\left(\bar{u}_{l,k} - \epsilon_{klr}\bar{\varphi}_r\right), \tag{12}$$

$$\bar{m}_{kl} = \bar{\beta}_0\epsilon_{rkl}\bar{\Phi}_r + \bar{\alpha}\bar{\varphi}_{r,r}\delta_{kl} + \bar{\beta}\bar{\varphi}_{k,l} + \bar{\gamma}\bar{\varphi}_{l,k}, \tag{13}$$

$$\bar{\beta}_k = \bar{\alpha}_0\bar{\Phi}_{,k} + \left(\bar{\beta}_0/3\right)\epsilon_{rkl}\bar{\varphi}_{r,l}, \tag{14}$$

and

$$(\bar{c}_1^2 + \bar{c}_3^2)\nabla(\nabla.\mathbf{u}) - (\bar{c}_2^2 + \bar{c}_3^2)\nabla\times(\nabla\times\mathbf{u}) + \bar{c}_3^2\nabla\times\boldsymbol{\phi} = \ddot{\mathbf{u}} \tag{15}$$

$$\left(\bar{c}_4^2 + \bar{c}_5^2\right)\nabla(\nabla.\boldsymbol{\phi}) - \bar{c}_4^2\nabla\times(\nabla\times\boldsymbol{\phi}) + \bar{\omega}_0^2\nabla\times\mathbf{u} - 2\bar{\omega}_0^2\boldsymbol{\phi} = \ddot{\boldsymbol{\phi}} \tag{16}$$

$$\bar{c}_6^2\nabla^2\bar{\Phi} - r_1\bar{\Phi} = \ddot{\bar{\Phi}} \tag{17}$$

where

$$\bar{c}_1^2 = \frac{\bar{\lambda} + 2\bar{\mu}}{\bar{\rho}}, \qquad \bar{c}_2^2 = \frac{\bar{\mu}}{\bar{\rho}}, \qquad \bar{c}_3^2 = \frac{\bar{\kappa}}{\bar{\rho}}, \qquad \bar{c}_4^2 = \frac{\bar{\gamma}}{\bar{\rho}\,\bar{j}}, \qquad \bar{c}_5^2 = \frac{\bar{\alpha} + \bar{\beta}}{\bar{\rho}\,\bar{j}},$$

$$\bar{\omega}_0{}^2 = \frac{\bar{c}_3^2}{\bar{j}} = \frac{\bar{\kappa}}{\bar{\rho}\,\bar{j}}, \qquad \bar{c}_6^2 = \frac{2\bar{\alpha}_0}{\bar{\rho}\,\bar{j}}, \qquad r_1 = \frac{2\bar{\eta}_0}{\bar{\rho}\,\bar{j}}, \tag{18}$$

where symbols $\lambda, \mu, \gamma, \kappa, \rho, j, \bar{\lambda}, \bar{\mu}, \bar{\kappa}, \bar{\gamma}, \bar{\alpha}_0, \bar{\beta}_0, \bar{\eta}_0, \bar{\rho}, \bar{j}$ have their usual meaning. $\mathbf{u}$, $\boldsymbol{\phi}$ and $\bar{\Phi}$ are displacement vector, micro-rotation vector and micro-stretch respectively. δ_{kl} is the kronecker delta. Superposed dots on the right hand side of (15), (16) and (17) stand for second partial derivative with respect to time.

Taking $\mathbf{u} = (\bar{u}, 0, \bar{w})$ and $\boldsymbol{\phi} = (0, \bar{\varphi}_2, 0)$ and introducing potentials $\bar{\varphi} = (x, z, t)$ and $\bar{\psi} = (x, z, t)$ which are related to displacement components as

$$\bar{u} = \left(\frac{\partial\bar{\varphi}}{\partial z}\right) + \left(\frac{\partial\bar{\psi}}{\partial x}\right), \qquad \bar{w} = \left(\frac{\partial\bar{\varphi}}{\partial x}\right) - \left(\frac{\partial\bar{\psi}}{\partial z}\right), \tag{19}$$

and components of stresses as

$$\bar{t}_{zz} = -(\bar{\lambda} + 2\bar{\mu} + \bar{\kappa})\frac{\partial^2\bar{\psi}}{\partial z^2} + (2\bar{\lambda} + 2\bar{\mu} + \bar{\kappa})\frac{\partial^2\bar{\varphi}}{\partial x\,\partial z} + \bar{\lambda}\frac{\partial^2\bar{\psi}}{\partial x^2}, \tag{20}$$

$$\bar{t}_{zx} = (\bar{\lambda} + \bar{\kappa})\frac{\partial^2\bar{\varphi}}{\partial z^2} + \bar{\mu}\frac{\partial^2\bar{\varphi}}{\partial x^2} + \bar{\kappa}\frac{\partial^2\bar{\psi}}{\partial z\,\partial x} - \bar{\kappa}\bar{\varphi}_2\,, \tag{21}$$

$$\bar{m}_{zy} = -\bar{\beta}_0\bar{\Phi}_{,x} + \bar{\beta}\bar{\varphi}_{z,y} + \bar{\gamma}\bar{\varphi}_{2,z}\,, \tag{22}$$

$$\bar{\beta}_z = \bar{\alpha}_0\bar{\Phi}_{,z} + (\bar{\beta}_0/3)\bar{\varphi}_{2,x}\,, \tag{23}$$

Substituting the displacement components given by (19) in (15) to (17), we obtained

$$\left(\nabla^2 - \frac{1}{(\bar{c}_2{}^2 + \bar{c}_3{}^2)}\frac{\partial^2}{\partial t^2}\right)\bar{\varphi} = 0, \tag{24}$$

$$\left(\nabla^2 - \frac{1}{(\bar{c}_2{}^2 + \bar{c}_3{}^2)}\frac{\partial^2}{\partial t^2}\right)\bar{\psi} - \bar{p}\bar{\varphi}_2 = 0, \tag{25}$$

$$\left(\nabla^2 - 2\bar{q} - \frac{1}{\bar{c}_4{}^2}\frac{\partial^2}{\partial t^2}\right)\bar{\varphi}_2 + \bar{q}\nabla^2\bar{\psi} = 0, \tag{26}$$

$$\left(\nabla^2 - 2r_1 - \frac{1}{\bar{c}_6{}^2}\frac{\partial^2}{\partial t^2}\right)\bar{\Phi} = 0, \tag{27}$$

where

$$\bar{p} = \frac{\bar{\mu}}{(\bar{\mu} + \bar{\kappa})}\,, \qquad \bar{q} = \frac{\bar{\kappa}}{\bar{\gamma}}\,, \tag{28}$$

Assume harmonic time variation as

$$\begin{aligned}
&\bar{\varphi}(x,z,t) = \bar{\varphi}(x,z)\exp(i\bar{\omega}t),\\
&\bar{\psi}(x,z,t) = \bar{\psi}(x,z)\exp(i\bar{\omega}t),\\
&\bar{\varphi}_2(x,z,t) = \bar{\varphi}_2(x,z)\exp(i\bar{\omega}t),\\
&\bar{\Phi}(x,z,t) = \bar{\Phi}(x,z)\exp(i\bar{\omega}t),
\end{aligned} \tag{29}$$

Substituting (29) in (24) to (27), we get

$$\left(\nabla^2 + \frac{\bar{\omega}^2}{\bar{V}_1^2}\right)\bar{\varphi} = 0, \tag{30}$$

$$\left(\nabla^4 + \bar{\omega}^2\bar{B}\nabla^2 + \bar{\omega}^4\bar{C}\right)(\bar{\psi}, \bar{\varphi}_2) = 0, \tag{31}$$

$$\left(\nabla^2 + \frac{\bar{\omega}^2}{V^2}\right)\bar{\Phi} = 0, \tag{32}$$

where

$$\bar{B} = \frac{\bar{q}(\bar{p} - 2)}{\bar{\omega}^2} + \frac{1}{(\bar{c}_2^2 + \bar{c}_3^2)} + \frac{1}{\bar{c}_4^2}\,,$$

$$\bar{C} = \frac{1}{(\bar{c}_2^2 + \bar{c}_3^2)}\left(\frac{1}{\bar{c}_4^2} - \frac{2\bar{q}}{\bar{\omega}^2}\right), \tag{33}$$

and

$$V^2 = \frac{\bar{c}_6^2}{\left(1 - \frac{r_1\bar{c}_6^2}{\bar{\omega}^2}\right)}, \qquad \bar{V}_1^2 = \bar{c}_1^2 + \bar{c}_3^2, \tag{34}$$

In an unbounded medium, the solution of (30) corresponds to modified longitudinal displacement wave propagating with velocity $\bar{V}_1$.

The solution of (31) can be written as

$$\bar{\psi} = \bar{\psi}_1 + \bar{\psi}_2 \tag{35}$$

where $\bar{\psi}_1$ and $\bar{\psi}_2$ satisfy

$$\left(\nabla^2 + \bar{\delta}_1^2\right)\bar{\psi}_1 = 0, \tag{36}$$

$$\left(\nabla^2 + \bar{\delta}_2^2\right)\bar{\psi}_2 = 0, \tag{37}$$

and

$$\bar{\delta}_1^2 = \bar{\lambda}_1^2\bar{\omega}^2\,, \qquad \bar{\delta}_2^2 = \bar{\lambda}_2^2\bar{\omega}^2\,, \tag{38}$$

$$\bar{\lambda}_{1,2}^2 = \frac{\left[\bar{B} \pm \sqrt{(\bar{B}^2 - 4\bar{C})}\right]}{2}. \tag{39}$$

From (31) we obtain

$$\bar{\varphi}_2 = \bar{E}\bar{\psi}_1 + \bar{F}\bar{\psi}_2,$$

where

$$\bar{E} = \frac{\left(\frac{\bar{\omega}^2}{(\bar{c}_2^2+\bar{c}_3^2)} - \bar{\delta}_1^2\right)}{\bar{p}}, \qquad \bar{F} = \frac{\left(\frac{\bar{\omega}^2}{(\bar{c}_2^2+\bar{c}_3^2)} - \bar{\delta}_2^2\right)}{\bar{p}}. \tag{40}$$

Thus there are two waves propagating with velocities $\bar{\lambda}_1^{-1}$ and $\bar{\lambda}_2^{-1}$ each consisting of transverse displacement $\bar{\psi}$ and transverse micro-rotation $\bar{\varphi}_2$. Following Parfitt & Eringen (1969), we call these waves the modified coupled transverse displacement wave and the transverse micro-rotational wave, respectively.

Equation (32) shows that there exists a wave propagating with velocity V, which we call a longitudinal micro-stretch wave in a micropolar viscoelastic medium with stretch.

This velocity V is real and finite if

$$1 - \left(\frac{r_1\bar{c}_6^2}{\bar{\omega}^2}\right) > 0. \tag{41}$$

The inequality (41) with help of (18) reduces to

$$\bar{\omega} > \bar{\omega}_c, \tag{42}$$

where

$$\bar{\omega}_c = \sqrt{2}\ \frac{\bar{\eta}_0}{\bar{\rho}\,\bar{j}}.$$

This is the condition for the existence of a modified longitudinal micro-stretch wave.

3. Formulation of the problem:

Consider a two dimensional problem by taking the z-axis pointing into the lower half-space and the plane interface z=0 separating the uniform micropolar elastic solid half space M_1 [z>0] and the micropolar viscoelastic solid half space with stretch M_2 [z<0]. A longitudinal wave propagates through the medium M_1 and incident at the plane z=0 and makes an angle θ_0 with normal to the surface. Corresponding to incident longitudinal wave, we get three reflected waves in the medium M_1 and four refracted waves in medium M_2. See fig.1

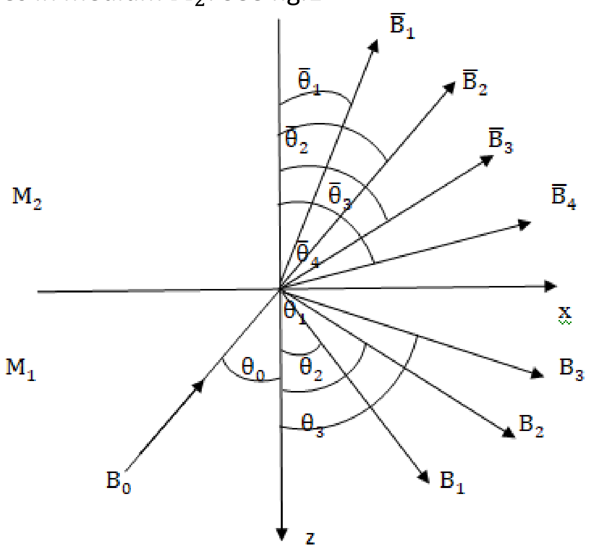

Fig.1 Geometry of the problem.

In medium M_1

$$\varphi = B_0 \exp\{ik_0 (x \sin\theta_0 - z \cos\theta_0) + i\omega_1 t\} + B_1 \exp\{ik_0 (x \sin\theta_1 + z \cos\theta_1) + i\omega_1 t\}, \quad (43)$$

$$\psi = B_2 \exp\{i\delta_1(x \sin\theta_2 + z \cos\theta_2) + i\omega_2 t\} + B_3 \exp\{i\delta_2(x \sin\theta_3 + z \cos\theta_3) + i\omega_3 t\}, \quad (44)$$

$$\Phi_2 = EB_2 \exp\{i\delta_1(x \sin\theta_2 + z \cos\theta_2) + i\omega_2 t\} + FB_3 \exp\{i\delta_2(x \sin\theta_3 + z \cos\theta_3) + i\omega_3 t\}, \quad (45)$$

where

$$E = \frac{\delta_1^2 \left(\delta_1^2 - \frac{\omega^2}{(c_2{}^2+c_3{}^2)} + pq\right)}{\text{deno.}}, \quad (46)$$

$$F = \frac{\delta_2^2 \left(\delta_2^2 - \frac{\omega^2}{(c_2{}^2+c_3{}^2)} + pq\right)}{\text{deno.}}, \quad (47)$$

and

$$\text{deno.} = p\left(2q - \frac{\omega^2}{c_4{}^2}\right), \quad \delta_1^2 = \lambda_1^2\omega^2, \ \delta_2^2 = \lambda_2^2\omega^2 . \quad (48)$$

where B_0, B_1, B_2, B_3 are amplitudes of incident longitudinal wave, reflected longitudinal wave, reflected coupled transverse and reflected micro-rotation waves respectively.

In medium M_2

$$\bar{\varphi} = \bar{B}_1 \exp\{i\bar{k}_0 (x \sin\bar{\theta}_1 - z \cos\bar{\theta}_1) + i\bar{\omega}_1 t\}, \quad (49)$$

$$\bar{\psi} = \bar{B}_2 \exp\{i\bar{\delta}_1(x \sin\bar{\theta}_2 - z \cos\bar{\theta}_2) + i\bar{\omega}_2 t\} + \bar{B}_3 \exp\{i\bar{\delta}_2(x \sin\bar{\theta}_3 - z \cos\bar{\theta}_3) + i\bar{\omega}_3 t\}, \quad (50)$$

$$\bar{\varphi}_2 = \bar{E}\,\bar{B}_2 \exp\{i\bar{\delta}_1(x \sin\bar{\theta}_2 - z \cos\bar{\theta}_2) + i\bar{\omega}_2 t\} + \bar{F}\bar{B}_3 \exp\{i\bar{\delta}_2(x \sin\bar{\theta}_3 - z \cos\bar{\theta}_3) + i\bar{\omega}_3 t\}, \quad (51)$$

$$\bar{\Phi} = G\bar{B}_0 \exp\{i\bar{k}_4 (x \sin\bar{\theta}_4 - z \cos\bar{\theta}_4) + i\bar{\omega}_4 t\}, \quad (52)$$

where $\bar{B}_1, \bar{B}_2, \bar{B}_3, \bar{B}_4$ are amplitudes of refracted longitudinal displacement wave, two refracted sets of two coupled waves (CD I and CD II) and refracted longitudinal micro-stretch wave (LMS-wave) respectively. G is the constant of dimension L^{-2}.

4. Boundary conditions

At the interface between micropolar elastic solid and micropolar viscoelastic solid with stretch, the appropriate boundary conditions are continuity of force stresses, displacements and microrotation, couple stresses and vector first moment respectively. Mathematically, these boundary conditions can be written as:

At the interface z = 0,

$$t_{zz} = \bar{t}_{zz}, \ t_{zx} = \bar{t}_{zx}, \ u = \bar{u}, \ w = \bar{w}, \ m_{zy} = \bar{m}_{zy}, \ \Phi_2 = \bar{\varphi}_2, \ \bar{\beta}_z = 0. \quad (53)$$

In order to satisfy the boundary conditions, the extension of the Snell's law will be

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{\lambda_1^{-1}} = \frac{\sin\theta_3}{\lambda_2^{-1}} = \frac{\sin\bar{\theta}_1}{\bar{V}_1} = \frac{\sin\bar{\theta}_2}{\bar{\lambda}_1^{-1}} = \frac{\sin\bar{\theta}_3}{\bar{\lambda}_2^{-1}} = \frac{\sin\bar{\theta}_4}{V}, \quad (54)$$

For longitudinal wave,

$$V_0 = V_1, \theta_0 = \theta_1, \quad (55)$$

Also

$$k_0V_1 = \delta_1\lambda_1^{-1} = \delta_2\lambda_2^{-1} = \bar{k}_0\bar{V}_1 = \bar{\delta}_1\bar{\lambda}_1^{-1} = \bar{\delta}_2\bar{\lambda}_2^{-1} = \bar{k}_4V = \omega, \quad \text{at } z = 0 \quad (56)$$

Making the use of potentials given by equations (43)-(45) and (49)-(52) in the boundary conditions given by (53) and using (54)-(56), we get a system of seven non homogeneous equations which can be written as

$$\sum_{j=1}^{7} a_{ij} Z_j = Y_i, \qquad (i = 1,2,3,4,5,6,7) \tag{57}$$

where

$$Z_1 = \frac{B_1}{B_0}, Z_2 = \frac{B_2}{B_0}, Z_3 = \frac{B_3}{B_0}, Z_4 = \frac{\bar{B}_1}{B_0}, Z_5 = \frac{\bar{B}_2}{B_0}, Z_6 = \frac{\bar{B}_3}{B_0}, Z_7 = \frac{\bar{B}_4}{B_0}, \tag{58}$$

where Z_1 to Z_7 are the amplitude ratios of reflected longitudinal wave, reflected coupled-wave at an angle θ_2, reflected coupled-wave at an angle θ_3, refracted longitudinal displacement wave, two refracted sets of two coupled waves (CD I and CD II) and refracted longitudinal micro-stretch waves (LMS-wave) respectively. Also a_{ij} and Y_i in non-dimensional form are as

$$a_{11} = \frac{\lambda}{\mu} + D_2\cos^2\theta_1, \qquad a_{12} = D_2\sin\theta_2\cos\theta_2\frac{\delta_1^2}{k_0^2}, \qquad a_{13} = D_2\sin\theta_3\cos\theta_3\frac{\delta_2^2}{k_0^2},$$

$$a_{14} = \left\{\frac{(2\bar{\lambda} + 2\bar{\mu} + \bar{\kappa})}{\mu}\sin\bar{\theta}_1\cos\bar{\theta}_1\right\}\frac{\bar{k}_0^2}{k_0^2}, \qquad a_{15} = \frac{(2\bar{\mu} + \bar{\kappa})}{\mu}(\cos^2\bar{\theta}_2 - \sin^2\bar{\theta}_2)\frac{\bar{\delta}_1^2}{k_0^2},$$

$$a_{16} = \frac{(2\bar{\mu} + \bar{\kappa})}{\mu}(\cos^2\bar{\theta}_3 - \sin^2\bar{\theta}_3)\frac{\bar{\delta}_2^{\,2}}{k_0^2}, \qquad a_{17} = 0, \qquad Y_1 = a_{11}.$$

$$a_{21} = D_2\sin\theta_1\cos\theta_1, \quad a_{22} = -\left\{(D_1\cos^2\theta_2 - \sin^2\theta_2) - \frac{\kappa}{\mu}\frac{E}{\delta_1^2}\right\}\frac{\delta_1^2}{k_0^2},$$

$$a_{23} = -\left\{(D_1\cos^2\theta_3 - \sin^2\theta_3) - \frac{\kappa}{\mu}\frac{F}{\delta_2^2}\right\}\frac{\delta_2^2}{k_0^2}, \qquad a_{24} = -\left\{\frac{(\bar{\mu} + \bar{\kappa}\cos^2\bar{\theta}_1)}{\mu}\right\}\frac{\bar{k}_0^2}{k_0^2},$$

$$a_{25} = \left(\bar{\delta}_1^2\sin\bar{\theta}_2\cos\bar{\theta}_2 - \bar{E}\right)\frac{\bar{\kappa}}{\mu\, k_0^2}, \qquad a_{26} = \left(\bar{\delta}_2^2\sin\bar{\theta}_3\cos\bar{\theta}_3 - \bar{F}\right)\frac{\bar{\kappa}}{\mu\, k_0^2},$$

$$a_{27} = 0, \qquad Y_2 = a_{21}.$$

$$a_{31} = 0, \qquad a_{32} = E\gamma\cos\theta_2\frac{\delta_1}{k_0}, \qquad a_{33} = F\gamma\cos\theta_3\frac{\delta_2}{k_0}, \qquad a_{34} = 0,$$

$$a_{35} = \bar{E}\,\bar{\gamma}\cos\bar{\theta}_2\frac{\bar{\delta}_1}{k_0}, \qquad a_{36} = \bar{F}\,\bar{\gamma}\cos\bar{\theta}_3\frac{\bar{\delta}_2}{k_0}, \qquad a_{37} = G\bar{\beta}_0\sin\bar{\theta}_4\frac{\bar{k}_4}{k_0}, \qquad Y_3 = a_{31}.$$

$$a_{41} = -\sin\theta_1, \qquad a_{42} = \cos\theta_2\frac{\delta_1}{k_0}, \qquad a_{43} = \cos\theta_3\frac{\delta_2}{k_0}, \qquad a_{44} = -\cos\bar{\theta}_1\frac{\bar{k}_0}{k_0},$$

$$a_{45} = -\sin\bar{\theta}_2\frac{\bar{\delta}_1}{k_0}, \qquad a_{46} = -\sin\bar{\theta}_3\frac{\bar{\delta}_2}{k_0}, \qquad a_{47} = 0, \qquad Y_4 = -a_{41}.$$

$$a_{51} = \cos\theta_1, \qquad a_{52} = \sin\theta_2\frac{\delta_1}{k_0}, \qquad a_{53} = \sin\theta_3\frac{\delta_2}{k_0}, \qquad a_{54} = -\sin\bar{\theta}_1\frac{\bar{k}_0}{k_0},$$

$$a_{55} = -\cos\bar{\theta}_2\frac{\bar{\delta}_1}{k_0}, \qquad a_{56} = -\cos\bar{\theta}_3\frac{\bar{\delta}_2}{k_0}, \qquad a_{57} = 0, \qquad Y_5 = a_{51}.$$

$$a_{61} = 0, \qquad a_{62} = -E, \qquad a_{63} = -F, \qquad a_{64} = 0, \qquad a_{65} = \bar{E}, \qquad a_{66} = \bar{F},$$

$$a_{67} = 0, \qquad Y_6 = a_{61}.$$

$$a_{71} = a_{72} = a_{73} = a_{74} = 0, \qquad a_{75} = \bar{E}\sin\bar{\theta}_2\frac{\bar{\beta}_0}{3}\frac{\bar{\delta}_1}{k_0}, \qquad a_{76} = \bar{F}\sin\bar{\theta}_3\frac{\bar{\beta}_0}{3}\frac{\bar{\delta}_2}{k_0},$$

$$a_{77} = -\bar{\alpha}_0 G\cos\bar{\theta}_4\frac{\bar{k}_4}{k_0}, \qquad Y_7 = a_{71}. \tag{59}$$

where

$D_1 = 1 + \frac{\lambda}{\mu}$, $D_2 = 1 + D_1$.

5. Numerical results and discussion:

The theoretical results obtained above indicate that the amplitude ratios $Z_i (i = 1,2,3,4,5,6,7)$ depend on the angle of incidence of incident wave and elastic properties of half spaces. In order to study in more detail the behaviour of various amplitude ratios. We take the case of aluminium-epoxy composite subject to viscous effect and stretch effect for our calculation. Following Gauthier (1982), the physical constants for micropolar elastic solid are

$$\lambda = 7.59 \times 10^{11}\text{dyne/cm}^2, \quad \mu = 1.89 \times 10^{11}\text{dyne/cm}^2,$$
$$\kappa = 0.0149 \times 10^{11}\text{dyne/cm}^2, \quad \rho = 2.19\text{gm/cm}^3$$
$$\gamma = 0.0268 \times 10^{11}\text{ dyne}, \quad j = 0.0196\text{ cm}^2, \quad \frac{\omega^2}{{\omega_0}^2} = 20. \tag{60}$$

For a particular modal micropolar viscoelastic solid with stretch, the physical constants are given as

$$\lambda^* = 6.8 \times 10^{11}\text{dyne/cm}^2, \quad \mu^* = 1.63 \times 10^{11}\text{dyne/cm}^2,$$
$$\kappa^* = 0.0134 \times 10^{11}\text{dyne/cm}^2, \quad \bar{\rho} = 2.06\text{gm/cm}^3,$$
$$\gamma^* = 0.0268 \times 10^{11}\text{ dyne}, \quad \bar{j} = 0.0196\text{ cm}^2, \quad \frac{\bar{\omega}^2}{{\bar{\omega}_0}^2} = 20. \tag{61}$$
$$\bar{\lambda} = \lambda^*(1 + iQ_1^{-1}), \quad \bar{\mu} = \mu^*(1 + iQ_2^{-1}),$$
$$\bar{\kappa} = \kappa^*(1 + iQ_3^{-1}), \quad \bar{\gamma} = \gamma^*(1 + iQ_4^{-1}).$$

where Q_i (i=1, 2, 3, 4) are chosen arbitrary as

$$Q_1 = 4, \quad Q_2 = 9, \quad Q_3 = 13, \quad Q_4 = 11.$$

and

$$\alpha_0 = 9.15 \times 10^5\text{dyne}, \quad \beta_0 = 7.26 \times 10^5\text{dyne}, \quad \eta_0 = 5.32 \times 10^5\text{dyne}.$$

A computer programme in MATLAB has been developed to calculate the modulus of amplitude ratios of various reflected and refracted waves for the particular model and to depict graphically.

In figures (2) – (4) and (5) – (8), solid lines shows the variations of amplitude ratios of reflected and refracted waves respectively, when medium-I is micropolar elastic solid (MPES) and medium-II is micropolar viscoelastic solid with stretch whereas dashed lines shows the variations of amplitude ratios of various reflected and refracted waves when medium-II becomes micropolar without viscous with stretch effects. In these figures, the effect of viscosity is clear. In these figures, MPESWS has been used for micropolar elastic solid with stretch and MPVS means micropolar viscoelastic solid.

In figures (9) – (11) and (12) – (15), solid lines shows the variations of amplitude ratios of reflected and refracted waves respectively, when medium-I is micropolar elastic solid and medium-II micropolar viscoelastic solid with stretch whereas dashed lines shows the variations of amplitude ratios of various reflected and refracted waves respectively, when medium-II becomes micropolar with viscous without effects of stretch. In these figures effects of stretch is negligible shown only except the figure (15) in whole phenomenon.

Figures (16) - (18) shows the variation of the modulus of the amplitude ratios of various reflected waves at free surface of micropolar elastic solid (MPES) medium-I. In figure (16) the amplitude ratio first decrease to their minimum value then increases and gets the maximum value. In figure (17) the amplitude ratio first rapidly increases to their maximum value at the angle of incidence 13^o and after that rapidly decrease at the angle of incidence 16^o and after all gradually approach to minimum value. In figure (18) the amplitude ratio first increases to their maximum value 0.35 at the angle of incidence 50^o and after then decreases and approaches to its minimum value.

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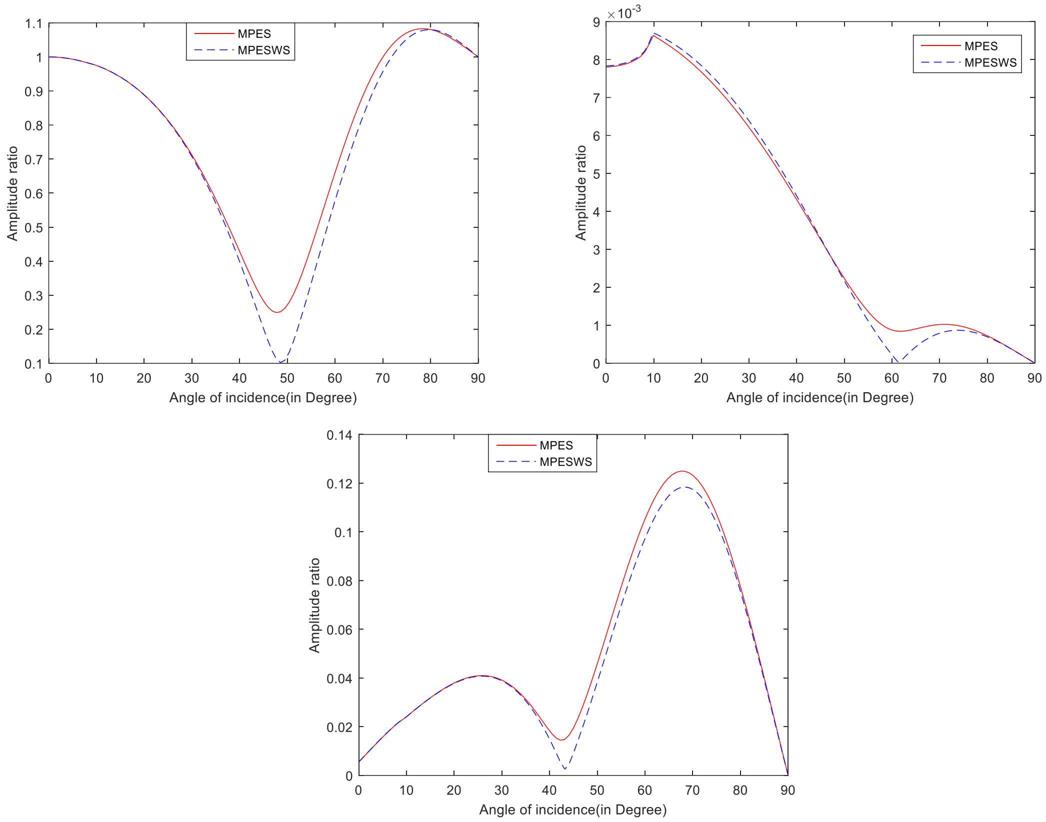

Fig.2-4.Variation of the amplitudes ratios $|Z_i|$, $(i = 1, 2, 3)$ with angle of incidence of the incident longitudinal wave.

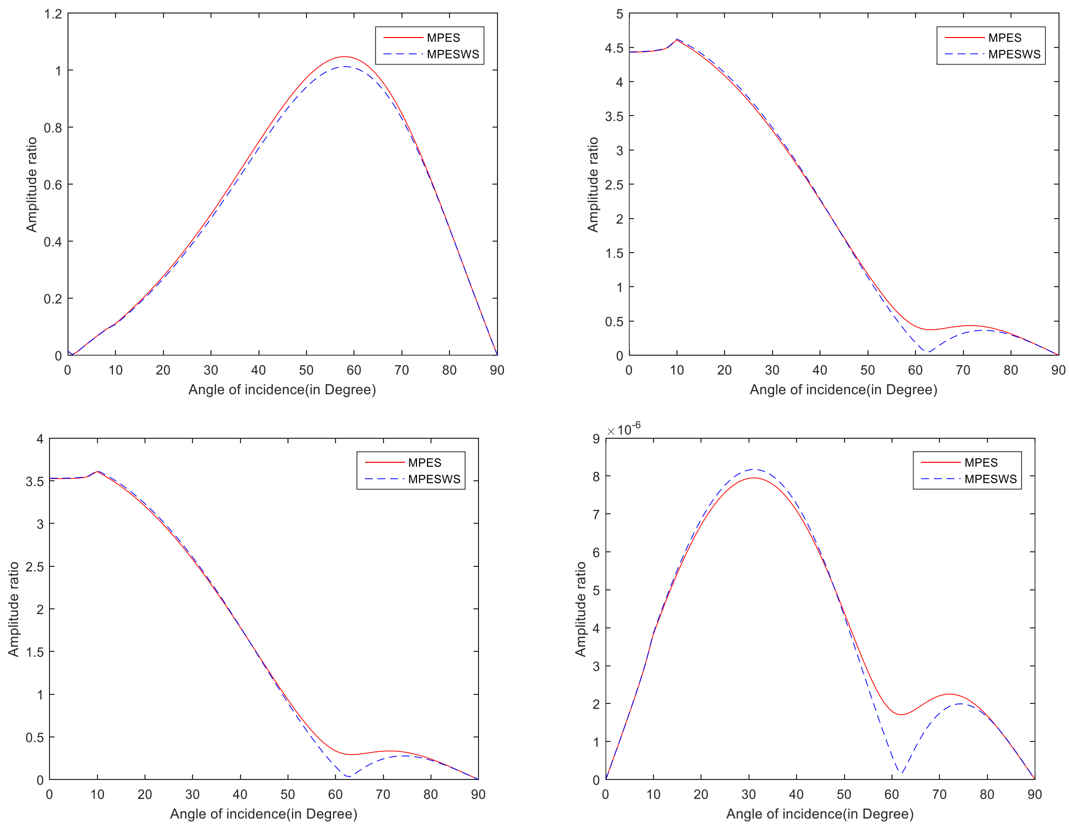

Fig.5-8.Variation of the amplitudes ratios $|Z_i|$, $(i = 4, 5, 6,7)$ with angle of incidence of the incident longitudinal wave.

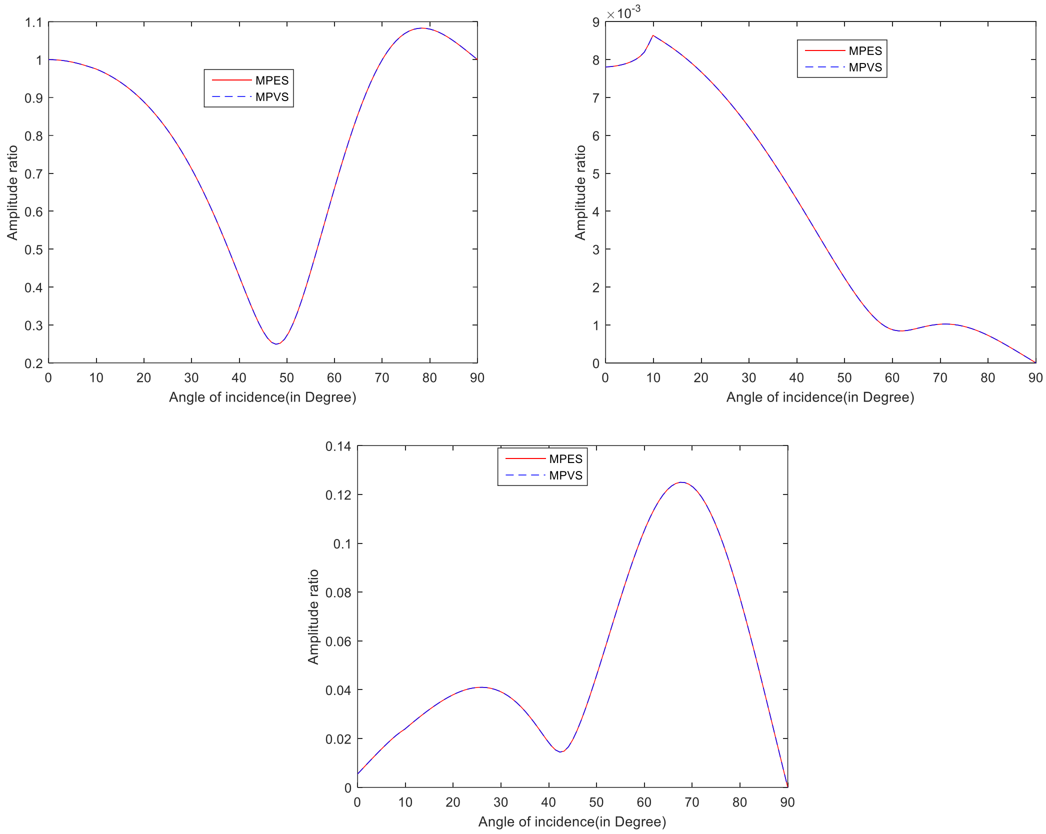

Fig.9-11.Variation of the amplitudes ratios $|Z_i|$, $(i = 1, 2, 3)$ with angle of incidence of the incident longitudinal wave.

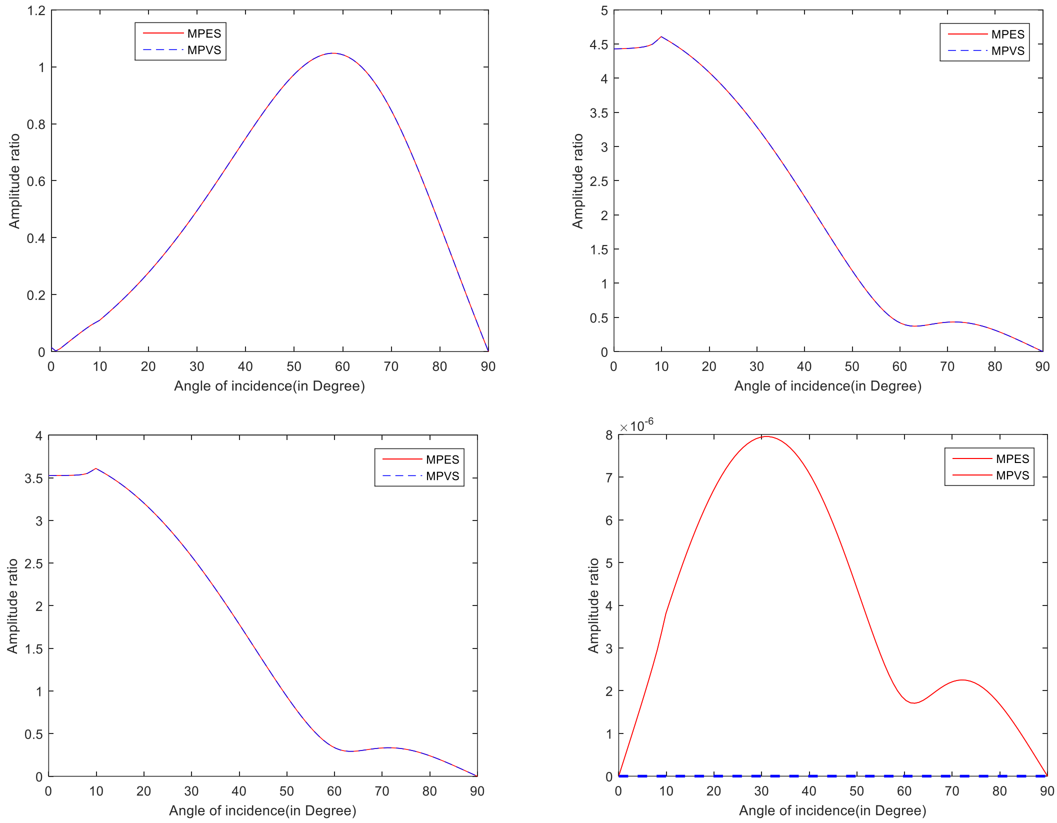

Fig.12-15.Variation of the amplitude ratios $|Z_i|$, $(i = 4, 5, 6, 7)$ with angle of incidence of the incident longitudinal wave.

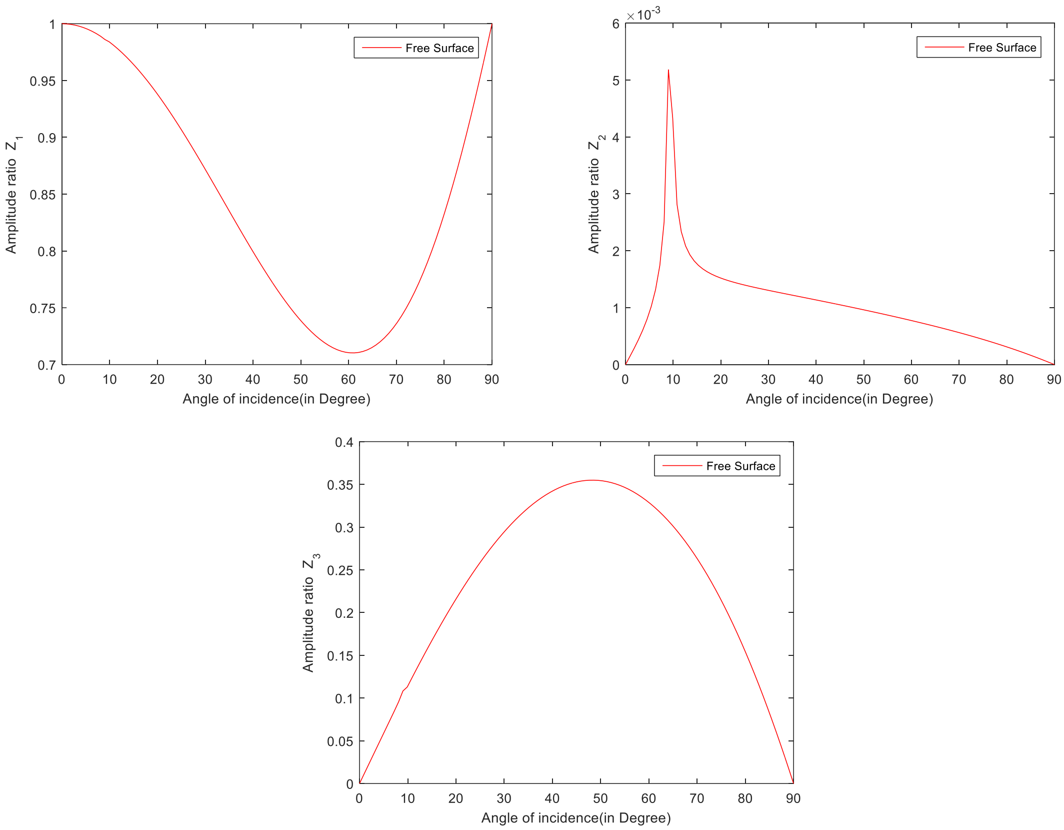

Fig.16-18.Variation of the amplitude ratios $|Z_i|$, $(i = 1, 2, 3.)$ with angle of incidence of the incident longitudinal wave.

6. Conclusion:

In conclusion, a mathematical study of reflection and refraction coefficients at interfaces separating micropolar elastic solid half space and micropolar viscoelastic solid with stretch half space is made when longitudinal wave is incident. It is observed that

1. The amplitudes ratios of various reflected and refracted waves are found to be complex valued.
2. The modulus of amplitudes ratios of various reflected and refracted waves depend on the angle of incidence of the incident wave and material properties of half spaces.
3. The effects of viscous and stretch are significant on the amplitudes ratios.
4. The effect of incident wave is significant on amplitude ratios. All the amplitudes ratios are found to depend on incident waves.
5. The solution leads to the existence of a new wave which named as longitudinal micro-stretch wave (LMS).

The model presented in this paper is one of the more realistic forms of the earth models. The present theoretical results may provide useful information for experimental scientists, researchers and seismologists are working in the area of wave propagation in micropolar elastic solid and micropolar viscoelastic solid with stretch.

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