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DUO NOETHERIAN SEMIGROUPS

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ABSTRACT

In this paper the terms Noetherian semigroup, closed semigroup and center of a semigroup are introduced. It is proved that if S is a noetherian semigroup containing proper ideals, then S has a maximal ideal. It is proved thatif H is the collection of all ideals in a duo semigroupS which are not principal and H ≠ ∅, then there exists a prime ideal of S which is not a principal ideal. It is proved that if every prime ideal including S is principal in a duo semigroup S, then every ideal in S is principal. Also it is proved that if S is a duo semigroup, which is a union of finite number of principal ideals and every proper prime ideal of S is principal and S = S^2 then every proper ideal is principal. If S is a duo semigroup such that S≠$S^2$and every maximal ideal is principal then it is proved that (1) S has atmost two maximal ideals and (2) if P is a proper prime ideal of S then either P is a principal ideal or P = *x*P for some $x \in S$. If every maximal ideal in a closed duo semigroup S is principal and S ≠S^2, $< x >^w = \emptyset$ for every $x \in S$, then it is proved that S is a union of two principal ideals and every ideal is an intersection of a prime ideal and an S-primary ideal. If S is a noetherian or archimedian duo semigroup such that S = $\bigcup_{i=1}^{n} < x_i >$ and suppose $a \notin < x_i a >$ for all $a \in S$, which is not a product of power of x_i`s,then it is proved that S is finitely generated and in particular if S is noetherian cancellative semigroup without identity then S is finitely generated. If S is a duo semigroup which is a union of finite number of principal ideals andif S =S^2, then it is proved that S contains idempotent elements. If S is a cancellable duo semigroup which is a union of finite number of principal ideals, then it is proved that S contains identity if and only if S = S^2.

KEY WORDS : Chained semigroup, duo chained semigroup,noetherian semigroup and center of a semigroup.

1. PRELIMINARIES :

DEFINITION 1.1 : Let S be any non-empty set. Then S is said to be a ***semigroup*** if there exist a mapping from S$\times$S to S which maps (*a*, *b*) $\rightarrow$*ab* satisfying the condition : (*ab*)*c* = *a(bc*) for all *a, b, c*∈ S.

NOTE 1.2 : Let S be a semigroup. If A and B are two subsets of S, we shall denote the set { *ab* : *a* ∈ A , *b* ∈ B } by AB.

DEFINITION 1.3: A nonempty subset A of a semigroup S is said to be a ***left ideal*** of S if $s \in S, a \in A$implies $sa \in A$.

NOTE 1.4:A nonempty subset A of a semigroup S is aleft ideal of S iff SA⊆A.

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DEFINITION1.5: A nonempty subset A of a semigroup S is said to be a ***right ideal*** of S if $s \in S, a \in A$ implies $as \in A$.

NOTE 1.6 :A nonempty subset A of a semigroup S is aright ideal of Siff AS⊆A.

DEFINITION 1.7: A nonempty subset A of a semigroup S is said to be a ***two sided ideal*** orsimply a***ideal*** of S if *s* ∈ S, *a* ∈ A imply *sa* ∈ A, *as* ∈ A.

NOTE 1.8 :A nonempty subset A of a semigroup S is atwo sided ideal iff it is both a left ideal and a rightideal of S.

THEOREM 1.9 : The nonempty intersection of any two (left or right) ideals of a semigroup S is a (left or right) ideal of S.

THEOREM 1.10 : The nonempty intersection of any family of (left or right) ideals of a semigroup S is a (left or right) ideal of S.

THEOREM 1.11 : The union of any two (left or right) ideals of a semigroup S is a (left or right) ideal of S.

THEOREM 1.12 : The union of any family of (left or right) ideals of a semigroup S is a(left or right) ideal of S.

DEFINITION 1.13 :A semigroup S is said to be a ***left duo semigroup*** provided every left ideal of S is a two sided ideal of S.

DEFINITION 1.14 :A semigroup S is said to bea ***right duo semigroup*** provided every right ideal of S is a two sided ideal of S.

DEFINITION 1.15 :A semigroup S is said to be a ***duo semigroup*** provided it is both a left duo Semigroup and a right duosemigroup.

THEOREM 1.16 : A semigroup S is a duo semigroup if and only if $xS^1 = S^1x$ for all *x*∈ S .

THEOREM 1.17 : Let A be a ideal in a duo semigroup S and *a*, *b* ∈ *S*. Then *ab*∈A if and only if$< a >< b > \subseteq A$.

COROLLARY 1.18 : Let A be a ideal in a duo semigroup S. Then for any natural number n, $a^n \in A$implies $< a >^n \subseteq A$.

DEFINITION 1.19 : An ideal A of a semigroup S is said to be a ***maximal ideal*** provided A is a proper ideal of S and A is not properly contained in any proper ideal of S.

DEFINITION 1.20 : An ideal P of a semigroup S is said to be a ***completely prime ideal*** provided *x, y*∈ S and *xy*∈P implies either *x*∈ P or *y*∈ P.

DEFINITION 1.21: An ideal P of a semigroup S is said to be a ***primeideal*** provided A, B are two ideals of S and AB ⊆ P ⇒ either A ⊆ P orB⊆ P.

COROLLARY 1.22: An ideal P of a semigroup S is a prime ideal iff*a, b* ∈ S such that *ab*∈P, then either *a* ∈P or *b* ∈P.

THEOREM 1.23 : Let S be a duo semigroup. An ideal P of S is prime ideal if and only if P is a completely prime ideal.

DEFINITION 1.24 : If A is an ideal of a semigroup S, then the intersection of all prime ideals of S containing A is called ***prime radical*** or simply ***radical*** of A and it is denoted by **√A** or ***rad A***.

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DEFINITION 1.25 : If A is an ideal of a semigroup S, then the intersection of all completely prime ideals of S containing A is called ***completeprime radical*** or simply ***completeradical*** of A and it is denoted by ***c. rad A***.

NOTE 1.26 : If A is an ideal of a semigroup S then ***rad* A = A_3** and ***c.rad* A = A_4.**

THEOREM 1.27 : If A is an ideal of a duo semigroup S, then *rad* A = *c.rad* A.

NOTATION 1.28 : If A is an ideal of a semigroup S, then we associate the following four types of sets.

A_1 = The intersection of all completely prime ideals of S containing A.

A_2 = {x∈S :x^n∈ A for some natural number n }

A_3 = The intersection of all prime ideals of S containing A.

A_4 = {x∈ S :<x>^{n}⊆ A for some natural number n }

THEOREM 1.29 : If A is an ideal of a semigroup S, then A ⊆ A_4⊆ A_3⊆ A_2⊆ A_1.

THEOREM 1.30 : If A is an ideal in a duo semigroup S then A_1 = A_2 = A_3 = A_4.

DEFINITION 1.31 :A semigroup S is said to be an ***archimedian semigroup***provided for any *a, b* ∈ S, there exists a natural number *n* such that a^n∈<*b* >.

DEFINITION 1.32 :A ideal A of a semigroup S is said to be a ***left primary ideal*** provided

i) If X, Y are two ideals of S such that XY ⊆ A and Y ⊈ A then X ⊆ √A.

ii) √A is a prime ideal of S.

DEFINITION 1.33 : An ideal A of a semigroup S is said to be a ***right primary ideal*** provided

i) If X, Y are two ideals of S such that XY ⊆ A and X ⊈ A then Y ⊆ √A.

ii) √A is a prime ideal of S.

DEFINITION 1.34 : An ideal A of a semigroup S is said to be a ***primary ideal*** provided A is both a left primary ideal and a right primary ideal.

THEOREM 1.35 : Let A be an ideal of a semigroup S. Then X, Y are two ideals of S such that XY ⊆ A and Y ⊈ A ⇒ X ⊆ √A if and only if *x*, *y*∈ S, <*x*><*y*>⊆ A and *y*∉ A ⇒*x*∈ √A.

THEOREM 1.36 : Let A be an ideal of a semigroup S. Then X, Y are two ideals of S such that XY ⊆ A and X ⊈ A ⇒ Y ⊆ √A if and only if *x*, *y*∈ S, <*x*><*y*>⊆ A and *x*∉ A ⇒*y*∈ √A.

DEFINITION 1.37 :A ideal A of a semigroup S is said to be ***semiprimary*** provided √A is a prime ideal of S.

DEFINITION 1.38 :A semigroup S is said to be a ***semiprimary semigroup*** provided every ideal of S is a semiprimary ideal.

THEOREM 1.39 : Every left primary or right primary ideal of a semigroup is a semiprimary ideal.

DEFINITION 1.40 : Let P be any prime ideal in a semigroup S. A primary ideal A in S is said to be ***P-primary*** or P is a ***prime ideal belongingto A*** provided $\sqrt{A}$ = P.

DEFINITION 1.41 : Let S be any prime ideal in a semigroup S. A primary ideal A in S is said to be S-***primary*** or S is a ***prime ideal belonging to A*** provided $\sqrt{A}$ = S.

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THEOREM 1.42 : If $A_1, A_2, \ldots, A_n$ are P-primary ideals in a semigroup S, then $\bigcap_{i=1}^{n} A_i$ is also a P-primary ideal.

DEFINITION 1.43 : An ideal A in a semigroup S is said to have a (***left***, ***right***) ***primary decomposition*** if $A = A_1 \cap A_2 \cap \ldots \cap A_n$ where each A_i is a (left, right) primary ideal. If no A_i contains $A_1 \cap A_2 \cap \ldots \cap A_{i-1} \cap A_{i+1} \cap \ldots \cap A_n$ and the radicals P_i of the ideals A_i are all distinct, then the primary decomposition is said to be ***reduced***. If P_i is minimal in the set
$\{ P_1, P_2, \ldots, P_n \}$ then P_i is said to be ***isolated prime***.

THEOREM 1.44 :Every ideal in a (left, right) duo noetherian semigroup S has a reduced (right, left) primary decomposition.

NOTE 1.45 : If S is a semigroup and $a \in$ S then we denote $< a >^{w} = \bigcap_{n=1}^{\infty} < a >^{n}$.

NOTE 1.46 : If S is a duo Γ-semigroup then $< a >^{w} = \bigcap_{n=1}^{\infty} < a^{n} > = \bigcap_{n=1}^{\infty} a^{n} S^{1}$.

DEFINITION1.47 : An element *a* of semigroup S is said to be ***semisimple*** provided
$a \in <a>^2$, that is $<a>^2 = <a>$.

DEFINITION 1.48 : An element *a* of semigroup S is said to be an ***idempotent*** or
if $a^2 = a$.

DEFINITION 1.49 :A semigroup S is said to be an ***idempotent semigroup*** provided every element of S is an idempotent.

DEFINITION 1.50 : An element *a* of a semigroup S is said to be ***regular*** provided
$a = axa$, for some $x \in$ S.

DEFINITION 1.51 :A semigroup S is said to be *a* ***regular semigroup*** provided every element of S is regular.

THEOREM 1.52 : If S is a duo semigroup, then*a* is regular if and only if*a* is semisimple for any element $a \in$ S.

THEOREM 1.53 : If a semigroup S contains regular elements then S contains idempotents.

THEOREM 1.54 : If a semigroup S regular then S contains idempotents.

DEFINITION 1.55 :A semigroup S is said to be a ***group*** if
(1) $\exists e \in$ S $\ni ae = ea = a$ for all $a \in$ S.
(2) every element $a \in$S has a inverse in S.

2. DUO NOETHERIAN SEMIGROUPS :

DEFINITION 2.1 :A semigroup S is said to be a ***noetherian semigroup*** if ascending chain of ideals becomes stationary.i.e., if $A_1 \subseteq A_2 \subseteq A_3 \subseteq \cdots$is an ascending chain of ideals of S, then there exists a natural number *m* such that $A_m = A_n$ for all natural numbers $n \geq m$.

NOTE 2.2 :A semigroup S is noetherian if and only if every ideal of S is a union of finite number of principal ideals of S.

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THEOREM 2.3 :If S is a noetheriansemigroup containing proper ideals then S has a maximal ideal.

Proof : Let A_1 be a proper ideal of S. If A_1 is not a maximal ideal of S, then there exists a proper ideal A_2 of S such that $A_1 \subset A_2$. If A_2 is not a maximal ideal of S, then there exists a proper ideal A_3 of S such that $A_1 \subset A_2 \subset A_3$. By continuing this process we get an ascending chain of proper ideals of S. Since S is noetherian, the chain $A_1 \subset A_2 \subset A_3$... is stationary. It is a contradiction. Therefore there exists a maximal ideal of S.

THEOREM 2.4 : Let H be the collection of all ideals in aduo semigroup S which are not principal. If H ≠ ∅ then there exists a prime ideal which is not a principal ideal.

Proof :Let H = { A_α : α ∈ Δ } be the collection of all ideals in a duo semigroup S, which are not principal. If $\bigcup_{\alpha\in\Delta} A_\alpha$ = <x> for some $x \in S$, then $x \in A_\beta$ for some $\beta \in \Delta$. Therefore <x>⊆A_β ⊆ $\bigcup_{\alpha\in\Delta} A_\alpha$ $= < x >$ and hence $A_\beta = < x >$. Then A_β∉ H. It is a contradiction. Hence $\bigcup_{\alpha\in\Delta} A_\alpha$ is not principal. So $\bigcup_{\alpha\in\Delta} A_\alpha$ ∈ H. Thus H satisfies all the conditions of Zorn`s lemma. By Zorn`s lemma, H has a maximal element say P. Suppose if possible P is not a prime ideal. Then there exists $a, b \in S$ such that $ab \subseteq P$and$a \notin P$and$b \notin P$. Since P is maximal in H, $P \cup < b > \notin H$. Therefore $P \cup < b >$ is a principal ideal. Then $P \cup < b > = < x >$ for some $x \in S$. If $x \in P$ then we get P = <x> and hence $b \in P$. It is not true. Hence $x \notin P$. Therefore $x \in < b >$and hence $< b >$= <x>. Hence P ⊆<b>. Now $P' = \{s \in S : sb \in P\}$ is an ideal of S. Then clearly $a \in P'$ and $a \notin P$. Therefore $P \subset P'$ and $P \neq P'$. By the maximality of P in H, we get $P' \notin H$. Therefore $P' = < y >$ for some $y \in S$. Now $y \in P' \Rightarrow yb \in P \Rightarrow < yb > \subseteq P$.Let $t \in P$. Since $P \subseteq < b >$, we have $t = sb$ for some $s \in S$.Now $sb \in P$. Hence $s \in P' = < y >$. Therefore $s = ry$ for some $r \in S$. Now $t = sb = (ry)b = r(yb) \in < yb > \Rightarrow t \in < yb > = < yb >$. Therefore we have $P \subseteq < yb >$. Hence $P = < yb >$. Thus P ∉ H. It is a contradiction. Therefore P is a prime ideal.

COROLLARY 2.5 : If H is the collection of all ideals in a duo semigroup S, which are not finitely generated and H ≠ ∅, then there exists a prime ideal which is not finitely generated.

THEOREM 2.6 : If every prime ideal including S is principal in a duo semigroup S, then every ideal in S is principal.

Proof : Let Hbe the collection of all ideals in S which are not principal. If H ≠ ∅ then by theorem 2.4, H contains a proper prime ideal which is not principal. It is a contradiction. Hence H = ∅. Therefore every ideal in S is principal.

COROLLARY 2.7 : If every prime ideal including S is finitely generated in a duo semigroup S, then every ideal in S is finitely generated.

THEOREM 2.8 : Let S be a duo semigroup, which is a union of finite number of principal ideals. If every proper prime ideal of S is principal and S = S^2 then every proper ideal is principal.

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Proof :Since S is a duo semigroup which is a union of finite number of principal ideals, S = $\bigcup_{i=1}^{n} < x_i >$ where $x_i \notin < x_j >$ for all $i \neq j$. Since S = S^2, $x_i \in < x_i^2 >$ for i = 1, 2, . . .,n. Thus x_i is semi simple and hence by theorem 1.52, x_i is regular. By theorem 1.54, $< x_i >$ = $< e_i >$ for some idempotent e_i in S. Let A be any proper ideal such that $\sqrt{A}$ = S. Therefore $e_i \in e_i^n$ for all i = 1, 2, . . ., n. Therefore $x_1, x_2, \ldots, x_n \in A$and hence S =A. It is a contradiction. Therefore there exists no ideal of A of S such that $\sqrt{A}$ = S. By theorem 2.6, every proper ideal is principal.

THEOREM 2.9 : If S is a duo semigroup such that S $\neq$ S^2 and **every maximal ideal is principal then S has atmost two maximal ideals.**

Proof **:** Let S be a duo semigroup such that S $\neq$ S^2. Suppose that every maximal ideal is principal. Let $a \in S \setminus S^2$. Then S\{a } is a maximal ideal. Therefore S\{a } = $<b>$ for some $b \in S$. Clearly $a \neq b$. Let $b \in S^2$. Then S\{a} = $<b> \subseteq S^2$ and hence S\{ a } = S^2. Let M be a maximal ideal of S. Then M = $< c >$ for some $c \in S$. If $c \in S^2$ then M$\subseteq$ S^2 Since M is maximal, M = S^2 = S\{a }. If $c \notin S^2$ then $c \notin$ S\{a } and hence c = a. Thus M = $< a >$. So if $b \in S^2$, S can have at most two maximal ideals, namely S\{ a } and $< a >$. Let $b \notin S^2$. Then S = $< b >$ U{ a } = { a }U{ b }U S^2. Let M = $< c >$ be a maximal ideal. If $c \notin S^2$ then c = a or c = b. Then
M = S\{a } or M = S\{ b }. If $c \in S^2$ then $M = S^2$ and hence M is properly contained in a proper ideal S\{ a }. It is a contradiction. Hence S has at most two maximal ideals.

THEOREM 2.10 : Let S be a duo semigroup such that S $\neq$ S^2 and every maximal ideal is principal. If P is a proper prime ideal of S then either P is a principal ideal or P = *x*P for some $x \in S$.

Proof **:** Let P be any proper prime ideal and $a \in S \setminus S^2$. Now S\{a } is a maximal ideal. Therefore S\{a } = $< b >$ for some $b \in S$. If $a \notin$ P then P $\subseteq$ S\{a } = $<b>$. If $b \in P$ then
P = $<b>$. If $b \notin P$ then P = bP, since P is a prime ideal. Let $a \in P$. If $b \in P$ then
P = S. If $b \notin$ P then P $\subseteq$ S\{b }. Since S\{b } is maximal ideal, we have P $\subseteq$ S\{ b }= $<x>$ for some $x \in S$. If $x \in P$ then P = $<x>$. If $x \notin$ P, let $y \in P$. Then $y \in < x >$. So $y \in xS \subseteq P$ for some $s \in S$. Since P is prime, $s \in P$. Hence $y \in P \subseteq xP$. Clearly $xP \subseteq$ P. Hence
P = $< x >$ or P = xP for some $x \in S$.

THEOREM 2.11 : If every maximal ideal in a duo semigroup S is principal and S $\neq$ S^2, $< x >^w = \emptyset$ for every $x \in S$, then S is a union of two principal ideals and every ideal is an intersection of a prime ideal and an S-primary ideal.

Proof **:**Let P be any proper prime ideal of S. By theorem 2.10, either P is a principal ideal or P = x P for some $x \in S$. If P = x P for some $x \in S$, then $x^n P = P$ for all natural numbers n. Thus

$$P = \bigcap_{n=1}^{\infty} x^n P \subseteq \bigcap_{n=1}^{\infty} < x^n > = < x >^w = \Phi$$

. It is a contradiction. Therefore P = $< x >$ for some $x \in S$. Thus every proper prime ideal is a principal ideal. If $a \in S \setminus S^2$ then by hypothesis, the maximal ideal S\{a} is of the form $<b>$ for some $b \in S$. Therefore S = { a } U$<b>$ = $<a>U<b>$. Then every ideal of S is an intersection of a prime ideal and an S-primary ideal of S.

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THEOREM 2.12 : Let S be a duo noetherian semigroup such that S = $\bigcup_{i=1}^{n} < x_i >$. Suppose $a \notin < x_i a >$ for all $a \in S$, which is not a product of power of x_i\`s. Then S is finitely generated. In particular if S is noetherian cancellative semigroup without identity then S is finitely generated.

Proof : Suppose that there exists an element *a* such that *a* is not a product of x_i\`s. If *a* = $x_i s_1$ where $a \neq s_1$ is not a product of power of $x_i$\`s. Hence $s_1 = x_j s_2$ where s_2 is not product of powers of x_i\`s. If $s_2 \in < s_1 >$ then $s_2 = s_1 r$ for some $r \in S^1$ and hence $s_1 = x_j(s_1 r) \in < x_j s_1 >$, which is not true. Hence $< s_1 > \subset < s_2 >$. By continuing this process, we get a nonterminating chain of ideals $< s_1 > \subset < s_2 > \subset < s_3 > \subset \cdots$ Since S is noetherian, it is a contradiction. So S is finitely generated. If S is a cancellative semigroup and if $a = a(ba)$, then ba is an identity in S. It is a contradiction. So $a \notin < x_i a >$ for all $a \in S$. As above, we have S is finitely generated.

THEOREM 2.13 : Let S be a duo semigroup which is a union of finite number of principal ideals. If S = S^2, then S contains idempotent elements.

Proof :Suppose that S = $\bigcup_{i=1}^{n} < x_i >$ and $x_i \notin < x_j >$ for $i \neq j$ and S =S^2. Since S =S^2, we have $x_i \in < x_i >^2$ for each *i* = 1, 2, 3, . . . , *n*. Therefore each x_i is semi simple in S. By theorem 1.52, x_i is regular in S and hence by theorem 1.53, S contains idempotents.

THEOREM 2.14 : Let S be a cancellable duo semigroup which is a union of finite number of principal ideals. Then S contains identity if and only if S = S^2.

Proof :Suppose that S is a cancellable duo semigroup and S =S^2. By theorem 2.13, S contains idempotent element say *e*. Let $a \in S$. Then $a(ee) = ae$. Since S is cancellative, $ae = a$. Similarly $ea = a$. Then *e* is the identity in S. Therefore S contains the identity. Conversely suppose that S contains the identity. Then clearly S = S^2.

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