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DYNAMICS OF DIRECTLY TRANSMITTED VIRAL MICRO PARASITE MODEL

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Abstract

We analyze the stability of directly transmitted viral micro parasite model consist of susceptible (x), invectives (y) and immune (z) populations. Here the total population (N) is kept constant by births and deaths. The model is characterized by the system of nonlinear ordinary differential equations. The model admits three equilibrium points and admits unique endemic equilibrium point if the basic reproduction rate R_0 exceeds one. We establish endemic equilibrium point is locally asymptotically stable if $(y^* > x^*)$. We show that the system is globally asymptotically stable. We identify the threshold population size N_c such that the parasite cannot maintain itself in the population eventually dies out the removable and infective class using Numerical simulation.

Keywords: SIR endemic model; Equilibrium points, Local stability, Global stability

1. Introduction: Epidemiology is the study of the dynamics of disease in a population of humans studied by Murry [1] R.M.Anderson et al [2]. Studies of human epidemiology usually treat the host (human) population as fixed in size and focus on the dynamics of disease within this population. Epidemiology is an acknowledgment that the dynamics of the parasite and the host populations may interact. The effects of the parasites on the dynamics of the hosts studied by H.W.Hethcote [3] R.M.Anderson and May [4]. Transmission of disease dynamics, in a very real sense, is the driving force behind the overall population dynamics of pathogens studied by N.J.T Bayley [5].

A micro parasite is an organism that can complete its life cycle within a single host. Most of the micro parasites are viruses, bacteria, or fungi; a few are protests' also. These parasites transmitted the diseases directly either through close contact or via environmental reservoir and those that require a vector or intermediate host for transmission studied by K. L. Cooke [8].The Qualitative analyses of transmissible disease models studied by H. W. Hethcote [7]. The viral transmission diseases are controlled by pulse vaccination. D. Nokes and J. Swinton [6] studied the control of childhood viral infections by pulse vaccination .The viral infection Irrespective of these distinctions, the rate of production of new infections in a population depends on the per capita transmission rate. The transmission rate depends on the infectiousness of the parasite, the susceptibility of the host, and so on, but it also depends on the contact rate between susceptible hosts and whatever it is that carries the infection. For directly transmitted parasites, we deal with the contact rate between infected hosts and susceptible hosts.

The dynamics of infectious disease has been derived from investigations of mathematical models of disease dynamics .The transmission of infection from infectious to susceptible hosts is arguably the driving force in the dynamics of any infectious disease. The dynamics of infectious disease model

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with time delay are dealt by Karuna et.all [9] and Ranjith kumar [10] dealt with some intersecting results in SIR epidemic models.

In this paper, we study the dynamics of directly transmitted viral micro parasite model which consists of susceptible (x), invectives (y) and immune (z) populations. Here the total population (N) is kept constant by births and deaths. The model is characterized by the system nonlinear ordinary differential equations .Two equilibrium points of the model are identified. The local and global stability analysis is carried out at endemic equilibrium point and find that it is locally asymptotically stable if $y^* > x^*$. The system is globally asymptotically stable is shown by constructing suitable Lyapunov function. Further, we identified the basic reproductive number (R_0) is given by $\frac{\beta N}{b+r}$ .The infection will be spread out i.e the transmission threshold if $R_0 > 1$, and an infection will eventually die out if $R_0 \le 1$. The threshold population size N_c is given by $\frac{b+r}{\beta}$ and if total population $N < N_c$ We identify the parameters for threshold transmission rate ($R_0 \le 1$) and threshold population size $N < N_c$,for which the parasite cannot maintain itself in the population eventually dies out the removable and infective class.

2. Basic equations:

The governing equations of the system are as follows

$$\frac{dx}{dt} = bN - \beta xy - bx$$
$$\frac{dy}{dt} = \beta xy - (b+r)y$$
$$\frac{dz}{dt} = ry - bz \qquad (2.1)$$

2.1 Nomenclature

N = Total population(x + y + z)

x= susceptible population

y = infectious population

z = Removable population

$\beta =$ Transmission rate

b= proportion rate to the total population

r = recovery rate of infection

3. Equilibrium states:

The system under investigation has two equilibrium points by equating $\frac{dx}{dt} = 0, \frac{dy}{dt} = 0 \,\&\, \frac{dz}{dt} = 0$ we get

E_1 : Disease free equilibrium point $(x^*, y^*, z^*) = (N, 0, 0)$

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E_2 : Endemic equilibrium point $(x^*, y^*, z^*) = \left[\frac{b+r}{\beta}, b\left(\frac{N\beta-b-r}{\beta(b+r)}\right), r\left(\frac{Nb-b-r}{\beta(b+r)}\right)\right]$

4. Local stability of analysis:

The positive equilibrium point $E_2(x^*, y^*, z^*)$ is locally asymptotically stable if $y^* > x^*$

The variational matrix is for the system is

$$J = \begin{bmatrix} -\beta y - b & -\beta x & 0 \\ \beta y & \beta x - (b+r) & 0 \\ 0 & r & -b \end{bmatrix} \quad (4.1)$$

With the characteristic equation $|J - \lambda I| = 0$

$$\lambda^3 + \lambda^2[\beta(y-x) + 2b + r] + \lambda[3b^2 + r\beta y + 2rb + 2b\beta(y-x)] + [b^3 + r\beta yb + rb^2 + b^2\beta(y-x)] = 0$$

(4.2)

With

$$a_0 = 1$$

$$a_1 = \beta(y-x) + 3b + r$$

$$a_2 = 2\beta(y-x) + 3b^2 + 2br + r\beta y \quad (4.3)$$

$$a_3 = b^2\beta(y-x) + rb^2 + b^3 + r\beta yb$$

By using Routh –Hurwitz criteria

Theorem: A set of necessary and sufficient condition that the all characteristic roots of $|A - \lambda I| = 0$ have negative real parts if, a_0 >0 and all the Hurwitz determinants are positive.

Where $|A - \lambda I| = a_0\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \ldots\ldots + a_n$

Hurwitz determinants: the Hurwitz determinates are defined as follows

$D_1 = a_1$, $D_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix}$, $D_3 = \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix}$,…. $D_n = |H|$.

Where

$D_1 = a_1 = \beta(y-x) + 3b + r > 0$ at the point (x^*, y^*, z^*) i.e if $y^* > x^*$

$$D_2 = \begin{vmatrix} \beta(y-x) + 3b + r & b^3 + rby\beta + rb^2 + \beta b^2(y-x) \\ 1 & 3b^2 + 2rb + r\beta y + 2\beta b(y-x) \end{vmatrix} > 0$$

$$D_2 = 3b^2\beta(y-x) + 2rb\beta(y-x) + r\beta^2 y(y-x)$$
$$+ 2b\beta^2(y-x) + 8b^3 + 6rb^2 + 2rby\beta + 5b^2\beta(y-x) + 2b^2r + 2r^2b + r^2\beta y + 2b\beta r(y-x) > 0$$

$D_2 > 0$ at the point (x^*, y^*, z^*) if $y^* > x^*$

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$$D_3 = \begin{vmatrix} \beta(y-x)+3b+r & b^3+rby\beta+rb^2+\beta b^2(y-x) & 0 \\ 1 & 3b^2+2rb+r\beta y+2\beta b(y-x) & 0 \\ 0 & \beta(y-x)+3b+r & b^3+rby\beta+rb^2+\beta b^2(y-x) \end{vmatrix} > 0$$

$$D_3 = [8b^2\beta(y-x)+4rb\beta(y-x)+r\beta^2 y(y-x)+2b\beta^2(y-x)^2 \\ +8b^3+8rb^2+2rby\beta+2r^2b+r^2\beta b][b^3+ryb\beta+rb^2+\beta b^2(y-x)]>0$$

i.e $D_3 > 0$ at the point (x^*, y^*, z^*) if $y^* > x^*$

Hence the interior equilibrium point $E_2(x^*, y^*, z^*)$ is locally asymptotically stable if $y^* > x^*$

5. Global Stability:

Theorem: The positive equilibrium point $E_2(x^*, y^*, z^*)$ is globally asymptotically stable

Proof: Let the Lyapunov function be

$$V(x,y,z) = \frac{1}{2}[(x-x^*)^2+(y-y^*)^2+(z-z^*)^2]$$

$$\dot{V}(x,y,z) = \frac{1}{2}[2(x-x^*)\frac{dx}{dt}+2(y-y^*)\frac{dy}{dt}+2(z-z^*)\frac{dz}{dt}]$$

$$\dot{V}(x^*,y^*,z^*) = [(x-x^*)(bN-\beta xy-bx)+(y-y^*)(\beta xy-by-ry)+(z-z^*)(ry-bz)]$$

By proper choice of

$$bN = \beta xy + bx^* \qquad \beta xy = (b+r)y^* \qquad ry = bz^*$$

Substitution equation in (**) we get

$$\dot{V}(x^*,y^*,z^*) = -b(x-x^*)^2-(b+r)(y-y^*)^2-b(z-z^*)^2$$

$$\dot{V}(x^*,y^*,z^*) = -[b(x-x^*)^2+(b+r)(y-y^*)^2+b(z-z^*)^2]$$

I e $\dot{V}(x^*,y^*,z^*) < 0$

Hence the interior equilibrium point $E_2(x^*, y^*, z^*)$ is globally asymptotically

6. Numerical example

Fig a: Time series responses of susceptible (x), infectious(y), Removable or immune (z) populations.

Fig b: Phase portrait susceptible (x), infectious(y), Removable or immune (z) populations.

Example1: b=0.1, r = 8, β = 0.2, x = 80, y = 40, z = 20

The behavior of the system exhibits oscillations and quenching to the equilibrium point E (41, 1, 98) shown in Fig 1.(a). The recovery or immune population is high due to the recovery rate of infection of Transmission coefficient is high and Transmission rate of diseases is low. The same is shown in phase portrait in fig 1(b).

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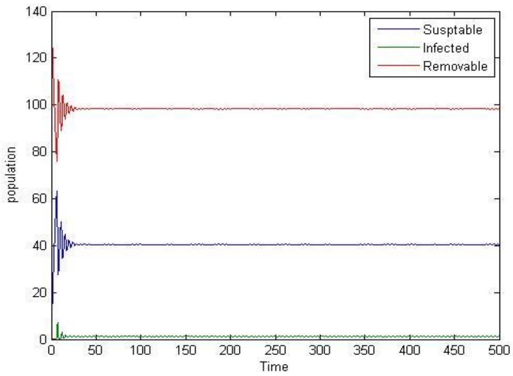
Example2: b=0.1, r = 8 , β=0.2 ,x=80,y=40,z =20.



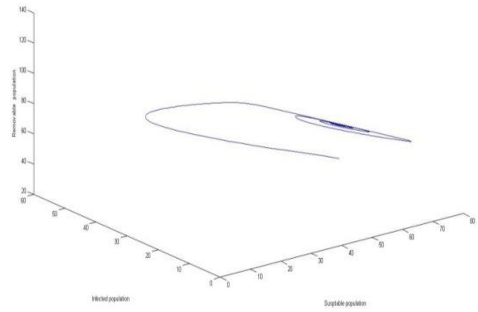
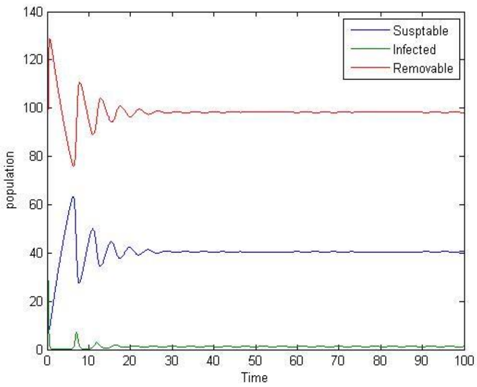


Fig:1(a) Fig:1(b)

The behavior of the system exhibits oscillations and quenching to the equilibrium point E (41, 1, 98) shown in Fig 2.(a). The recovery or immune population is high due to the recovery rate of infection of Transmission coefficient is high and Transmission rate of diseases is low. The infection rate is very low almost vanishes. The phase portrait of this case is shown in fig 2(b).



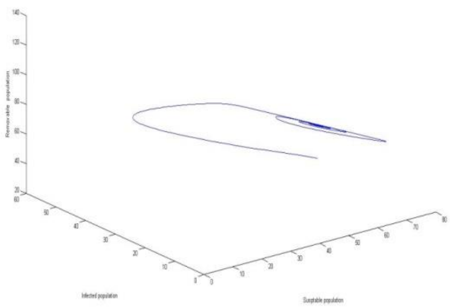


Fig:2(a)

Fig:2(b)

Example3: b=0.5, r = 1 , β = 0.5 ,x=30,y=10,z =5

For the above parametric values, the system is converging to the equilibrium point E (3, 14, 28) and does not exhibit any oscillations. Recovery and infected populations are quite high when compared with susceptible population due the interaction coefficients are almost same is shown in Fig 3(a) and phase portrait analysis in Fig 3(b).

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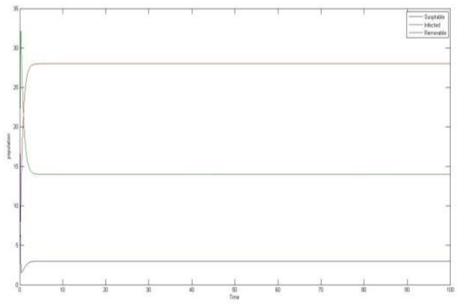

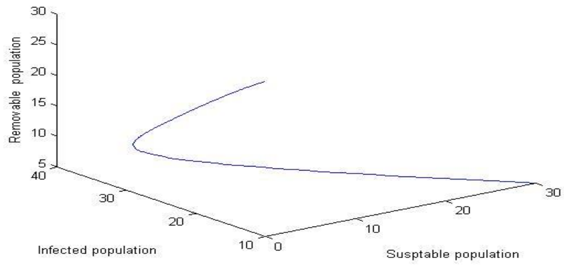

Fig:3(a) Fig:3(b)

Example 4: b=0.02, r = 1, β=0.05 ,x=30,y=10,z =5

Due to the low rate of transmission of disease the infected population is almost vanish. Hence the system dynamics oscillates for susceptible and removable populations shown in the fig 4(a) and its phase analysis shown in fig 4(b). The system is converging to its equilibrium point E (21 , 0 , 24)

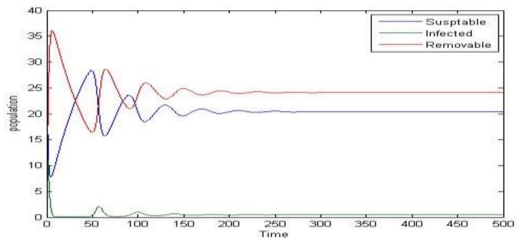

Fig:4(a)

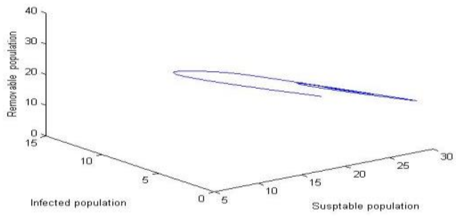

Fig:4(b)

Example5: b=0.2, r =15, β =0.3 , x = 30, y=10,z = 5.

In this case the removable and infected class population eventually dies out. This is due to the condition that $N < N_c$. Where $N_c = (b+r)/ \beta$. Here N = (x+y+z) = 45 and $N_c = 51$. This is called threshold population where the parasite cannot maintain itself in the population. Hence the population remains only susceptible.

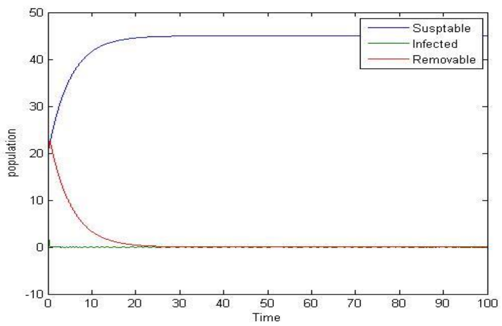

Fig:5(a)

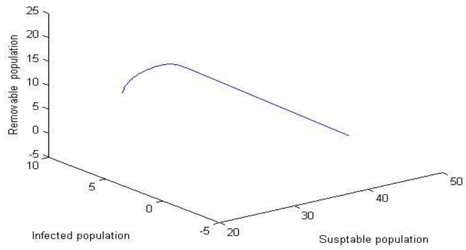

Fig:5(b)

Example 6: b=0.4, r =15 , β=0.3 ,x=30,y=10,z =5.

In this case the removable and infected class population eventually dies out. This is due to the

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condition that $N < N_c$. Where $N_c = (b+r)/ \beta$. Here N = (x+y+z) = 45 and N_c = 52 . This is called threshold population where the parasite cannot maintain itself in the population. Hence the population remains only susceptible.

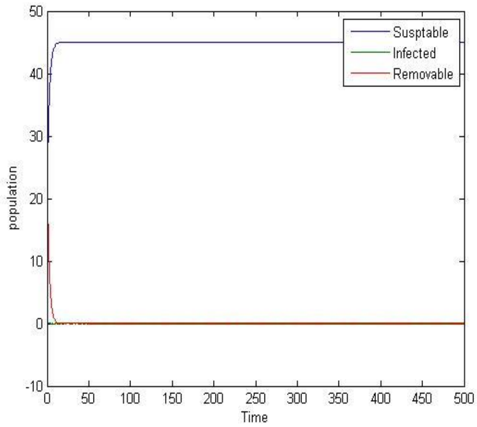

Fig:6(a)

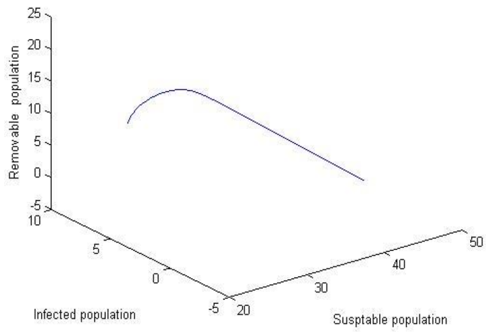

Fig:6(b)

Example 7: b =0.04, r = 0.05, β = 3 , x=30,y=10, z =5.

The quite intersecting fact is that the susceptible population almost vanishes since the recovery rate is low and transmission of disease is high hence infected class and removable populations are increases from its initial population strengths and after certain time period t=20 , the growth rate is linear and attain equilibrium to the point E(0,20,25)

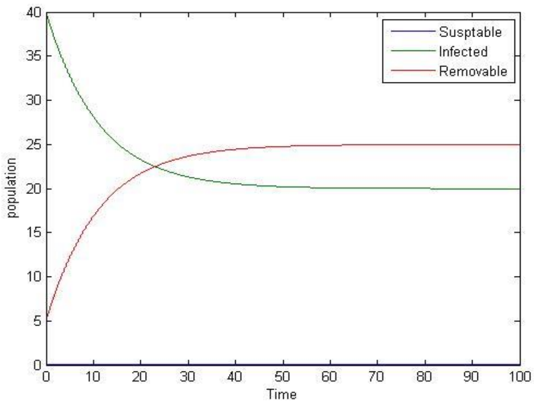

Fig:7(a)

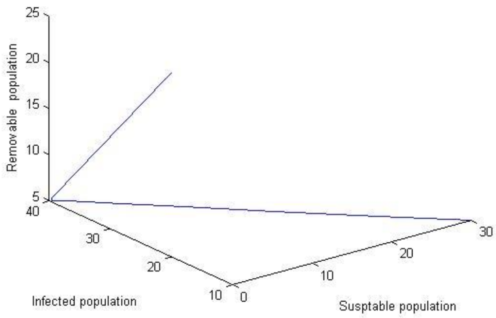

Fig:7(b)

Example8: b=0.2,r = 0.01 ,β=0.02 ,x = 80,y =40,z=20.

The disease transmission rate is high and low recovery rate makes the infected population is high and susceptible and recovery rates are low and converging to the equilibrium point E(10,124,6) shown in the following graphs

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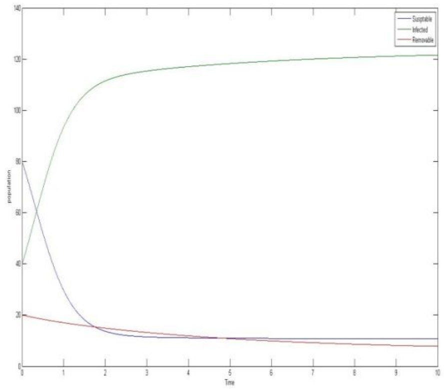

Fig;8(a)

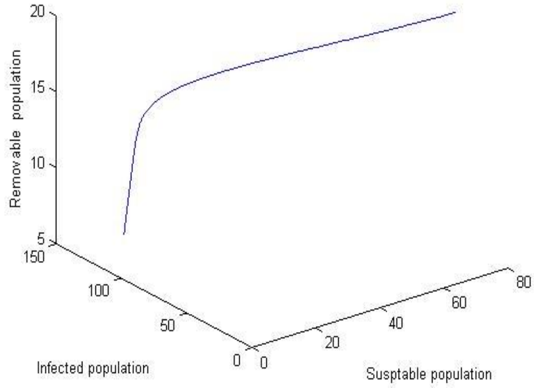

Fig:8(b)

Example9: b = 2, r = 0.5, β=1, x=30, y =10, z =5

The above parametric values the system dynamics shows that the infected population is high due to the transmission coefficient is high comparatively with recovery coefficient. The susceptible and recovery populations exhibit linear growth and tend to equilibrium point E(2, 35,8) .The result is shown in the following graphs

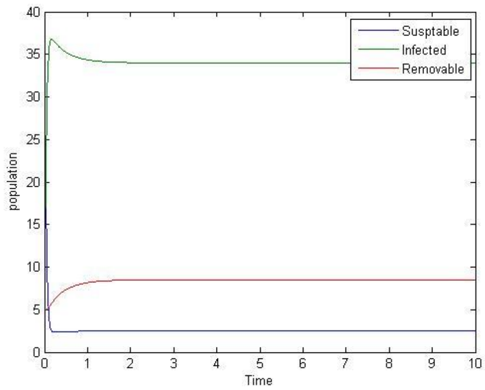

Fig:9(a)

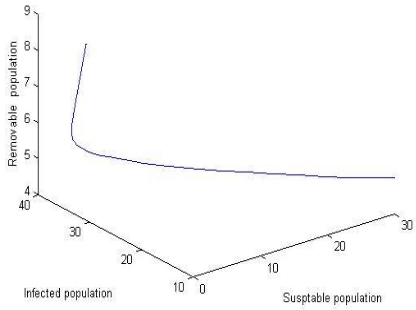

Fig:9(b)

Example10: b=0.5, r = 0.5 , β=0.5 ,x=30,y=10,z = 5.

All the interaction coefficients are equal , the system is stable and converging to the equilibrium point E(3, 21,21) . After t = 10 the infected and removable populations are coincide which shows linear growth.

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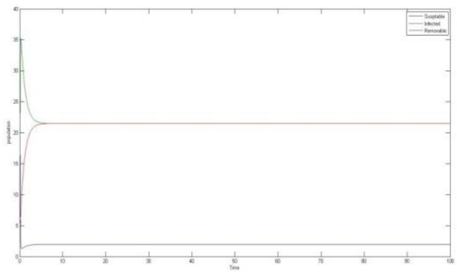

Fig:10(a)

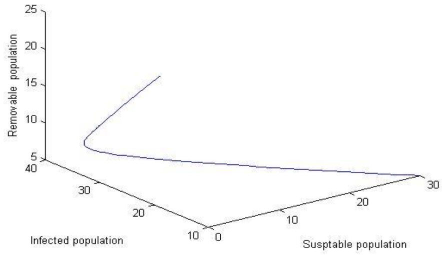

Fig:11(b)

7. Conclusion

We analyze the stability of directly transmitted viral micro parasite model consist of susceptible (x), invectives (y) and immune (z) populations. Here the total population (N) is kept constant by births and deaths. The model is characterized by the system of nonlinear ordinary differential equations. The model admits three equilibrium points and admits unique endemic equilibrium point if the basic reproduction rate R_0 exceeds one. We establish endemic equilibrium point is locally asymptotically stable if $\bar{y} > \bar{x}$. Moreover we show that the system is globally asymptotically stable. From the Numerical simulation for different parametric values, the following observations are made .

(1) If the parametric relation is r < b for fixed population size with x ,y z , then the infected population is high comparatively with susceptible and removable populations

(2) If the parametric relation is r > b for fixed population size with x ,y z , then the removable population is high comparatively with susceptible and infected populations

(3) If N < Nc = $\frac{b+r}{\beta}$, The parasite cannot maintain itself in the population eventually dies out the removable and infective class i.e only susceptible population exist . In other words , there is spread of infection in the population

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