FUZZY LOGIC

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Abstract

Fuzzy logic is a superset of conventional (Boolean) logic that has been extended to handle the concept of partial truth -- truth values between "completely true" and "completely false". It was introduced by Dr. Lotfi Zadeh of UC/Berkeley in the 1960's as a means to model the uncertainty of natural language.

Zadeh says that rather than regarding fuzzy theory as a single theory, we should regard the process of ``fuzzification'' as a methodology to generalize ANY specific theory from a crisp (discrete) to a continuous (fuzzy) form. Thus recently researchers have also introduced "fuzzy calculus", "fuzzy differential equations", and so on.

Key Words: Fuzzy subsets, Boolean logic, truth-truth values, completely true, completely false statements

Introduction

Fuzzy Subsets: Just as there is a strong relationship between Boolean logic and the concept of a subset, there is a similar strong relationship between fuzzy logic and fuzzy subset theory.

Preliminaries: In classical set theory, a subset U of a set S can be defined as a mapping from the elements of S to the elements of the set $\{0, 1\}$, $U: S \rightarrow \{0, 1\}$

This mapping may be represented as a set of ordered pairs, with exactly one ordered pair present for each element of S. The first element of the ordered pair is an element of the set S, and the second element is an element of the set $\{0, 1\}$. The value zero is used to represent non-membership, and the value one is used to represent membership. The truth or falsity of the statement x is in U is determined by finding the ordered pair whose first element is x. The statement is true if the second element of the ordered pair is 1, and the statement is false if it is 0.

Similarly, a fuzzy subset F of a set S can be defined as a set of ordered pairs, each with the first element from S, and the second element from the interval [0,1], with exactly one ordered pair present for each element of S. This defines a mapping between elements of the set S and values in the interval [0,1]. The value zero is used to represent complete non-membership, the value one is used to represent complete membership, and values in between are used to represent intermediate DEGREES OF MEMBERSHIP. The set S is referred to as the UNIVERSE OF DISCOURSE for the fuzzy subset F. Frequently, the mapping is described as a function, the MEMBERSHIP FUNCTION of F. The degree to which the statement x is in F is true is determined by finding the ordered pair whose first element is x. The DEGREE OF TRUTH of the statement is the second element of the ordered pair.

In practice, the terms "membership function" and fuzzy subset get used interchangeably.

That's a lot of mathematical baggage, so here's an example. Let's talk about people and "tallness". In this case the set S (the universe of discourse) is the set of people. Let's define a fuzzy subset TALL,

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which will answer the question "to what degree is person x tall?" Zadeh describes TALL as a LINGUISTIC VARIABLE, which represents our cognitive category of "tallness". To each person in the universe of discourse, we have to assign a degree of membership in the fuzzy subset TALL. The easiest way to do this is with a membership function based on the person's height.

tall(x) = { 0, if height(x) < 5 ft.,

        (height(x)-5ft.)/2ft., if 5 ft. <= height (x) <= 7 ft.,

        1, if height(x) > 7 ft. }

Given this definition, here are some example values:

Person	Height	degree of tallness
Billy	3' 2"	0.00 [I think]
Yoke	5' 5"	0.21
Drew	5' 9"	0.38
Erik	5' 10"	0.42
Mark	6' 1"	0.54
Kareem	7' 2"	1.00 [depends on who you ask]

Expressions like "A is X" can be interpreted as degrees of truth,

e.g., "Drew is TALL" = 0.38.

Note: Membership functions used in most applications almost never have as simple a shape as tall(x). At minimum, they tend to be triangles pointing up, and they can be much more complex than that. Also, the discussion characterizes membership functions as if they always are based on a single criterion, but this isn't always the case, although it is quite common. One could, for example, want to have the membership function for TALL depend on both a person's height and their age (he's tall for his age). This is perfectly legitimate, and occasionally used in practice.

It's referred to as a two-dimensional membership function, or a "fuzzy relation". It's also possible to have even more criteria, or to have the membership function depend on elements from two completely different universes of discourse.

Logic Operations:

Now that we know what a statement like "X is LOW" means in fuzzy logic, how do we interpret a statement like   X is LOW and Y is HIGH or (not Z is MEDIUM)

The standard definitions in fuzzy logic are:

truth (not x)   = 1.0 - truth (x)

truth (x and y) = minimum (truth(x), truth(y))

truth (x or y) = maximum (truth(x), truth(y))

Some researchers in fuzzy logic have explored the use of other interpretations of the AND and OR operations, but the definition for the NOT operation seems to be safe.

Note that if you plug just the values zero and one into these definitions, you get the same truth tables as you would expect from conventional Boolean logic. This is known as the **EXTENSION PRINCIPLE**, which states that the classical results of Boolean logic are recovered

from fuzzy logic operations when all fuzzy membership grades are restricted to the traditional set {0, 1}. This effectively establishes fuzzy subsets and logic as a true generalization of classical set theory and logic. In fact, by this reasoning all crisp (traditional) subsets ARE fuzzy subsets of this very special

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type; and there is no conflict between fuzzy and crisp methods.

Some examples -- assume the same definition of TALL as above, and in addition, assume that we have a fuzzy subset OLD defined by the membership function:

old (x) = { 0, if age(x) < 18 yr.

(age(x)-18 yr.)/42 yr., if 18 yr. <= age(x) <= 60 yr.

1,   if age(x) > 60 yr. }

And for compactness, let

   a = X is TALL and X is OLD

   b = X is TALL or X is OLD

   c = not (X is TALL)

Then we can compute the following values.

height	age	X is TALL	X is OLD	a	b	c
3' 2"	65	0.00	1.00	0.00	1.00	1.00
5' 5"	30	0.21	0.29	0.21	0.29	0.79
5' 9"	27	0.38	0.21	0.21	0.38	0.62
5' 10"	32	0.42	0.33	0.33	0.42	0.58
6' 1"	31	0.54	0.31	0.31	0.54	0.46
7' 2"	45	1.00	0.64	0.64	1.00	0.00
3' 4"	4	0.00	0.00	0.00	0.00	1.00

For those of you who only grok the metric system, here's a dandy little conversion table:

Feet+Inches = Meters

   3' 2"   0.9652

   3' 4"   1.0160

   5' 5"   1.6510

   5' 9"   1.7526

5' 10"   1.7780

   6' 1"   1.8542

   7' 2"   2.1844

Where is fuzzy logic used?

fuzzy logic is used directly in very few applications. The Sony PalmTop apparently uses a fuzzy logic decision tree algorithm to perform handwritten (well, computer lightpen) Kanji character recognition.Most applications of fuzzy logic use it as the underlying logic system for fuzzy expert systems

What is a fuzzy expert system?

A fuzzy expert system is an expert system that uses a collection of fuzzy membership functions and rules, instead of Boolean logic, to reason about data. The rules in a fuzzy expert system are usually of a form similar to the following:

if x is low and y is high then z = medium

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where x and y are input variables (names for know data values), z is an output variable (a name for a data value to be computed), low is a membership function (fuzzy subset) defined on x, high is a membership function defined on y, and medium is a membership function defined on z.

The antecedent (the rule's premise) describes to what degree the rule applies, while the conclusion (the rule's consequent) assigns a membership function to each of one or more output variables. Most tools for working with fuzzy expert systems allow more than one conclusion per

rule. The set of rules in a fuzzy expert system is known as the rulebase or knowledge base.

The general inference process proceeds in three (or four) steps.

1. Under FUZZIFICATION, the membership functions defined on the input variables are applied to their actual values, to determine the degree of truth for each rule premise.

2. Under INFERENCE, the truth value for the premise of each rule is computed, and applied to the conclusion part of each rule. This results in one fuzzy subset to be assigned to each output variable for each rule. Usually only MIN or PRODUCT are used as inference rules. In MIN inferencing, the output membership function is clipped off at a height corresponding to the rule premise's computed degree of truth (fuzzy logic AND). In PRODUCT inferencing, the output membership function is scaled by the rule premise's computed degree of truth.

3. Under COMPOSITION, all of the fuzzy subsets assigned to each output variable are combined together to form a single fuzzy subset for each output variable. Again, usually MAX or SUM are used. In MAX composition, the combined output fuzzy subset is constructed by taking the point wise maximum over all of the fuzzy subsets assigned to variable by the inference rule (fuzzy logic OR). In SUM composition, the combined output fuzzy subset is constructed by taking the point wise sum over all of the fuzzy subsets assigned to the output variable by the

inference rule.

4. Finally is the (optional) DEFUZZIFICATION, which is used when it is useful to convert the fuzzy output set to a crisp number. There are more defuzzification methods than you can shake a stick at (at least 30). Two of the more common techniques are the CENTROID and MAXIMUM

methods. In the CENTROID method, the crisp value of the output variable is computed by finding the variable value of the center of gravity of the membership function for the fuzzy value. In the MAXIMUM method, one of the variable values at which the fuzzy subset has its maximum truth value is chosen as the crisp value for the output variable.

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