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INTEGRATION IN PHYSICS

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ABSTRACT

The concepts of integration are important tools for solving real world problems .Integration analyses changes the results, and physics is more concerned with changes. Integration provides evidence for the existence of symbolic form in reasoning about differential and integrals. Integral equation arise in many scientific and engineering problems .The large class of initial and boundary value problem can be converted to integral equation and solved.

Mathematical physics models such as diffraction problems and scattering in quantum mechanics are solved using integration methods. The application of integration in physics helps to find areas when the sides are curved .Using indefinite integral displacement and velocity can be determined. Moment of inertia section explains how to find the resistance of a rotating body when the shape of the body is curved by using integration methods. Work done by the variable force can be calculated using integration .HIC (head injury criterion) based on the average value of an acceleration helps to describe the risk of head injury in a crash. By integration method force due to the liquid pressure can be solved. Work done when electric charges move towards each other (or when they are separated) can be calculated.

Categorizing the different ways in which mathematics can be used in solving physics integration problems.

Keywords: Integration ,force due to liquid pressure, Head injury criterion.

Introduction

The concepts of integration are important tools for solving real world problems .Integration analyses changes the results, and physics is more concerned with changes. A very useful application of calculus is calculating displacement ,velocity and acceleration. Applying integration to find the volume of an object having curved sides.

Instantaneous velocity

Instantaneous velocity can be determined by differentiating the expression for displacement

$$V=ds\backslash dt$$

Acceleration by differentiating the expression velocity

$$a=dv\backslash dt$$

Since integration is opposite process to differentiation, to obtain displacement's' of an object at time



t we use

$$S = \int v \, dt$$

Similarly velocity of an object

$$V = \int a \, dt$$

Ex: A car starts from rest at $s=4\text{m}$ from the origin and has acceleration at time t given by $a = 4t - 5 \text{ m/s}^2$. Find the velocity and displacement of the car at $t=5\text{s}$

Sol: $v = \int a \, dt$

$$V = \int (4t - 5) \, dt$$

$$= 2t^2 - 5t + k$$

When $t=0, v=0$ so $k=0$

So the expression for the velocity as a function of time

$$V = 2t^2 - 5t$$

When $t=4\text{s}$; $v = 2(4)^2 - 5(4) = 12 \text{ m/s}$

For finding displacement

$$S = \int v \, dt$$

$$S = \int (2t^2 - 5t) \, dt$$

$$S = 2t^3/3 - 5t^2/2 + k$$

$S=3, t=0$ so $k=3$

$$S = 2t^3/3 - 5t^2/2 + 3$$

$t=4\text{s}$

$$s = 2(4)^3/3 - 5(3)^2/2 + 3 = 23.1\text{m}$$

Volume of an object (curved sides)

Finding the volume especially when solid objects having circular cross section and curved sides by Integration and we use disk method

Disk method: to find this volume consider slices of disc each dx wide and radius y

$$V = \pi r^2 h (r=y)$$

$$V = \pi y^2 dx \text{ (for each disc)}$$

Now adding volumes of the disk

$$V = \pi \int_a^b y^2 \, dx$$

This means $v = \pi \int_a^b [f(x)]^2 \, dx$

Ex: find the volume if the area bounded by the curve $y = x^3 + 1$, the x -axis and the limit of $x=0$ and $x=3$ is rotated around the axis

$$V = \pi \int_a^b y^2 \, dx$$

$$= \pi \int_0^3 (x^3 + 1)^2 \, dx$$

$$= 1118.2 \text{ units}^3$$

Moment of inertia

The moment of inertia is a measure of resistance of a rotating body to a change in motion

The moment of inertia of a particle of mass m rotating about a particular point is given by

$$\text{Moment of inertia} = md^2$$



If a group of particles

$$I=(m_1+m_2+m_3+\dots+m_n)R$$

R is radius of gyration

Ex: Find the moment of inertia and radius of gyration w.r.t the origin (0,0) of a system which has masses at the pt given

M	2	3	4
Point	(-2, 0)	(-1, 0)	(1, 0)

The moment of inertia is

$$I=2(-2)^2+3(-1)^2+4(1)^2=15gm\text{-}cm^2$$

To find R

$$I=(m_1+m_2+m_3+\dots+m_k)R^2$$

$$15=(2+3+4)R^2=1.3$$

Work done by a variable force :

If the force varies (eg compressing a spring) we need to calculate to find to work done

$$W=\int_a^b f(x)dx$$

Ex Find the work done on a spring when you compress it from its natural length of 1m to a length of 1m to a length of 0.75 of the spring constant k=18N/m

Using the formula F=kx

$$W=\int_0^{0.25} 18x dx$$

$$=0.5Nm$$

HIC (head injury criterion)

Which is based on the average value of acceleration ove the critical part of the deceleration?

$$\text{The average value } a=1/(t_2-t_1)\int_{t_1}^{t_2} a(t)dt$$

For HIC this was modified based on the experimental data

$$(t_2-t_1)[1/t_2-t_1\int a(t) dt]^{2.5}$$

Where t₁ and t₂ are the initial and final limits of interval during which HIC attains a maximum value and acceleration is measured in gs (standard gravity acceleration).The maximum time duration of HIC, (t₂-t₁) is limited to a specific value between 3 and 36ms.This means that the HIC includes the effects of head acceleration and the duration of the acceleration .Large acceleration may be evaluated for very short time.

Force due to liquid pressure

$$F=w\int_a^b xydy$$

Where x is the length (in m) of the element of area Y is the depth (in m) of the element of area W is the density of the liquid (in Nm⁻³) a =depth at top of the area in question (in m),b = depth at the bottom of the area in question(in m)

References

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