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FIXED POINT THEOREMS FOR WEAK K-QUASI CONTRACTIONS ON A GENERALIZED METRIC SPACE

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ABSTRACT
In this paper we obtain conditions for a k- quasi contraction on a generalized metric space to have a fixed point. Using this we derive certain known results as corollaries.

Key *Words*: Generalized metric space, weak k-quasi contraction, fixed point.
AMS: Subject classification (2010): 54H25, 47H10.

1. Introduction

The concept of metric was introduced by Frechet [10] as an extension of the distance on the real line. Banach contraction principle was given a shape in the context of metric spaces. Later several generalizations of Banach contraction principle were obtained. Also many generalizations of metric spaces were obtained and Banach contraction principle was extended to such spaces. Some of the generalizations of Banach contraction principle were also extended to the generalized versions of metric spaces. In recent years, several generalizations of metric spaces have appeared. In 1993, Czerwik [8] introduced the concept of a b-metric space. Since then, several works have dealt with fixed point theory in such spaces; see [4,5,9,11,15,16] and references therein. In 2000, Hitzler and Seda [12] introduced the notion of dislocated metric spaces in which self distance of a point need not be equal to zero. Such spaces play a very important role in topology and logical programming. Fixed point theory in dislocated metric spaces was studied extensively in [2,6,11,14]. Combining several generalizations of metric spaces Jleli and Samet [13] obtained a new generalization in 2015. They termed it as a generalized metric space. Jleli and Samet extended many results available in fixed point theory such as Banach contraction principle and Ciric [6].

In this paper we deal with fixed point results in generalized metric spaces, extend Ciric theorem [6] in generalized metric spaces with less stringent conditions and obtain Banach contraction principle in generalized metric spaces as a corollary.

2. Preliminaries

In this section we give the definition of a generalized metric space, obtain certain properties of generalized metric which we use in the later development. We also give some examples of generalized metric spaces. We give various definitions of generalizations of metric spaces which are included in the notion of generalized metric spaces.

2.1 Definition

A metric on a non- empty set X is a mapping $d : X \times X \to [0,+\infty)$ satisfying the following conditions:

$(2.1.1)$ for every $(x, y) \in X \times X$,we have

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$d(x,y)=0 \Leftrightarrow x=y;$

$(2.1.2)$ for every $(x,y)\in X\times X$,we have

$d(x,y)=d(y,x);$

$(2.1.3)$ for every $(x,y,z)\in X\times X\times X$, we have

$d(x,y)\le d(x,z)+d(z,y).$

2.2 Definition:- (Czerwik[8]) Let X be a non-empty set and $d: X\times X \to [0,+\infty)$ be a given mapping.

We say that d is a b-metric on X if it satisfies the following conditions:

$(2.2.1)$ for every $(x,y)\in X\times X$,

$d(x,y)=0 \Leftrightarrow x=y;$

$(2.2.2)$ for every $(x,y)\in X\times X$,we have

$d(x,y)=d(y,x);$

$(2.2.3)$ there exists $s\ge 1$ such that ,for every $(x,y,z)\in X\times X\times X$, we have

$d(x,y)\le s[d(x,z)+d(z,y)]$

In this case , (X,d) is said to be a b-metric space.

The concept of convergence in such spaces is similar to that of metric spaces.

2.3 Definition:-(Hitzler and Seda [12]) Let X be a non-empty set and $d: X\times X \to [0,+\infty)$ be a given mapping.

We say that d is a dislocated metric on X if it satisfies the following conditions:

$(2.3.1)$ for every $(x,y)\in X\times X$,

$d(x,y)=0 \Rightarrow x=y;$

$(2.3.2)$ for every $(x,y)\in X\times X$,we have

$d(x,y)=d(y,x);$

$(2.3.3)$ for every $(x,y,z)\in X\times X\times X$,we have

$d(x,y)\le d(x,z)+d(z,y).$

In this case (X,d) is said to be a dislocated metric space.

The motivation of defining this new notion is to get better results in logic programming semantics.

The concept of convergence in such spaces is similar to that of metric spaces.

2.4 Notation (Jleli and Samet [13]) : Let X be a non-empty set and $D: X\times X \to [0,+\infty]$ be a given mapping. For every $x\in X$, let us define the set

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$C(D,X,x)=\left\{\{x_n\}\subset X \ \lim_{n\to\infty} D(x_n,x)=0\right\}.$

2.5 Definition: (Jleli and Samet [13])

We say that D is a generalized metric on X if it satisfies the following conditions:

$(2.5.1)$ for every $(x,y)\in X\times X$,

$D(x,y)=0\Rightarrow x=y;$

$(2.5.2)$ for every $(x,y)\in X\times X$,we have

$D(x,y)=D(y,x);$

$(2.5.3)$ there exists $\lambda>0$ such that

If $(x,y)\in X\times X, \{x_n\}\in C(D,X,x)$, then $D(x,y)\le\lambda \limsup_{n\to\infty} D(x_n,y)$.

In this case , we say the pair (X,D) is a generalized metric space.

We also say that λ is a coefficient of X . Thus we say that (X,D) is a generalized metric space with coefficient λ. In general we drop λ.

(i) Remark :- Obviously, if the set $C(D,X,x)$ is empty for every $x\in X$,then (X,D) is a generalized metric space if and only if $(2.5.1)$ and $(2.5.2)$ are satisfied.

(ii) Remark :- It may be observed that metrc spaces, b-metric spaces and dislocated metric spaces are included in the class of generalized metric spaces.

2.6 Definition :- Let (X,D) be a generalized metric space. Let $\{x_n\}$ be a sequence in X and $x\in X$. We say that $\{x_n\}$ D-converges to x if $\{x_n\}\in C(D,X,x)$.

2.7 Proposition :- Let (X,D) be a generalized metric space. Let $\{x_n\}$ be a sequence in X and $(x,y)\in X\times X$. If $\{x_n\}$ D-converges to x and $\{x_n\}$ $D-$converges to y then $x=y$.

Proof:- Using the property $(2.5.3)$, we have $D(x,y)\le\lambda \limsup_{n\to\infty} D(x_n,y)$ = 0,

which implies from the property $(2.5.1)$ that $x=y$.

2.8 Definition :- Let (X,D) be a generalized metric space. Let $\{x_n\}$ be a sequence in X . We say that $\{x_n\}$ is a D-Cauchy sequence if $\lim_{m,n\to\infty} D(x_n,x_{n+m})=0$.

2.9 Definition :- Let (X,D) be a generalized metric space. It is said to be D-complete if every D-Cauchy sequence in X is convergent to some element in X .

2.10 Definition:- Let $f:X\to X$ be a self map and $x\in X$. Write $f^1(x)=f(x)$ and $f^{n+1}(x)=f(f^n(x))$ for n= 1, 2, 3, ... Then $\{f^n(x)\}$ is called the sequence of iterates of f at x .

Jleli and Samet [13] extended Banach contraction principle to generalized metric spaces as follows.

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2.11 Theorem :- (Jleli and Samet [13] ,Proposition 3.2)(Banach contraction principle for generalized metric spaces).

Let (X,D) be a complete generalized metric space and $f:X\to X$

be such that $D\big(f(x),f(y)\big)\le k\,D(x,y)$ for some $k\in[0,1)$ and for all $x,y\in X$.

Suppose there exists $x_0\in X$ such that $\alpha=\sup_n D\big(x_0,f^n(x_0)\big)<\infty$ Then $\{f^n(x_0)\}$ converges to some w $\in X$ and w is a fixed point of *f*. Further if w' is another fixed point of *f* with D(w,w') < ∞ then w'=w.

2.12 Theorem: Let (X,D) be a generalized metric space. Suppose $\{x_n\}\subseteq X, x\in X$ and $x_n\to X$. Then $D(x,x)$=0

Proof:- We have $D(x,x)\le \lambda \limsup_{n\to\infty}$ D (x_n,x) = λ lim D (x_n,x) = λ .0 (since $x_n \to x$)= 0

Hence $D(x,x)$= 0.

2.13 Theorem :- *Let (X,D) be a generalized metric space and $x\in X$.*

Suppose $C(D,X,x)\ne\phi$. Then $D(x,x)$ = 0.

Proof :- $C(D,X,x)\ne\phi\Rightarrow\exists\{x_n\}\in C(D,X,x)\Rightarrow x_n\to x\Rightarrow D(x,x)$ = 0.

(by Theorem2.11)

2.14 Examples :

Example 1 :- *Let $\mathbb{R}$ be the set of real numbers and $X=\{x\ge 0\}$. Define $D:X\times X\to\mathbb{R}$ by*

$$D(x,y)=\begin{cases}|x-y|\ if & x \text{ and y are rational}\\ \infty, otherwise\end{cases}$$

Then (X,D) is a generalized metric space.

Example 2:- *Let X = [0,1]. Define D:X×X →$\mathbb{R}$ by*

$$D(x,y)=\begin{cases}|x-y|+1 \text{ if } x\ne y\\ 0 \text{ ,otherwise}\end{cases}$$

Then (X ,D) is a generlized metric space with λ =2.

Example 3:- Let X =[0,1] .Define $D:X\times X\to\mathbb{R}$ by

$$D(x,y)=\begin{cases}|x-y|+1 \text{ if } x\ne y\\ 2 \text{ ,otherwise}\end{cases}$$

Then (X ,D) is a generalized metric space

It may be noted that in this example 0< D(x, y)< ∞ ∀ x, y∈X and C $(D,X,x)=\phi\ \forall\ x\in X$.

Example 4:- Let X = {x ≥ 0}. Define $D:X\times X\to\mathbb{R}$ by

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$$D(x,y)=\begin{cases}|x-y|+1 \text{ if } x\neq y\\ 2x, \text{ if } x=y\end{cases}$$

Then (X,D) is a generalized metric space with $\lambda\geq 1$. In this case, if $x\neq 0$ then C (D,X,x)= ϕ. Further $0\leq D(x,x)<\infty\ \forall\ x\in X$.

Then (2.5.3) is satisfied with $\lambda\geq 1$

Hence (X,D) is a generelised metric space.

In this case $0\ \leq\ D(x,\ x)<\infty\ \forall\ x\in X$.

3 Main Results:

In this section we define k-quasi contraction on a generalized metric space. We obtain conditions for a k-quasi contraction on a generalized metric space to have a fixed point, thus extending the result of Ciric [6] to generalized metric spaces. Rhoades [17] introduced R-type contraction for non self mappings of a closed subset of a Banach space X into X and proved fixed point results using such contraction. CIRIC [7] also obtained results for R-type contractions. This notion of R-type contraction is extended to a generalized metric spaces for self maps by Vats et al.[18] We also show that corresponding result for R-type contraction follows as a corollary.

3.1 Definition:- Let (X,D) be a generalized metric space and $f:X\to X$ be a self map. Suppose $0\leq k<1$.

We say that

(i) f is a k-contraction if $D(fx,fy)\leq k\ D(x,y)\forall x,y\in X$.

(ii) f is a R-type contraction if $\exists\ q>0$ with $q\geq 1+2k$ and

$$D(fx,fy)\leq k\ \max\left\{\frac{1}{2}D(x,y),D(x,fx),D(y,fy),\frac{D(x,fy),D(y,fx)}{q}\right\}\forall\ x,y\in X,x\neq y$$

(iii) f is a k-quasi contraction if for some $k\in(0\ \ )$,

$$D(fx,fy)\leq k\ \max\left\{D(x,y),D(x,fx),D(y,fy),D(x,fy),D(y,fx)\right\}\forall\ x,y\in X$$

We observe that every k-contraction is a k-quasi contraction.

Now we state and prove our main theorem, which is an extension of a result of Ciric [6] to generalized metric spaces.

3.2 Theorem:- Suppose that the following conditions hold:

(i) (X,D) is complete generalized metric space, with coefficient λ .

(ii) f is a k-quasi contraction for some $k\in(0,1)$ with $k\lambda<1$.

(iii) there exists $x_0\in X$ such that $\alpha=\sup_n\ D(x_0,f^n(x_0))<\infty$ (1)

and $D(f^n(x_0),f^{n+1}(x_0))<k^n\alpha$ for $n=0,1,2,...$ (2)

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Then $\left\{f^{n}(x_0)\right\}$ converges to some $w \in X$. If $\limsup_n D\left(f^{n}(x_0), f(w)\right) < \infty$ then w is a fixed point of f. Moreover , if $w^1 \in X$ is also fixed point of f such that $D\left(w, w^1\right) < \infty$ and $D\left(w^1, w^1\right) < \infty$, then $w = w^1$.

Proof:- We first show that $D\left(f^{n}(x_0), f^{n+m}(x_0)\right) < k^n\alpha$ for $n = 0,1,2,\ldots$ and m=0,1,2,... The result is true if n=0 by (1)

Now assume the truth for $n.(i.e)$ $D\left(f^{n}(x_0), f^{n+m}(x_0)\right) < k^n\alpha$ for $m = 0,1,2,\ldots$ (3)

We show $D\left(f^{n}(x_0), f^{n+m}(x_0)\right) < k^{n+1}\alpha$ for $m = 0,1,2,\ldots$ (4)

Now consider

$$D\left(f^{n+1}(x_0), f^{n+1}(x_0)\right) \le k \max \left\{D\left(f^{n}(x_0), f^{n}(x_0)\right), D\left(f^{n}(x_0), f^{n+1}(x_0)\right), D\left(f^{n}(x_0), f^{n+1}(x_0)\right),\right.$$
$$\left. D\left(f^{n}(x_0), f^{n+1}(x_0)\right), D\left(f^{n}(x_0), f^{n+1}(x_0)\right)\right\}$$
$$\le k\; k^n\alpha \;\ldots\ldots\ldots \text{ by } (3)$$
$$= k^{n+1}\alpha$$

From this it follows that (4) is true when m= 0

Assume the truth of (4) form. (5)

Now

$$D\left(f^{n+1}(x_0), f^{n+1+m+1}(x_0)\right) \le k \max \left\{D\left(f^{n}(x_0), f^{n+m+1}(x_0)\right), D\left(f^{n}(x_0), f^{n+1}(x_0)\right),\right.$$

$$\left. D\left(f^{n+m+1}(x_0), f^{n+1+m+1}(x_0)\right), D\left(f^{n}(x_0), f^{n+1+m+1}(x_0)\right), D\left(f^{n+m+1}(x_0), f^{n+1}(x_0)\right)\right\}$$

$\le k \max\left\{k^n\alpha, k^n\alpha, k^{n+1+m}\alpha, k^n\alpha, k^{n+1}\alpha,\right\}$ .... by (3) and (5)

$$= k.k^n\alpha = k^{n+1}\alpha$$

Hence (4) holds for m=0,1,2,...

Consequently (3) holds (i.e)

$D\left(f^{n}(x_0), f^{n+m}(x_0)\right) \le k^n\alpha$ for n=0,1,2,... and m=0,1,2,...

This shows that $\left\{f^{n}(x_0)\right\}$ is a Cauchy sequence and hence converges to a limit, say, w.

$$D\left(f^{n+1}(x_0), f(w)\right) \le k \max\left\{D\left(f^{n}(x_0), w\right), D\left(f^{n}(x_0)\right), f^{n+1}(x_0), D\left(w, f(w)\right),\right.$$
$$\left. D\left(f^{n}(x_0), f(w)\right), D\left(w, f^{n+1}(x_0)\right)\right\}$$

Let $\in > 0$. Then there exists N such that $D\left(f^{n}(x_0), w\right) < \in$ for $n \ge N$.

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Hence for $n \geq N$ we have

$D\left(f^{n+1}(x_0), f(w)\right) \leq k \max\left\{\in, k^n\alpha, \lambda \limsup_n D\left(f^n(x_0), f(w)\right), D\left(f^n(x_0), f(w)\right), \in\right\}$ ....(6)

Suppose $k\lambda < 1$,

Threfore $\limsup_n D\left(f^{n+1}(x_0), f(w)\right) \leq \max\{k\in, 0, k\lambda \limsup_n D\left(f^n(x_0), f(w)\right),$

$k \limsup_n D\left(f^n(x_0), f(w)\right), k\in\}$ (from (6))

$\leq \max\{\max\{(k, k\lambda)\}.\limsup_n D\left(f^n(x_0), f(w)\right), k\in\}$ (since $k\lambda < 1$ and $k < 1$)

Therefore $\limsup_n D\left(f^n(x_0), f(w)\right)$=0 (since $\in$ is arbitrary and

$\limsup_n D\left(f^n(x_0), f(w)\right) < \infty$,(by hypothesis)

$\therefore f^n(x_0) \to f(w)$

$\therefore f(w) = w$

Therefore $D(w, w) = 0$.

Suppose w^1 is also a fixed point of f with $D(w, w^1) < \infty$ and $D(w^1, w^1) < \infty$.

Then $D(w, w^1) = D\left(f(w), f(w^1)\right)$

$\leq k \max\left\{D(w, w^1), D(w, f(w)), D(w^1, f(w^1)), D(w, f(w^1)), D(w^1, f(w))\right\}$

$= k \max\left\{D(w, w^1), D(w, w), D(w^1, w^1), D(w, w^1), D(w^1, w)\right\}$

$= k \max\left\{D(w, w^1), 0, D(w^1, w^1)\right\}$

$= k \max\left\{D(w, w^1), 0, 0\right\}$

(w^1 being a fixed point)

$= k\, D(w, w^1)$

$\therefore D(w, w^1)$ = 0.

Therefore $w = w^1$

Note : Jleli and Samet [13] claimed the above theorem without the condition $k\lambda < 1$ but imposing stronger conditions than(1) and (2) of (iii).

3.3 Corollary:- Banach contraction principle for generalized metric spaces (Theorem 2.11) is a simple corollary of Theorem 3.1 .

3.4 Theorem:- Let (X, D) be complete generalized metric space and $f : X \to X$.

Suppose the following conditions hold

(3.4.1) f is a R-type contraction (i.e) $\exists\, k \in [0,1]$ and $q > 0$ with $q \geq 1 + 2k$ and $k\lambda < 1$ such that

$$D(fx, fy) \le k \max\left\{\frac{1}{2}D(x,y), D(x,fx), D(y,fy), \frac{\left(D(x,fy)+D(y,fx)\right)}{q}\right\} \text{for all } x, y \in X$$

(3.4.2) $q \ge 2$ and $k\lambda < 1$

(3.4.3) There exists $x_0 \in X$ such that

$$\alpha = \sup_n D\left(x_0, f^n(x_0)\right) < \infty$$

and $D\left(f^n(x_0), f^{n+1}(x_0)\right) \le k^n \alpha$ for $n = 0,1,2,\ldots$

Then $\left\{f^n(x_0)\right\}$ converges to some $w \in X$.

If $\limsup_n \left(f^n(x_0), f(w)\right) < \infty$ then w is a fixed point of f .

Moreover if $w^1 \in X$ is another fixed point of f such that

$D\left(w, w^1\right) < \infty$ and $D\left(w^1, w^1\right) < \infty$ then $w^1 = w$.

Proof : Under the given conditions (3.4.1), (3.4.2) and (3.4.3) follows that f is a $k-quasi$ contraction and hence the conclusion follows from Theorem 3.2

Note: VATS et al. [19] claimed the above theorem without condition (3.4.2), but the proof is defective.

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