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EFFECT OF QUADRATIC DENSITY TEMPERATURE VARIATION ON CONVECTIVE HEAT TRANSFER FLOW IN VERTICAL CHANNEL

[1]Dr.B.SUSEELAVATHY, [2]Y.ANITHA, [3]Prof.D.R.V.PRASADA RAO
[1]Department of Mathematics, S.S. & N College, Narasaraopet.[2]J.K.C College Guntur Dist, A.P, India. Email:lathabyneni@gmail.com
[3]Department of Mathematics, S.K.University, Anantapur, A.P., India
Email : **drv_atp@yahoo.in**

ABSTRACT

We investigate the study convective heat transfer flow of a viscous electrically conducting fluid in a vertical wavy channel under the influence of an inclined magnetic fluid with heat generating sources. The walls of the channels are maintained at constant temperature. The equations governing the flow and heat are solved by employing perturbation technique with the slope δ of the wavy wall as a perturbation parameter. The velocity and temperature distributions are investigated for different values of G, M, m, N, N_1, α and x. The rate of heat transfer is numerically evaluated for different variations of the governing parameters.

Keywords: Vertical Wavy Channel, Quadratic Density Temperature Variation, Hall Effects, Radiation

1.INTRODUCTION

It has been established [9] that channels with diverging – converging geometries augment the transportation of heat transfer and momentum. As the fluid flows through a tortuous path viz., the dilated – constricted geometry, there will be more intimate contact between them. The flow takes place both axially (primary) and transversely (secondary) with the secondary velocity being towards the axis in the fluid bulk rather than confining within a thin layer as in straight channels. Hence it is advantageous to go for converging-diverging geometries for improving the design of heat transfer equipment. Keeping these applications in view several authors (23,21,22,10,5,12,20,6,9,2,7) have studies heat transfer in wavy channel under vertical conditions.

In all these investigations, the effects of Hall currents are not considered. However, in a partially ionized gas, there occurs a Hall current [3] when the strength of the impressed magnetic field is very strong. These Hall effects play a significant role in determining the flow features. Sato [14],Sherman and Sutton [17] have discussed the Hall effects on the steady hydromagnetic flow between two parallel plates. These effecs in the unsteady cases were discussed by Pop [11]. Debnath [4] has studied the effects of Hall currents on unsteady hydromagnetic flow past a porous plate in a rotating fluid system and the structure of the steady and unsteady flow is investigated. Alam *et. al.,*[1] have studied unsteady free convective heat and mass transfer flow in a rotating system with Hall currents, viscous dissipation and Joule heating. Taking Hall effects in to account Krishan *et. al.,*[8] have investigated Hall effects on the unsteady hydromagnetic boundary layer flow. Rao *et. al.,* [12] have analyzed Hall effects on unsteady Hydromagnetic flow. Siva Prasad *et. al.,* [19] have studied Hall effects on unsteady MHD free and forced convection flow in a porous rotating channel. Recently Seth *et. al.,* [15] have investigated the effects of Hall currents on heat transfer in a rotating MHD channel

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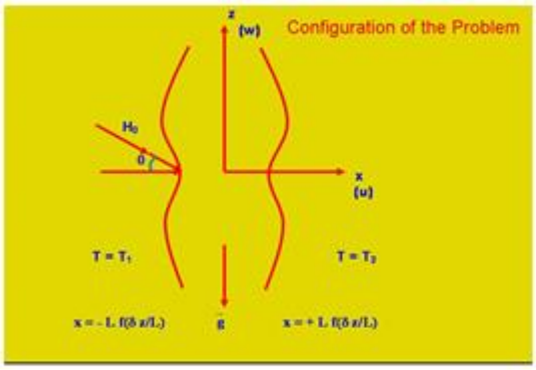
flow in arbitrary conducting walls. Sarkar *et. al.,* [13] have analyzed the effects of mass transfer and rotation and flow past a porous plate in a porous medium with variable suction in slip flow region. Reddy(18) has studied the effect of Hall currents on the free convection flow of a viscous fluid in a vertical channel with asymmetric temperatures .

In this paper we investigate the study convective heat transfer flow of a viscous electrically conducting fluid in a vertical wavy channel under the influence of an inclined magnetic fluid with heat generating sources. The walls of the channels are maintained at constant temperature. The equations governing the flow and heat are solved by employing perturbation technique with the slope δ of the wavy wall as a perturbation parameter. The velocity and temperature distributions are investigated for different values of G, M, m, N, N_1, α and x. The rate of heat transfer is numerically evaluated for different variations of the governing parameters.

2.FORMULATION AND SOLUTION OF THE PROBLEM

We consider the steady flow of an incompressible, viscous, electrically conducting fluid confined in a vertical channel bounded by two wavy walls under the influence of an inclined magnetic field of intensity Ho lying in the plane (y-z). The magnetic field is inclined at an angle α to the axial direction k and hence its components are $(0, H_0 Sin(\alpha), H_0 Cos(\alpha))$.In view of the waviness of the wall the velocity field has components (u,0,w)The magnetic field in the presence of fluid flow induces the current $(J_x, 0, J_z)$.We choose a rectangular cartesian co-ordinate system O(x,y,z) with z-axis in the vertical direction and the walls at $x = \pm f(\frac{\delta z}{L})$.



When the strength of the magnetic field is very large we include the Hall current so that the generalized Ohm's law is modified to

$$\bar{J} + \omega_e \tau_e \bar{J} x \bar{H} = \sigma(\bar{E} + \mu_e \bar{q} x \bar{H}) \quad (2.1)$$

where q is the velocity vector. H is the magnetic field intensity vector E is the electric field, J is the current density vector, ω_e is the cyclotron frequency, τ_e is the electron collision time,σ is the fluid conductivity and μ_e is the magnetic permeability. Neglecting the electron pressure gradiention-slip and thermo-electric effects and assuming the electric field E=0,equation (2.1) reduces

$$j_x - m H_0 J_z Sin(\alpha) = -\sigma\mu_e H_0 w Sin(\alpha) \quad (2.2)$$

$$J_z + m H_0 J_x Sin(\alpha) = \sigma\mu_e H_0 u\, Sin(\alpha) \quad (2.3)$$

where m=$\omega_e \tau_e$ is the Hall parameter.

On solving equations (2.2)&(2.3) we obtain

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$$j_x = \left(\frac{\sigma\mu_e H_0 Sin(\alpha)}{1+m^2H_0^2Sin^2(\alpha)}\right)(mH_0Sin(\alpha)-w) \tag{2.4}$$

$$j_z = \left(\frac{\sigma\mu_e H_0 Sin(\alpha)}{1+m^2\, H_0^2Sin^2(\alpha)}\right)(u+mH_0 w\,Sin(\alpha)) \tag{2.5}$$

where u,w are the velocity components along x and z directions respectively.
Substituting J_x and J_z from equations (2.4)&(2.5) we obtain

$$\begin{aligned}\left(u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right) &= -\left(\frac{\partial p}{\partial x}\right)+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)\\ &-\left(\frac{\sigma\mu_e H_{0\,0}^2 Sin^2(\alpha)}{1+m^2H_0^2Sin^2(\alpha)}\right)(u+mH_0wSin(\alpha))\end{aligned} \tag{2.6}$$

$$\begin{aligned}\left(u\frac{\partial W}{\partial x}+w\frac{\partial W}{\partial z}\right) &= -\left(\frac{\partial p}{\partial z}\right)+\mu\left(\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial z^2}\right)-\\ &-\left(\frac{\sigma\mu_e H_0^2 Sin^2(\alpha)}{1+m^2H_0^2Sin^2(\alpha)}\right)((w-mH_0uSin(\alpha))-\rho g\end{aligned} \tag{2.7}$$

The energy equation is

$$\rho C_p\left(u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}\right)=k_f\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)-\left(\frac{\partial(q_r)}{\partial x}\right) \tag{2.8}$$

The equation of state is

$$\rho-\rho_0=-\beta_0(T-T_o)-\beta_1(T-T_o)^2 \tag{2.9}$$

Where T, is the temperature and concentration in the fluid.k_f is the thermal conductivity, Cp is the specific heat constant pressure, β is the coefficient of thermal expansion, Q is the strength of the heat source and q_r is the radiative heat flux.
Followed by Rosseland approximation(Brewester(1a))
The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$q=\frac{1}{L}\int_{-Lf}^{Lf} w\,dx \tag{2.10}$$

The boundary conditions are

u= 0 ,w=0 T=T1 , on $x=-f\left(\frac{\delta z}{L}\right)$ (2.11)

w=0, w=0, T=T_2 ,on $x=f\left(\frac{\delta z}{L}\right)$ (2.12)

Eliminating the pressure from equations(2.6)&(2.7) and introducing the Stokes Stream function ψ as

$$u=-\left(\frac{\partial\psi}{\partial z}\right),\quad w=\left(\frac{\partial\psi}{\partial x}\right) \tag{2.13}$$

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the equations (2.6)-(2.8) in terms of ψ is

$$\left(\frac{\partial\psi}{\partial z}\frac{\partial(\nabla^2\psi)}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial(\nabla^2\psi)}{\partial z}\right)=\mu\nabla^4\psi+\beta_0 g\frac{\partial(T-T_e)}{\partial x}+2\beta_1 g(T-T_e)\frac{\partial(T-T_e)}{\partial x}-\left(\frac{\sigma\mu_e^2H_0^2Sin^2(\alpha)}{1+m^2H_0^2\,Sin^2(\alpha)}\right) \quad (2.14)$$

$$\rho C_p\left(\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial z}-\frac{\partial\psi}{\partial z}\frac{\partial T}{\partial x}\right)=k_f\left(\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial z^2}\right)+\left(\frac{16\sigma^{\bullet}T_e^3}{3\beta_R}\frac{\partial^2T}{\partial x^2}\right) \quad (2.15)$$

On introducing the following non-dimensional variables

$$(x',z')=(x,z)/L,\ \psi'=\frac{\psi}{qL},\ \theta=\frac{T-T_2}{T_1-T_2}$$

the equation of momentum and energy in the non-dimensional form are

$$\nabla^4\psi-M_1^2\nabla^2\psi+\frac{G}{R}\left(\frac{\partial\theta}{\partial x}+2\gamma\theta\frac{\partial\theta}{\partial x}\right)=R\left(\frac{\partial\psi}{\partial z}\frac{\partial(\nabla^2\psi)}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial(\nabla^2\psi)}{\partial z}\right) \quad (2.16)$$

$$PR\left(\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial z}-\frac{\partial\psi}{\partial z}\frac{\partial\theta}{\partial x}\right)=\nabla^2\theta+\alpha+\left(\frac{4}{3N_1}\frac{\partial^2\theta}{\partial x^2}\right) \quad (2.17)$$

where

$G=\dfrac{\beta g\,\Delta T_e L^3}{\nu^2}$ (Grashof Number) $M^2=\dfrac{\sigma\mu_e^2H_o^2L^2}{\nu^2}$ (Hartman Number)

$M_1^2=\dfrac{M^2Sin^2(\alpha)}{1+m^2}$ $R=\dfrac{qL}{\nu}$ (Reynolds Number)

$P=\dfrac{\mu C_p}{K_f}$ (Prandtl Number) $N_1=\dfrac{3\beta_R K_f}{4\sigma^{\bullet}T_e^3}$ (Radiation parameter)

$\gamma=\dfrac{\beta_1(T_1-T_2)L^3}{\beta_o}$ (Density ratio)

The corresponding boundary conditions are

$$\psi(f)-\psi(-f)=1,\qquad \frac{\partial\psi}{\partial z}=0,\frac{\partial\psi}{\partial x}=0,\theta=1,\ at\ x=-f(\delta z)$$

$$\frac{\partial\psi}{\partial z}=0,\frac{\partial\psi}{\partial x}=0,\theta=0\quad at\ x=+f(\delta z)$$

3.ANALYSIS OF THE FLOW

Introduce the transformation such that

$$\bar{x}=\delta x,\frac{\partial}{\partial x}=\delta\frac{\partial}{\partial\bar{x}}\qquad \text{Then}\qquad \frac{\partial}{\partial x}\approx O(\delta)\to\frac{\partial}{\partial\bar{x}}\approx O(1)$$

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Assuming the slope δ of the wavy boundary to be small we take

$$\psi(x,z)=\psi_0(x,y)+\delta\psi_1(x,z)+\delta^2\psi_2(x,z)+\dots\dots$$
$$\theta(x,z)=\theta_o(x,z)+\delta\theta_1(x,z)+\delta^2\theta_2(x,z)+\dots\dots\dots. \quad (3.1)$$

Let $\eta=\dfrac{x}{f(\bar{z})}$ (3.2)

Substituting (3.1) in equations (2.16)&(2.17) and using (3.2) and equating the like powers of δ the equations and the respective boundary conditions to the zeroth order are

$$\frac{\partial^2\theta_0}{\partial\eta^2}=-(\alpha_1 f^2) \quad (3.3)$$

$$\frac{\partial^4\psi_0}{\partial\eta^4}-\left(M_1^2 f^2\right)\frac{\partial^2\psi_0}{\partial\eta^2}=-\frac{Gf^3}{R}\left(\frac{\partial\theta_0}{\partial\eta}+2\gamma\theta_0\frac{\partial\theta_0}{\partial\eta}\right) \quad (3.4)$$

with

$$\psi_0(+1)-\psi_0(-1)=1$$
$$\frac{\partial\psi_0}{\partial\eta}=0,\ \frac{\partial\psi_0}{\partial\bar{z}}=0,\quad \theta_0=1 \quad at \quad \eta=-1 \quad (3.5)$$
$$\frac{\partial\psi_0}{\partial\eta}=0,\ \frac{\partial\psi_0}{\partial\bar{z}}=0,\quad \theta_0=0 \quad at \quad \eta=+1$$

and to the first order are

$$\frac{\partial^2\theta_1}{\partial\eta^2}{}_1=P_1Rf\left(\frac{\partial\psi_0}{\partial\eta}\frac{\partial\theta_0}{\partial\bar{z}}-\frac{\partial\psi_0}{\partial\bar{z}}\frac{\partial\theta_0}{\partial\eta}\right) \quad (3.6)$$

$$\frac{\partial^4\psi_1}{\partial\eta^4}-\left(M_1^2 f^2\right)\frac{\partial^2\psi_1}{\partial\eta^2}=-\frac{Gf^3}{R}\left(\frac{\partial\theta_1}{\partial\eta}+2\gamma(\theta_0\frac{\partial\theta_1}{\partial\eta}+\theta_1\frac{\partial\theta_0}{\partial\eta})\right)+$$
$$+Rf\left(\frac{\partial\psi_0}{\partial\eta}\frac{\partial^3\psi_0}{\partial\bar{z}^3}-\frac{\partial\psi_0}{\partial\bar{z}}\frac{\partial^3\psi_0}{\partial x\partial\bar{z}^2}\right) \quad (3.7)$$

with

$$\psi_1(+1)-\psi_1(-1)=0$$
$$\frac{\partial\psi_1}{\partial\eta}=0,\ \frac{\partial\psi_1}{\partial\bar{z}}=0,\quad \theta_1=0, \quad at \quad \eta=-1 \quad (3.8)$$
$$\frac{\partial\psi_1}{\partial\eta}=0,\ \frac{\partial\psi_1}{\partial\bar{z}}=0,\quad \theta_1=0\ .at \quad \eta=+1$$

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