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COMMON FIXED POINT THEOREMS FOR TWO SELF MAPS ON A PARTIALLY ORDERED b-METRIC-LIKE SPACES

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ABSTRACT
In this paper we establish common fixed point theorems fortwo self maps on a partially ordered b-metric-like spaces. Incidentally we obtain results of Nannan Fang [12] as corollaries.
Keywords: fixed point, b-metric-like space, partially ordered set, altering distance function.
Mathematics Subject Classification (2010): Primary 54H25; Secondary 47H10, 54C30

Introduction and Preliminaries

Many generalizations of metric spaces have been introduced [1-5] and were studied with reference to fixed point theorem. The concept of b-metric spaces was introduced by Bahtin[6] and was used by Czersik[7] to study contraction mapping in b-metric spaces. Harandi[8] introduced the notion of metric like spaces and studied the existence of fixed points in such spaces. Alghamdi[9] introduced the notion of b-metric like spaces as natural generalization of metric spaces and metric like spaces. NannanFang [12]obtained common fixed point theorems in ordered b-metric spaces. In this paper we further investigate the existence of common fixed point for a pair of self maps on a partially ordered b-metric-like spaces. Incidentally we obtain the results of NannanFang [12]as corollaries.

We begin with some known definitions

Definition(Bahtin[6]): A b-metric on a non empty set X is a function $d: X \times X \to [0, \infty)$ such that for all $x, y, z \in X$ and a constant $b \geq 1$ the following conditions hold.

i. $d(x, y) = 0 \iff x = y$.

ii.$d(x, y) = d(y, x)$.

iii.$d(x, y) \leq b(d(x, z) + d(z, y))$.

The pair (X, d) is a called b-metric space.

Definition(Harandi [8]): A metric-like on a non empty set X is a function $d: X \times X \to [0, \infty)$ such that for all $x, y, z \in X$ the following conditions hold.

i. $d(x, y) = 0 \Rightarrow x = y$.

ii.$d(x, y) = d(y, x)$.

iii.$d(x, y) \leq d(x, z) + d(z, y)$.

The pair (X, d) is a called metric-like space.

Definition(Alghamdi [9]):A b-metric-like on a non empty set X is a function $d: X \times X \to [0, \infty)$ such that for all $x, y, z \in X$ and a constant $b \geq 1$ the following conditions hold.

i. $d(x, y) = 0 \Rightarrow x = y$.

ii.$d(x, y) = d(y, x)$.

iii.$d(x, y) \leq b(d(x, z) + d(z, y))$.

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The pair (X, d) is a called b-metric-like space.

Definition(Alghamdi [9]):Let(X, d) be a b-metric-like space and let $\{x_n\}$ be a sequence of points of X. A point $x \in X$ is said to be limit of the the sequence $\{x_n\}$ if $\lim_{n\to\infty} d(x, x_n) = d(x, x)$ and we say that the sequence $\{x_n\}$ is convergent to x and it is denoted by $x_n \to x$as $n \to \infty$.

Definition(Alghamdi [9]):Let (X, d) be a b-metric-like space.

i.A sequence $\{x_n\}$ is said to be Cauchy iflim$_{m,n\to\infty} d(x_m, x_n)$ exists and is finite.

ii. A b-metric-like space $(X, d)$is said to be complete if every Cauchy sequence $\{x_n\}$in X converges to $x \in X$, so that

$$\lim_{m,n\to\infty} d(x_m, x_n) = d(x, x) = \lim_{n\to\infty} d(x_n, x)$$

Definition (Khan,Swaleh,Sessa[10]):A function $\varphi: [0, \infty) \to [0, \infty)$ is called an altering distance function if the following properties hold.

i.φ is continuous and non-decreasing.

ii.$\varphi(t) = 0 \Leftrightarrow t = 0$.

Definition (Ciric,Abbas,Saadati,Hussain[11]): Let $(X, \leq)$ be a partially ordered set. Then two mappings $f, g: X \to X$are said to be weakly increasing if $fx \leq gfx$and $gx \leq fgx\ \forall x \in X$.

Main result

Before we go to the main result, we introduce the function $d^s: X^2 \to [0, \infty)$ on a b-metric-like space (X, d) as follows:

$$d^s(x, y) = |2d(x, y) - d(x, x) - d(y, y)|\ \forall\ x, y \in X \text{.....................................(A)}$$

We observe that $d^s(x, x) = 0\ \forall x \in X$.

Theorem 1. Let$(X, \leq)$ be a partially ordered set and (X, d) bea complete b-metric-like space. Let $f, g: X \to X$ be two weakly increasing mappings with respect to $\leq$. Suppose

$d(x, fx) \geq d(x, x)$and $d(x, gx) \geq d(x, x)\ \forall x \in X$.Suppose there exists $k \geq 1$ such that

$\psi(kd(fx, gy) \leq \psi(M_b(x, y)) - \varphi(M'_b(x, y)) + L\psi(N(x, y))$.................................(1.1)

whenever$x, y$are comparable elements of X,

where$M_b(x, y) = \max\{d(x, y), d(x, fx), d(y, gy), \frac{d(x,gy)+d(y,fx)}{4b}\}$$M'_b(x, y) = \max\{d(x, y), d(x, fx), d(y, gy), \frac{d(x,gy)+d(y,fx)}{6b}\}$and $N(x, y) = \min\{d^s(x, fx), d^s(y, fx), d^s(x, gy)\}$,

where$d^s$is defined in (A), $L \geq 0, \psi$ and φ are altering distance functions. Let $x_0 \in X$, define the sequence$\{x_n\}$ inductively as follows:

$x_1 = fx_0,\ x_2 = gx_1$ and $x_{2n+1} = fx_{2n}$ and $x_{2n+2} = gx_{2n+1}$ for $n = 0,1,2 \dots \dots$

Then $\{x_n\}$ is an increasing Cauchy sequences.

Further $f$and g have a common fixed point, if

(i)f or$g$ is continuous or (ii) $(X, \leq, d)$ satisfies the following property:

$\{z_n\}$is an increasing sequences and converges to $z \Rightarrow z_n \leq z$ for every$n$.

Proof.

We first observe that(1) *u* is fixed point of *f*if and only if *u* is a fixed point of g.

(2)$M'_b(x, y) \leq M_b(x, y)\ \forall x, y \in X$

Now $x_1 = fx_0 \leq gfx_0 = gx_1 = x_2 \Rightarrow x_1 \leq x_2$

$x_2 = fx_1 \leq fgx_1 = fx_2 = x_3 \Rightarrow x_2 \leq x_3$

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Thus $x_1 \leq x_2 \leq x_3$

Inductively we can show that $x_n \leq x_{n+1}$ for $n = 1,2,3 \ldots \ldots$

Thus $\{x_n\}$ is an increasing sequence.

Suppose $x_{2n} = x_{2n+1}$ for some n, so that

$$x_{2n} = fx_{2n}$$

$\therefore$ x_{2n} is a fixed point of f and hence is a fixed point of g.

Thus x_{2n} is a common fixed point of f and g.

Similarly we can show that

$x_{2n+1} = x_{2n+2}$ for some n

$\Rightarrow$ x_{2n+1} is a common fixed point of f and g.

Hence we may suppose without loss of generality that if $x_n \neq x_{n+1}$ for $n = 1,2,3 \ldots \ldots$

Now $\psi(d(x_{2n+1}, x_{2n+2}) \leq \psi(kd(x_{2n+1}, x_{2n+2}))$

$$= \psi(kd(fx_{2n}, gx_{2n+1}))$$

$$\leq \psi(M_b(x_{2n}, x_{2n+1})) - \varphi(M'_b(x_{2n}, x_{2n+1})) + L\psi(N(x_{2n}, x_{2n+1}))\ldots(1.2)$$

Now

$$M_b(x_{2n}, x_{2n+1}) = \max\left\{d(x_{2n}, x_{2n+1}), d(x_{2n}, fx_{2n}), d(x_{2n+1}, gx_{2n+1}), \frac{d(x_{2n}, gx_{2n+1}) + d(x_{2n+1}, fx_{2n})}{4b}\right\}$$

$$= \max\left\{d(x_{2n}, x_{2n+1}), d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2}), \frac{d(x_{2n}, x_{2n+2}) + d(x_{2n+1}, x_{2n+1})}{4b}\right\}$$

Since $d(x_{2n}, x_{2n+2}) \leq bd(x_{2n}, x_{2n+1}) + bd(x_{2n+1}, x_{2n+2})$

and $d(x_{2n+1}, x_{2n+1}) \leq bd(x_{2n+1}, x_{2n}) + bd(x_{2n}, x_{2n+1}) = 2bd(x_{2n+1}, x_{2n})$

we have, $\frac{d(x_{2n}, x_{2n+2}) + d(x_{2n+1}, x_{2n+1})}{4b} \leq \frac{3bd(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})}{4b}$

$$= \frac{3}{4} d(x_{2n}, x_{2n+1}) + \frac{1}{4} d(x_{2n+1}, x_{2n+2})$$

$$= \max\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\}$$

Hence $M_b(x_{2n}, x_{2n+1}) = \max\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\}$

Similarly $M'_b(x_{2n}, x_{2n+1}) = \max\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\}$

and

$$N(x_{2n}, x_{2n+1}) = \min\{d^s(x_{2n}, fx_{2n}), d^s(x_{2n+1}, fx_{2n}), d^s(x_{2n}, gx_{2n+1})\}$$

$$= \min\{d^s(x_{2n}, x_{2n+1}), d^s(x_{2n+1}, x_{2n+1}), d^s(x_{2n}, x_{2n+2})\}$$

$$= 0 \quad (\because d^s(x_{2n+1}, x_{2n+1}) = 0)$$

$$\therefore \psi(d(x_{2n+1}, x_{2n+2}) \leq \psi(\max\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\})$$

$$-\varphi(\max\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.3)$$

If $d(x_{2n}, x_{2n+1}) \leq d(x_{2n+1}, x_{2n+2})$, we get from (1.3)

$$\psi(d(x_{2n+1}, x_{2n+2})) \leq \psi\big(d(x_{2n+1}, x_{2n+2})\big) - \varphi(d(x_{2n+1}, x_{2n+2}))$$

$$\Rightarrow \varphi(d(x_{2n+1}, x_{2n+2})) \leq 0$$

$$\Rightarrow d(x_{2n+1}, x_{2n+2}) = 0$$

$\Rightarrow x_{2n+1} \neq x_{2n+2}$, a contradiction.

Hence $d(x_{2n+1}, x_{2n+2}) < d(x_{2n}, x_{2n+1})$ for $n = 1,2,3 \ldots \ldots$

Similarly we can show that $d(x_{2n}, x_{2n+1}) < d(x_{2n-1}, x_{2n})$ for $n = 1,2,3 \ldots \ldots$

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Thus $d(x_{n+1},x_{n+2}) < d(x_n,x_{n+1})$for $n = 1,2,3 \ldots \ldots ..$

$\therefore \{d(x_n,x_{n+1})\}$isa strictly decreasing and hence diverges to a limit $r \geq 0$

Now from (1.3), we have

$$\psi(d(x_{2n+1},x_{2n+2})) \leq \psi\big(d(x_{2n+1},x_{2n+2})\big) - \varphi(d(x_{2n+1},x_{2n+2}))$$

On letting $n \to \infty$, we get $\psi(r) \leq \psi(r) - \varphi(r)(\because \psi,\varphi$ are continuous)

$$\Longrightarrow \varphi(r) = 0$$

$$\Longrightarrow r = 0$$

Thus $\{d(x_n,x_{n+1})$ decreases to 0……………………………………………….(1.4)

Claim: $\{x_n\}$ is a Cauchy sequence.

Case (i): $\lim_{m,n\to\infty} d(x_{2m},x_{2n})$ exists and is finite say *l.*

So $\exists\, N \ \ni d(x_{2m},x_{2n}) > \frac{l}{2}$ for $m,n \geq N$

Inparticular,$d(x_{2m},x_{2m+2}) > \frac{l}{2}$ for $m \geq N$

$$\therefore \frac{l}{2} < d(x_{2m},x_{2m+2}) \leq b.d(x_{2m},x_{2m+1}) + b.d(x_{2m+1},x_{2m+2})$$

$$\to 0 \ as \ \mathrm{m} \to \infty(\because \text{by } (1.4))$$

$$\therefore \frac{l}{2} = 0.$$

$$\therefore l = 0.$$

Suppose $\varepsilon > 0$. Then $\exists\, N \ \ni d(x_n,x_{n+1}) < \frac{\varepsilon}{2} \ if \ n \geq N$

$$d(x_{2n},x_{2n+1}) < \frac{\varepsilon}{2b} \ if \ n \geq N$$

$$d(x_{2m},x_{2n}) < \frac{\varepsilon}{2b} \ if \ m,n \geq N$$

Suppose $m,n \geq 2N$.Then

$$d(x_{2m},x_{2n+1}) \leq bd(x_{2m},x_{2n}) + bd(x_{2n},x_{2n+1})$$

$$b\left(\frac{\varepsilon}{2b} + \frac{\varepsilon}{2b}\right) = \varepsilon$$

Similarly $d(x_{2m+1},x_{2n+1}) \leq bd(x_{2m+1},x_{2n}) + bd(x_{2n},x_{2n+1})$

$$< b\left(\frac{\varepsilon}{b} + \frac{\varepsilon}{2b}\right) = \frac{3}{2}\varepsilon$$

$$\therefore d(x_{2m+1},x_{2n+1}) \to 0 \ as \ m,n \to \infty$$

$$\therefore d(x_m,x_n) \to 0 \ as \ m,n \to \infty$$

$\therefore \lim_{n\to\infty} d(x_m,x_n)$exists and is finite

$\therefore \{x_n\}$is a Cauchy sequence

Case (ii)$\lim_{m,n\to\infty} d(x_{2m},x_{2n})$does not exist.

So $\exists\, \varepsilon > 0$ and $\{m_i\}\{n_i\} \ni n_i > m_i > i \ and \ d(x_{2m_i}x_{2n_i}) \geq \varepsilon$……………………………….(1.5)

This means $d\big(x_{2m_i}x_{2n_i-2}\big) < \varepsilon$…………………………………………………………………….(1.6)

From (1.5) and (1.6)

$$d\big(x_{2m_i}x_{2n_i+1}\big) \leq b\, d\big(x_{2m_i}x_{2n_i-2}\big) + bd\big(x_{2n_i-2}x_{2n_i+1}\big)$$

$$\leq b\varepsilon + bd\big(x_{2n_i-2}x_{2n_i+1}\big)$$

$$\therefore \overline{\lim_{i\to\infty}} d\big(x_{2m_i}x_{2n_i+1}\big) \leq b\varepsilon$$

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Also from (1.5) and (1.6)

$d(x_{2m_i}x_{2n_i+1}) \leq b\, d(x_{2m_i}x_{2n_i}) + bd(x_{2n_i}, x_{2n_i+1})$

$= bd(x_{2n_i}, x_{2n_i+1}) + b(b\, d(x_{2m_i}x_{2n_i-1}) + d(x_{2n_i-1}, x_{2n_i}))$

$\leq bd(x_{2n_i}, x_{2n_i+1}) + b^2 d(x_{2m_i}, x_{2n_i-1}) + b^2 d(x_{2n_i-1}, x_{2n_i})$

$\leq bd(x_{2n_i}, x_{2n_i+1}) + b^2 d(x_{2n_i-1}, x_{2n_i}) + b^2(bd(x_{2m_i}, x_{2n_i-2}) + bd(x_{2n_i-2}, x_{2n_i-1}))$

$=bd(x_{2n_i}, x_{2n_i+1}) + b^2 d(x_{2n_i-1}, x_{2n_i}) + b^3(bd(x_{2m_i}, x_{2n_i-2}) + b^3 d(x_{2n_i-2}, x_{2n_i-1}))$

$< bd(x_{2n_i}, x_{2n_i+1}) + b^2 d(x_{2n_i-1}, x_{2n_i}) + b^3 d(x_{2n_i-2}, x_{2n_i-1})) + b^3\varepsilon$

Letting $i \to \infty$,

$\lim_{i\to\infty} d(x_{2m_i}x_{2n_i+1})$

$< b\lim_{i\to\infty} d(x_{2n_i}, x_{2n_i+1}) + b^2 \lim_{i\to\infty} d(x_{2n_i-1}, x_{2n_i}) + b^3 \lim_{i\to\infty} d(x_{2n_i-2}, x_{2n_i-1})) + b^3\varepsilon$

$\therefore \lim_{i\to\infty} d(x_{2m_i}x_{2n_i+1}) \leq b^3\varepsilon$

From (1.5)

$\varepsilon \leq d(x_{2m_i}x_{2n_i}) \leq bd(x_{2m_i}, x_{2n_i+1}) + bd(x_{2n_i+1}, x_{2n_i})$

By (1.4) $\frac{\varepsilon}{b} \leq \underline{\lim}_{i\to\infty} d(x_{2m_i}x_{2n_i+1}) \leq \overline{\lim}_{i\to\infty} d(x_{2m_i}x_{2n_i+1}) \leq \varepsilon b$

Similarly $\frac{\varepsilon}{b} \leq \underline{\lim}_{i\to\infty} d(x_{2m_i-1}, x_{2n_i}) \leq \overline{\lim}_{i\to\infty} d(x_{2m_i-1}x_{2n_i}) \leq \varepsilon b$

$\varepsilon \leq \overline{\lim}_{i\to\infty} d(x_{2m_i}, x_{2n_i}) \leq \varepsilon b^2$

$\frac{\varepsilon}{b} \leq \overline{\lim}_{i\to\infty} d(x_{2m_i}, x_{2n_i}) \leq \varepsilon b^2$

Since x_{2n_i} and x_{2m_i-1} are comparable,

$\psi(\varepsilon b) + \varphi\left(d(x_{2n_i}, x_{2m_i-1})\right) \leq \psi(\varepsilon b) - \varphi(M_b(x_{2n_i}, x_{2m_i-1}))$

$\leq \psi(M_b(x_{2n_i}, x_{2m_i-1})) + L\psi N(x_{2n_i}, x_{2m_i-1})$

On letting $i \to \infty$

$\psi(\varepsilon b) + \varphi\left(\frac{\varepsilon}{b}\right) \leq \psi(\varepsilon b) - \varphi(\underline{\lim}_{i\to\infty} d(x_{2m_i-1}, x_{2n_i}))$

$\leq \psi(\varepsilon b) + 0$

$\therefore \varphi\left(\frac{\varepsilon}{b}\right) \leq 0$

$\therefore \frac{\varepsilon}{b} = 0$

$\therefore \varepsilon = 0$

This is a contradiction.

$\therefore \{x_n\}$isa Cauchy sequence in X.

So $\exists x \ni x_n \to x \; as \; n \to \infty$

$$\therefore \lim_{m,n\to\infty} d(x_m, x_n) = d(x,x) = \lim_{n\to\infty} d(x_n, x)$$

Inparticular $\lim_{n\to\infty} d(x_n, x_{n+1}) = d(x,x)$

$\therefore d(x,x) = 0$ ($\because$ by (1.4))

i. Suppose f is continuous. Then

$$\lim_{n\to\infty} x_{2n+1} = \lim_{n\to\infty} f x_{2n} = f\left(\lim_{n\to\infty} x_{2n}\right) = fx(\because f \text{ is continuous})$$

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∴ x is a fixed point of f and hence is a common fixed point of f and g.

ii. Suppose (ii) holds

Since $\{x_n\}$ is an increasing sequence, it converges say to x.

By (ii) we have $x_n \le x$ for every n.

Now $\psi(kd(x_{2n+1}, gx)) = \psi(kd(fx_{2n}, gx))$

$$\le \psi(M_b(x_{2n}, x)) - \varphi(M_b{}'(x_{2n}, x)) + L\psi(N(x_{2n}, x)$$

$$M_b(x_{2n}, x) = \max\left\{d(x_{2n}, x), d(x, fx_{2n}), d(x, gx), \frac{d(x_{2n}, gx) + d(x, fx_{2n})}{4b}\right\}$$

$$= \max\left\{d(x_{2n}, x), d(x, x_{2n+1}), d(x, gx), \frac{d(x_{2n}, gx) + d(x, x_{2n+1})}{4b}\right\}$$

Since $\frac{d(x_{2n}, gx) + d(x, x_{2n+1})}{4b} \le \frac{bd(x_{2n}, x) + b.d(x, gx) + b.d(x, gx)}{4b}$

$$= \frac{1}{4} d(x_{2n}, x) + \frac{1}{4} d(x, gx) + \frac{1}{4} d(x, gx)$$

$$M_b(x_{2n}, x) = \max\{d(x_{2n}, x), d(x, x_{2n+1}), d(x, gx)\}$$

Similarly $M_b{}'(x_{2n}, x) = \max\{d(x_{2n}, x), d(x, x_{2n+1}), d(x, gx)\}$

and $N(x_{2n}, x) = \min\{d^s(x_{2n}, fx_{2n}), d^s(x, fx_{2n}), d^s(x_{2n}, gx)\}$

$$= \min\{d^s(x_{2n}, x_{2n+1}), d^s(x, x_{2n+1}), d^s(x_{2n}, gx)\}$$

Now

$$\psi\big(d(x_{2n+1}, gx)\big) \le$$
$$\psi(\max\{d(x_{2n}, x), d(x, x_{2n+1}), d(x, gx)\}) - \varphi(\max\{d(x_{2n}, x), d(x, x_{2n+1}), d(x, gx)\}) + L\psi(N(x_{2n}, x))$$

On letting $n \to \infty$, we get

$$\psi\big(d(x, gx)\big) \le \psi d(x, gx) - \varphi\big(d(x, gx)\big) + L\psi(0)$$

$$\therefore \varphi\big(d(x, gx)\big) = 0$$

$$\therefore d(x, gx) = 0$$

$$\therefore gx = x$$

∴ x is a fixed point of g and hence is a common fixed point of f and g.

Theorem 2. Under the hypothesis of theorem 1, no two common fixed points of f and g are comparable.

Proof: Suppose x and y are common fixed points of f and g and $x \le y$

Then $\psi(d(fx, gy) \le \psi(M_b(x, y)) - \varphi(M_b{}'(x, y)) + L\psi(N(x, y))$

$$= \psi(d(x, y)) - \varphi(d(x, y))$$

$$\therefore \varphi\big(d(x, y)\big) = 0$$

$$\therefore d(x, y) = 0$$

$$\therefore x = y.$$

∴ Any two comparable common fixed points of f and g are equal.

Corollary 3. Let $(X, \le)$ be a partially ordered set and suppose that there exists a b-metric-like d on X such that (X, d) is a complete b-metric-like space and let $f, g: X \to X$ be two weakly increasing mappings with respect to $\le$. Suppose f satisfies $d(x, fx) \ge d(x, x)$ and g satisfies $d(x, gx) \ge d(x, x), \forall x \in X$. Suppose there exists $k \ge 1$ such that

$$\psi(kd(fx, gy) \le \psi(M_b(x, y)) - \varphi(M_b(x, y)) + L\psi(N(x, y)) \quad \text{……………………………(3.1)}$$

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where$M_b(x,y) = \max\{d(x,y), d(x,fx), d(y,gy), \frac{d(x,gy)+d(y,fx)}{4b}\}$ and $N(x,y) = \min\{d^s(x,fx), d^s(y,fx), d^s(x,gy)\}$, where$d^s$is defined in (A)for all comparable elements $x,y \in X, L \geq 0, \psi$ and φ are altering distance functions.Then f and g have a common fixed point.

Proof: Since $M'_b(x,y) = M_b(x,y)$

$\psi(kd(fx,gy) \leq \psi(M_b(x,y)) - \varphi(M_b(x,y)) + L\psi(N(x,y)) \Rightarrow$

$$\psi(kd(fx,gy) \leq \psi(M_b(x,y)) - \varphi(M'_b(x,y)) + L\psi(N(x,y))$$

Hence the result follows from Theorem 1.

Corollary 4. Let$(X,\leq)$ be a partially ordered set and suppose that there exists a b-metric-like d on X such that (X,d) is a complete b-metric-like space and let $f,g: X \to X$ be two weakly increasing mappings with respect to $\leq$. Suppose f satisfies $d(x,fx) \geq d(x,x)$ and g satisfies $d(x,gx) \geq d(x,x), \forall x \in X$. Suppose there exists $k \geq 1$ such that

$$\psi(kd(fx,gy) \leq \psi(M_b{}'(x,y)) - \varphi(M_b{}'(x,y)) + L\psi(N(x,y)) \quad \text{.................................(4.1)}$$

where$M_b{}'(x,y) = \max\{d(x,y), d(x,fx), d(y,gy), \frac{d(x,gy)+d(y,fx)}{6b}\}$ and $N(x,y) = \min\{d^s(x,fx), d^s(y,fx), d^s(x,gy)\}$, where$d^s$is defined in (A)for all comparable elements $x,y \in X, L \geq 0, \psi$ and φ are altering distance functions.Then f and g have a common fixed point.

Proof:

$\psi(kd(fx,gy) \leq \psi(M_b{}'(x,y)) - \varphi(M_b{}'(x,y)) + L\psi(N(x,y)) \Rightarrow$

$$\psi(kd(fx,gy) \leq \psi(M_b(x,y)) - \varphi(M_b(x,y)) + L\psi(N(x,y))$$

andhence the result follows from Corollary3.

Corollary5. If in Corollary3,k is replaced by b^2, then$f$ and$g$ have a common fixed point.

Corollary 6.(Nannan Fang[12], Theorem 1): If in Corollary3, $k$is replaced by b^4,then f and g have a common fixed point.

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