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STUDY ON FUZZY GRAPHS AND APPLICATIONS

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Abstract

In this paper we study fuzzy graph is the generalization of the ordinary graph, here fuzzy graph is a simple fuzzy graph. A necessity of fuzzy graphs is introduced. Define regular fuzzy graphs, totally edge regular fuzzy graphs, partially edge regular fuzzy graphs, fuzzy dual graphs derive the properties as dual of fuzzy bipartite graph is Eulerian fuzzy graph. We are using fuzzy logic in so many areas like as modeling traffic and transportation problems, telecommunications, job allocation and at ATM centers.

Key words: *Fuzzy graph, regular fuzzy graph, fuzzy bipartite graph, Eulerian graph*

1. INTRODUCTION

One of the remarkable mathematical inventions of the 20th century is that of Fuzzy sets by Lotfi.A.Zadeh in 1965. His aim was to develop a mathematical theory to deal with uncertainty and imprecision. Researches on the theory of fuzzy sets has been witnessing an exponential growth both within mathematics and in its applications, this ranges from traditional mathematical subjects like logic, topology, algebra, analysis, etc. to pattern recognition, information theory, artificial intelligence, operation research, neural networks and planning, etc. The advantage of replacing the classical sets by Zadeh's fuzzy sets is that it gives more accuracy and precision in theory and more efficiency and system complatability in applications. So in systems with imprecision, a fuzzy set model is more valuable than a classic model. The distinction between set and fuzzy set is that the set divide the universal set into two subsets, namely members and non-members while fuzzy set assigns a sequence of membership values to elements of the universal set ranging from 0 to 1. That is partial memberships are allowed in tha latter. Also fuzzy sets can be used effectively to study quality variables like intelligence, beauty, consistency, etc., Zadeh's paper "Fuzzy sets" also proved the way to a new philosophical thinking of Fuzzy logic which now, is an essential concept in artificial intelligence.

Fuzzy graphs are useful to represent relationships which deal with uncertainty and it differs greatly from classical graph. The first definition of Fuzzy graph by Kaufman(1973) was based on Zadeh's fuzzy relations(1971). After that Rosenfeld(1975)[1] who considered fuzzy relation on fuzzy sets and developed the theory of fuzzy graphs. The author introduced fuzzy analogues of several graph theoretic concepts such as subgraphs, paths and connectedness, cliques, bridges and cut nodes, forest and trees, etc. During the same time, Yeh and Bang(1975) also introduced fuzzy graphs independently and studied various connectedness concepts such as connectivity matrix, reachability matrix, degree of connectivity, edge connectivity, vertex connectivity etc. These results are applied directly to clustering analysis include fuzzy trees, fuzzy line graphs, operations on fuzzy graphs, automorphism of fuzzy graph fuzzy interval graphs cycles and cocycles of fuzzy graphs, bipartite

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fuzzy graph. In this paper we define some basic definitions and we will see some properties like as dual of dual fuzzy graph is the fuzzy graph itself, and the dual of bipartite fuzzy graph is an Eulerian fuzzy graph.

2. **Preliminaries**

2.1. Definition: Let X be any nonempty set. A mapping M:x[0,1] is called a fuzzy subset of X.

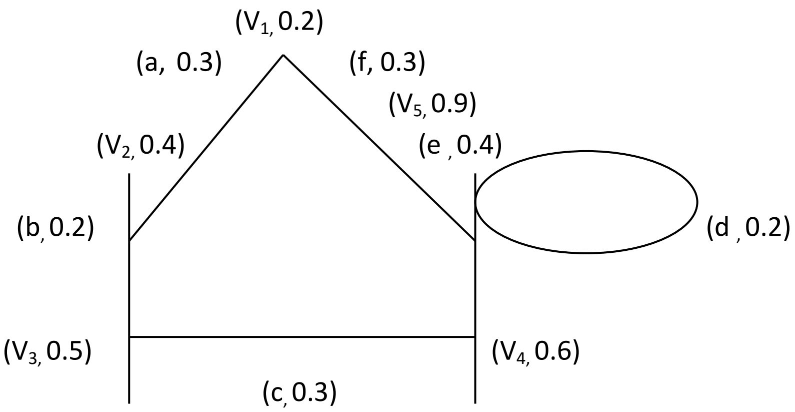
2.2. Example: A fuzzy subset A = { (a, 0.3), (b, 0.4), (c, 0.6)} of a set X ={a, b,c}.

2.3. Definition: Let M be a fuzzy subset in a set S, the strongest fuzzy relation on S, that is a fuzzy relation V with respect to M given by $V_{(X, Y)}$ = min {M(x), M(y)} for all X and Y in S.

Theory of fuzzy graphs – definitions and basic concepts

2.4. Definition: Let V be any nonempty set, E be any set and f: E→ V X V be any function. Then A is a fuzzy subset of V, S is a fuzzy relation on V with respect to A and B is a fuzzy subset of E such that B(e) ≤ S(X, Y) where e ∈ f^{-1} (x, y). Then the ordered triple F (A, B, f) is called a fuzzy graph. Where the elements of A are called fuzzy points or fuzzy vertices and the elements of B are called fuzzy lines or fuzzy edges of the fuzzy graph. If f(e) = (x, y), then the fuzzy points(x, A(x)), (y, A(y)) are called fuzzy adjacent points and fuzzy points (x, A(x)), fuzzy line (e, B(e)) are called incident with each other. If two distinct fuzzy lines (e_1, B(e_1)) and (e_2, B(e_2)) are incident with a common fuzzy point, then they are called fuzzy adjacent lines.

2.5. Example:



In this figure, (i). (V_1, 0.2) is a fuzzy point. (ii). (a, 0.3) is a fuzzy edge. (iii). (V_2, 0.4) and (V_3, 0.5) are fuzzy adjacent points. (iv). (a, 0.3) join with (V_1, 0.2) and (V_2, 0.4) and therefore it is incident with (V_1, 0.2) and (V_2, 0.4) (v). (b, 0.2) and (c, 0.3) are fuzzy adjacent lines. (vi). (e, 0.4) is a fuzzy loop

2.6. Definition: A fuzzy line joining a fuzzy point to itself is called a fuzzy loop.

2.7. Definition: Let F=(A, B,f) be a fuzzy graph. If more than one fuzzy line joining two fuzzy vertices is allowed, then the fuzzy graph F is called a fuzzy pseudo graph.

2.8. Definition: F= (A, B, f) is called a fuzzy simple graph if it has neither fuzzy multiple lines nor fuzzy loops.

3. **Types of graphs**

3.1. Definition: The fuzzy graph H =(C, D, f) is called a fuzzy subgraph of F=(A, B, f) if C≤A and D ≤ B.

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3.2. Definition: The fuzzy subgraph H=(C, D, f) is said to be a fuzzy spanning subgraph of F=(A, B, f) if C=A.

3.3. Definition: The fuzzy subgraph H=(C, D, f) is said to be a fuzzy induced subgraph of F=(A, B, f) if H is the maximal fuzzy subgraph of F with fuzzy point set C.

3.4. Definition: A fuzzy graph F =(A, B, f) is called fuzzy regular graph if d(v) = k for all v in V.

3.5. Definition: A fuzzy graph F= (A, B, f) is called a fuzzy complee graph if every pair of distinct fuzzy vertices are fuzzy adjacent and B(e) = S(X, Y) where $e \in f^{-1}$ (x, y) for all x, y in v.

Definiton: A planar graph is a graph drawn in the plane in such a way that no two edges intersect geometrically except at a vertex to which they are both incident, each closed region in a planar graph called face. In fuzzy planar these edges must have membership values. A fuzzy planar graph is called strong fuzzy planar graph if the fuzzy planarity value of the graph is greater than 0.5. otherwise it's called weak fuzzy planar graph.

3.6. Definition: A connected graph G is called an Eulerian graph if there exists a circuit which includes every edge of G; such a circuit is called an Eulerian circuit.

3.7. Definition: The maximal fuzzy planar graph is the fuzzy planar graph with maximum number of edges such that, when we add any other edge, the graph be fuzzy non planar graph. Note that the maximal fuzzy bipartite planar graph having even cycles of length four.

4 . Results and applications

4.1. Proposition (1)

A given connected graph G is an Eulerian Graph if and only if all vertices of G are of even degree.

4.2. Proposition (2)

The dual of fuzzy bipartite graph is an Eulerian fuzzy graph.

Proof:

In this proof we consider the maximal fuzzy bipartite graph with the degree of membership and for the vertices σ(v), μ (e) and $\leftthreetimes$(f) for the vertices , edges and faces respectively.Since it is maximal fuzzy bipartite planar graph then by definition each cycles of even length, such that every face f in G have even length.

So the dual of these faces for all i=1,2,…k obtained a vertices of even degree such that by proposition(1) the dual graph is Eulerian fuzzy graph, with the degree of membership $\sigma(v^*)$ = $\leftthreetimes$(f), $\mu(e^*) = \mu(e)$ and $\leftthreetimes(f^*) = \sigma$(v).

4.3. TRAFFIC LIGHTS PROBLEM AND ITS SOLUTION BY FUZZY

The traffic lights problem consists of controlling a traffic lights system in such a way that certain level of security will be attained. This problem has been studied as an intersection graph. The traffic lights problem can be modeled by graph coloring. Vehicles will go from one direction to another direction and each intersecting point shows the possible accidental zones of two arrows. But number of vehicles in all paths are not always equal. Due to this reason, we consider it as fuzzy set whose membership value depends upon on vehicles number. If the number of vehicles in any path is high then its membership value will be high and if the number of vehicles in any path is low then its membership value will be low.

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4.4. A New Model of Telecommunication Network, FTN

A Telecommunication network is a social network. In **this** system we propose a method to represent telecommunication network by fuzzy graph.

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