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AN EXAMPLE WHICH SHOWS THAT A WEAK K-CONTRACTION IN A COMPLETE GENERALIZED METRIC SPACE MAY FAIL TO HAVE UNIQUE FIXED POINT

[1]K.P.R. SASTRY,[2]B. RAMU NAIDU, [3]K .SUJATHA*, [4]D.NARAYANA RAO,
*Lecturer, Department of Mathematics, St. Joseph's College for Women(A), Visakhapatnam, kambhampati.sujatha@gmail.com

In this paper an example is provided to show that a weak k-quasi contraction in a complete generalized metric space may fail to have unique fixed point.

K.P.R Sastry, G. Appala naidu, Ch. Srinivasa Rao, and B. Ramunaidu [1] have shown that a weak k-quasi contraction f in a complete generalized metric space has a unique fixed point w, in the sense that if w' any fixed point with $D(w,w')<\infty$ with $D(w',w')<\infty$ then $w=w'$.

In the following example we show that this result may fail if there does not exist w' with the specified condition. ((i.e) $D(w,w')<\infty$ and $D(w',w')<\infty$).

Ex:- Let $\square$ be set of real numbers and $X=\{x\geq 0\}$. Define $D: X\times X\to\square$ by

$$D(x,y)=\begin{cases}|x-y| & \text{if } x \text{ and } y \text{ are rational}\\ \infty, & \text{otherwise}\end{cases}$$

Then (X,D) is a generalized metric space .

Define $f: X\to X$ by $f(x)=\begin{cases}\frac{1}{2}x, & \text{if } x \text{ is rational}\\ x, & \text{if } x \text{ is irrational}\end{cases}$

Then $D(fx,fy)\leq\frac{1}{2}\max\{D(x,y),D(x,fx),D(y,fy),D(x,fy),D(y,fx)\}\ \forall\ x,y\in X\to(1)$

Clearly (1) holds if both x and y are rational.

Further (1) also holds if one of x,y is irrational, since in this case L.H.S of $(1)=\infty$ and also the R.H.S of $(1)=\infty$.

Thus f is a weak $\frac{1}{2}-$quasi contraction and '0' is a fixed point .

But every positive irrational number is also a fixed point.

We observe that for any other fixed point w' of f, we have $D(0,w')=\infty$ and $D(w',w')=\infty$.

Reference

[1] KPRSastry, G.Appala naidu, Ch.Srinivasa Rao and B.Ramunaidu: Fixed point theorems for weak k-contractions on a generalized metric space