BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH
A Peer Reviewed International Journal,
Contents available on www.bomsr.com
Vol.4. S1.2016; ISSN: 2348-0580
Email:editorbomsr@gmail.com
RADMAS– 2016

EXPRESSION OF FIBONACCI SEQUENCES IN PLANTS AND ANIMALS

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ABSTRACT

The Fibonacci numbers are Nature's numbering system discovered by the Italian mathematician Leonardo Fibonacci. They appear everywhere in Nature, from the leaf arrangement in plants, to the pattern of the florets of a flower, the bracts of a pinecone, or the scales of a pineapple. The Fibonacci numbers are therefore applicable to the growth of every living thing, including a single cell, a grain of wheat, a hive of bees, and even all of mankind. In the seeming randomness of the natural world, we can find many instances of mathematical order involving the Fibonacci numbers themselves and the closely related "Golden ratio" elements. The series of numbers 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 and so on is known as the Fibonacci numbers or the Fibonacci sequence. The ratio between the numbers (1.618034) is frequently called the golden ratio or golden number. Fibonacci numbers appear in nature often enough to prove that they reflect some naturally occurring patterns which are commonly evident by studying the manner in which various plants and animals grow. The golden ratio is expressed in spiraling shells where areas of the shell's growth are mapped out in squares. Most of human body parts follow the numbers one, two, three and five. DNA molecules follow this sequence, measuring 34 angstroms long and 21 angstroms wide for each full cycle of the double helix. Many natural patterns reflect the Fibonacci sequence whose correlation may just be a coincidence but in other situations, the ratio exists because that particular growth pattern evolved as the most effective. Fibonacci spiral patterns appear in many plants, such as pinecones, pineapples, and sunflowers. The patterns consist of spirals that curve around a surface in both the "sinister" form (clockwise) and the "dexter" form (anticlockwise). Fibonacci numbers or spirals in leaf arrangement, or phyllotaxis, in some of the cases may be related to maximizing the space for each leaf, or for the average amount of light falling on each of them. While the aesthetics and symmetry of Fibonacci spiral patterns has often attracted scientists, but a mathematical or physical explanation for their common occurrence in nature is yet to be discovered. The objective of this study is to understand the significance of Fibonacci growth patterns in nature. By observing the growth patterns of some selected plants and animals, this paper discusses the expression of Fibonacci numbers and spirals in nature and how they can be correlated to golden ratio.

Keywords: Fibonacci numbers, Fibonacci sequence, Fibonacci spiral, growth patterns, Golden ratio

1. INTRODUCTION

It has been said that art imitates nature, and it appears that nature is where Fibonacci numbers have always been. In plants, the arrangement of seeds is frequently a Fibonacci number. Although not a rule of nature, Fibonacci numbers are an interesting tendency. Examples of these numbers and their corresponding ratios, rectangles and spirals are endless and are all around us. The Fibonacci appears

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in the smallest to the largest objects in nature. It is a way for information to flow in a very efficient way. Spirals are the most common galaxy shape. Although not a rule of nature, Fibonacci numbers are an interesting tendency. Examples of these numbers and their corresponding ratios, rectangles and spirals are endless and are all around us.

Leonardo Pisano, later known as Fibonacci, "son of Bonacci," was a brilliant mathematician, born in Italy about 1175 AD. He was the first to mention the following sequence of numbers that now bears his name. Add the first two numbers together and they equal the third number. Add the second and third numbers and they equal the fourth, and so on. Fibonacci series can go on infinitely, as it begins with 0, 1, 1 and each subsequent number is created by adding the previous two together: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34 55, 89, 144 … $F_n = F_{n-1} + F_{n-2}$; with $F_0 = 0$, $F_1=1$ The Italian mathematician Leonardo Fibonacci is widely regarded as the greatest European mathematician of the middle Ages. Fibonacci introduced the Hindu-Arabic numerals (the decimal system) to the Western world. This system replaced the inefficient and clumsy Roman numerals. Fibonacci published Liber abaci in the year 1202, a book that revolutionized everything from economics and commerce to education and science. It is quite impossible to imagine how some of the great turning points in Western history could have been achieved without Fibonacci's Book of Calculation. In Liber Abaci, the Italian mathematician also introduced a very interesting sequence of numbers with intriguing characteristics.

While the aesthetics and symmetry of Fibonacci spiral patterns has often attracted scientists, a mathematical or physical explanation for their common occurrence in nature is yet to be discovered. Recently, scientists have successfully produced Fibonacci spiral patterns in the lab, and found that an elastically mismatched bi-layer structure may cause stress patterns that give rise to Fibonacci spirals. The discovery may explain the widespread existence of the pattern in plants.

2. Fibonacci Sequences in Plants:

Some plants express the Fibonacci sequence in their growth points, the places where tree branches form or split. One trunk grows until it produces a branch, resulting in two growth points. The main trunk then produces another branch, resulting in three growth points. Then the trunk and the first branch produce two more growth points, bringing the total to five. This pattern continues, following the Fibonacci numbers. Additionally, if counted, the number of petals on a flower often found the total to be one of the numbers in the Fibonacci sequence. For example, lilies and irises have three petals, buttercups and wild roses have five, delphiniums have eight petals and so on. Seed heads, pinecones, fruits and vegetables show the similar patterns. When looked at the array of seeds in the center of a sunflower and one will notice what looks like spiral patterns curving left and right. Amazingly, if these spirals counted, the total will be a Fibonacci number. Divide the spirals into those pointed left and right and it will result in two consecutive Fibonacci numbers. One can decipher spiral patterns in pinecones, pineapples and cauliflower that also reflect the Fibonacci sequence in this manner. In plants, this may mean maximum exposure for light-hungry leaves or maximum seed arrangement.

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2. 1. Leaf and flower arrangements

Based on a survey of the literature encompassing 650 species and 12,500 specimens, R. Jean estimated that, among plants displaying spiral or multijugate phyllotaxis, about 92 percent of them have Fibonacci phyllotaxis (from the Greek: Phyllo means leaf, and taxis means arrangement). In most Aroids, a vast group of beautiful ornamental plants, flowers are arranged in a mathematical series. Clear spirals are visible and the numbers of these spirals are usually a pair of Fibonacci numbers. For example, all the spadices of Anthurium macrolobium present floral spirals matching the Fibonacci numbers eight and five. The Indian Statistical Institute and the Royal Agri-Horticultural Society dedicated an entire study to this topic. It provides solid evidence to support these claims.

Locate the lowest leaf of a green plant that hasn't been pruned. Count both the number of times you circle the stem of the plant before arriving at the leaf located directly above the first one (pointing in the same direction), as well as the number of leaves above the lowest located leaf. The number of rotations, of turns in each direction and the number of leaves met will be Fibonacci numbers! Of course, leaf arrangements vary from species to species, but they should all be Fibonacci numbers.

If the number of turns is x and the number of leaves is y, specialists commonly call the leaf arrangement x/y phyllotaxis or x/y spiral. The following ratios are the phyllotaxis ratios of different plants:

1/2 phyllotaxis: elm, basswood, lime, some grasses etc.
1/3 phyllotaxis: beech, hazel, blackberry etc.
2/5 phyllotaxis: oak, apricot, cherry, apple, some roses etc.
3/8 phyllotaxis: cabbages, poplar, pear, hawkweed etc.
5/13 phyllotaxis: willow, almond etc.
13/34 phyllotaxis: some pine trees

2. 2. Pine cones – scale arrangement

Pine cones are good at Fibonacci numbers! There are many species of pine, about 115, all with different characteristics, but what most of them have in common are the Fibonacci spirals of the cones. If you look closely at a pine cone, you will see two distinct spirals. The number of spirals going in each direction typically corresponds to successive Fibonacci numbers. The spirals of the cone from the above picture correspond to 8 and 13. When this occurs, the angle between consecutive scales gradually approximates the golden angle – which is about 137.5 degrees. This angle is responsible for the optimal packing of scales.

2. 3. Fruits

Some fruits like the pineapple, banana, Sharon fruit, apple and more exhibit patterns following the Fibonacci sequence. Expecting all pineapples to display the Fibonacci sequence is too much to ask, but beautifully grown pineapples with spirally arranged fruit-lets still follow a simple rule that illustrates perfection in nature, the Fibonacci sequence. The pineapple's hexagonal bracts form three distinct sets of spirals. If we count the number of rows formed in each direction, we get the Fibonacci numbers. 5-8-13 and 8-13-21 are the typical Fibonacci sequences pineapples exhibit.

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2. 4. Arrangement of seeds on flower heads

Nature tries to optimize the arrangement of seeds on flower heads. To fill space efficiently, with no gaps from start to end, seeds must grow in a pattern that is not well approximated by a fraction. This number is exactly ϕ = 1.6180 with the corresponding angle (α) of 137.5 degrees. Each new seed grows at a certain angle in relation to the previous one. What's so special about α = 137.5 degrees? It's the golden angle. According to Dr. Mark Freitag from the University of Georgia, *"if a circle is divided into two arcs in the proportion of the golden ratio, the central angle of the smaller arc marks off the golden angle, which is 137.5 degrees."* Only with this angle can one obtain the optimal filling. Eg., sunflower seeds. For example, if we look at fig.2, the angle of seeds growing equals 10 degrees, and the flower in fig.3 has seeds growing 14 degrees apart.

Sunflower seeds are arranged in a beautiful spiral pattern, both in a clockwise and counter-clockwise manner. Add up the number of clockwise and counter-clockwise spirals and in 82% of the cases they will be consecutive Fibonacci numbers: 21:34, 34:55, 55:89, or 89:144. As they mature, sunflowers form more spirals. The disk flowers perfectly arranged in the center of the head of a purple coneflower follow the same pattern as the sunflower seeds.

2. 5. Fractal pattern in Vegetables

Mathematics, rightly viewed, possesses not only truth, but supreme beauty — a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show," said Bertrand Russell. Number one goes to the ultimate fractal pattern! Fractals are the result of an iterative process. They contain self-similar arrangements with different magnification factors. If a fractal pattern breaks down into pieces, it will get a small-size copy of the whole.

Not all vegetables are connected to the Fibonacci numbers and spirals, but those that are, are quite fascinating. For example, Romanesque broccoli (Roman cauliflower),one of the most beautiful vegetables ever seen. The peaked florets are arranged in beautiful Fibonacci spirals, 8 and 13 in this case. Chinese leaves and lettuce are other vegetables that display Fibonacci phyllotaxis.

3. Fibonacci Sequences in Animals:

Plants are not the only example of this natural phenomenon; animals also exhibit Fibonacci numbers. To understand this fully we must first do some geometric constructions. Begin with a golden rectangle (width and length are Fibonacci numbers) and divide it into a perfect square and another golden rectangle. Keep dividing the rectangle into successively smaller squares and rectangles. Drawing an arc through each new square result in an equiangular spiral. Wild sheep horns, hurricane clouds, snails, and growing ferns all exhibit the equiangular spiral, which is seen abundantly in nature. This spiral has the distinctive characteristic of increasing in size while retaining the same shape. The need for this feature can best be seen in a growing mollusk, which grows larger but does not change shape.

3. 1. The ancestral tree of a male honeybee

The ancestral tree of a male honeybee follows Fibonacci's sequence. Female honeybees (either workers or queens) hatch from an egg that has been fertilized by a male honeybee, but male

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honeybees or drones are produced by the queen's unfertilized eggs, so they have a mother but no father. As the above picture clearly shows, the sequence that gives us the number of honeybees in each generation of the ancestral tree of a male bee is 1, 1, 2, 3, 5, 8, 13…

1st generation……..1 offspring
2nd generation…….1 parent
3rd generation…….2 grandparents
4th generation…….3 great-grandparents
5th generation…….5 great-great-grandparents…

Therefore, Fibonacci numbers express a drone's family tree in that he has one parent, two grandparents, three great-grandparents and so forth. The pattern that emerged was reduced to a mathematical model that can be used to find the number of bees in any generation back from the very first honeybee.

3. 2. Fibonacci sequence in Rabbits

Given optimal conditions, how many pairs of rabbits can be produced from a single pair of rabbits in one year? This **thought experiment** dictates that the female rabbits always give birth to pairs, and each pair consists of one male and one female.

When two newborn rabbits are placed in a fenced-in yard and left to, breed like rabbits. Rabbits can't reproduce until they are at least one month old, so for the first month, only one pair remains. At the end of the second month, the female gives birth, leaving two pairs of rabbits. When month three rolls around, the original pair of rabbits produce yet another pair of newborns while their earlier offspring grow to adulthood. This leaves three pairs of rabbit, two of which will give birth to two more pairs the following month. The order goes as follows: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 and on to infinity.

4. Fibonacci spirals and the Golden ratio

The ratio between the Fibonacci numbers (1.618034) is frequently called the golden ratio or golden number. The golden ratio is expressed in spiraling shells. In the above illustration, areas of the shell's growth are mapped out in squares. If the two smallest squares have a width and height of 1, then the box to their left has measurements of 2. The other boxes measure 3, 5, 8 and 13. Many plants and animals express different number sequences. And just because a series of numbers can be applied to an object, that doesn't necessarily imply there's any correlation between figures and reality. But, Fibonacci numbers appear in nature often enough to prove that they reflect some naturally occurring patterns.

Fibonacci sequences have countless applications in modern mathematics and computer science, but what is really fascinating is their discreet but striking appearance in nature. This series of fabulous numbers is also the mathematical first cousin of ϕ (Phi), the golden ratio – another special number that has fascinated the human culture for millennia. The golden ratio is the limit of the ratios of consecutive Fibonacci numbers.

$\phi = 1.6180339887498948482045868343656$

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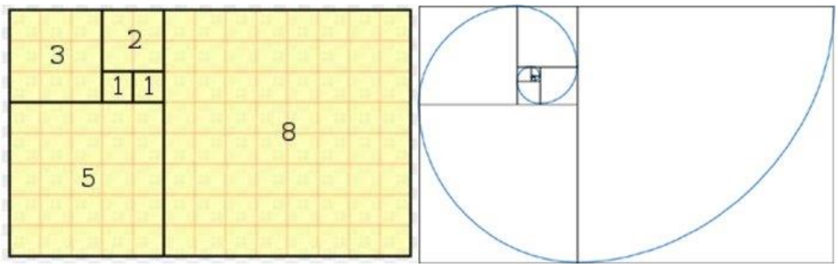
If a Fibonacci number is divided by the one before it, the quotient approximates Phi; e.g. 4181/2584 ≈ 1.6180340557275541795665634674923... The golden ratio is supposed to be the ideal proportion, the most aesthetically pleasing shape to the human eye.

Many plants and animals express different number sequences. And just because a series of numbers can be applied to an object, that doesn't necessarily imply there's any correlation between figures and reality. But, Fibonacci numbers appear in nature often enough to prove that they reflect some naturally occurring patterns.



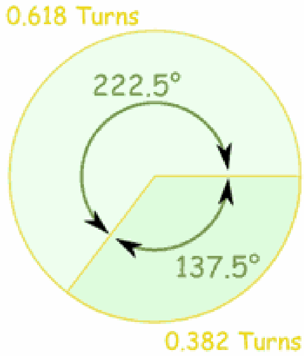


Fig.: 1. Fibonacci spiral and Golden Ratio Fig.: 2.

4. 1. Spiraling shells

The golden ratio is expressed in spiraling shells. The areas of the shell's growth are mapped out in squares. If the two smallest squares have a width and height of 1, then the box to their left has measurements of 2. The other boxes measure 3, 5, 8 and 13.

4. 2. The human body: Most of your body parts follow the numbers one, two, three and five. Humans have one nose, two eyes, three segments to each limb and five fingers on each hand. The proportions and measurements of the human body can also be divided up in terms of the golden ratio. DNA molecules follow this sequence, measuring 34 angstroms long and 21 angstroms wide for each full cycle of the double helix.

4.3. The human X chromosome inheritance tree

The number of possible ancestors on the X chromosome inheritance line at a given ancestral generation follows the Fibonacci sequence. (After Hutchison, L. "Growing the Family Tree: The Power of DNA in Reconstructing Family Relationships".

Luke Hutchison noticed that number of possible ancestors on the X chromosome inheritance line at a given ancestral generation also follows the Fibonacci sequence.[66] A male individual has an X chromosome, which he received from his mother, and a Y chromosome, which he received from his father. The male counts as the "origin" of his own X chromosome (F1 = 1), and at his parents' generation, his X chromosome came from a single parent (F2 = 1). The male's mother received one X chromosome from her mother (the son's maternal grandmother), and one her father (the son's maternal grandfather), so two grandparents contributed to the male descendant's X chromosome (F3 = 2). The maternal grandfather received his X chromosome from his mother, and the maternal grandmother received X chromosomes from both of her parents, so three great-grandparents contributed to the male descendant's X chromosome (F4 = 3). Five great-great-grandparents

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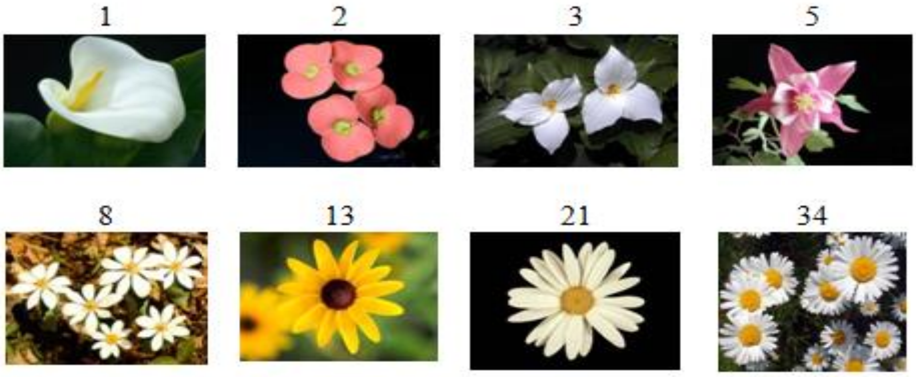
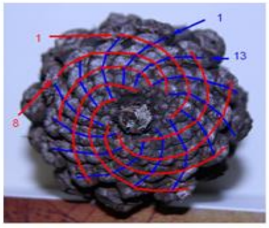
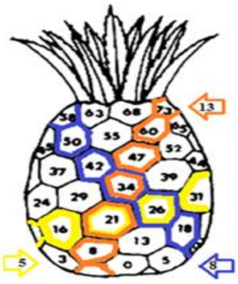
contributed to the male descendant's X chromosome (F5 = 5), etc. (Note that this assumes that all ancestors of a given descendant are independent, but if any genealogy is traced far enough back in time, ancestors begin to appear on multiple lines of the genealogy, until eventually a population founder appears on all lines of the genealogy.



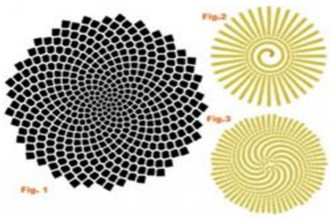
(Figure 3: Fibonacci Numbers and Petals of Flowers)

Table 1. Images of Fibonacci numbers and Spirals in Plants and Animals


Pine cone


Pine apple

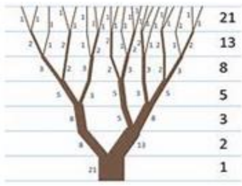

Sunflower


Sunflower seeds

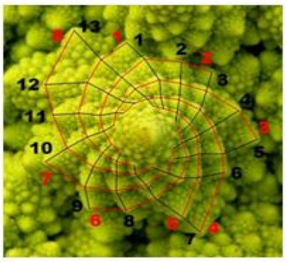

Marlborough Rock Daisy


Purple cone flower


Leaf spiral


Branching


Romanesco cauliflower


its spiral follows the Fibonacci sequence


Spiral aloe


Spiral aloe

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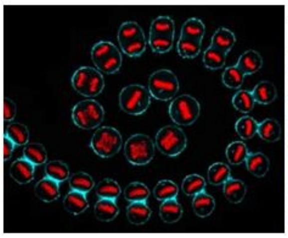
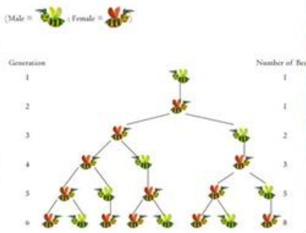
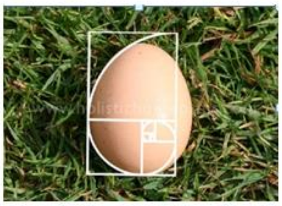


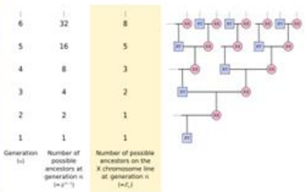



Cancer cell (HeLa undergoing cell division ovary of an Anglerfish. Honey bee tree Snail spiral




Egg American giant millipede Snails and fingerprints Water falls into the shapes of a fibonacci



Fibonacci in spores A fiddlehead or koru Fibonacci and armor A monarch caterpillar about to form a chrysalis

Applications

- The Fibonacci numbers are important in the computational run-time analysis of Euclid's algorithm to determine the greatest common divisor of two integers: the worst case input for this algorithm is a pair of consecutive Fibonacci numbers.
- Brasch et al. 2012 show how a generalised Fibonacci sequence also can be connected to the field of economics. In particular, it is shown how a generalised Fibonacci sequence enters the control function of finite-horizon dynamic optimisation problems with one state and one control variable. The procedure is illustrated in an example often referred to as the Brock–Mirman economic growth model.
- Yuri Matiyasevich was able to show that the Fibonacci numbers can be defined by a Diophantine equation, which led to his original solution of Hilbert's tenth problem.
- The Fibonacci numbers are also an example of a complete sequence. This means that every positive integer can be written as a sum of Fibonacci numbers, where any one number is used once at most.
- Moreover, every positive integer can be written in a unique way as the sum of *one or more* distinct Fibonacci numbers in such a way that the sum does not include any two consecutive Fibonacci numbers. This is known as Zeckendorf's theorem, and a sum of Fibonacci numbers that

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satisfies these conditions is called a Zeckendorf representation. The Zeckendorf representation of a number can be used to derive its Fibonacci coding.

- Fibonacci numbers are used by some pseudorandom number generators.
- They are also used in planning poker, which is a step in estimating in software development projects that use the Scrum (software development) methodology.
- Fibonacci numbers are used in a polyphase version of the merge sort algorithm in which an unsorted list is divided into two lists whose lengths correspond to sequential Fibonacci numbers – by dividing the list so that the two parts have lengths in the approximate proportion φ. A tape-drive implementation of the polyphase merge sort was described in *The Art of Computer Programming*.
- Fibonacci numbers arise in the analysis of the Fibonacci heap data structure.
- The Fibonacci cube is an undirected graph with a Fibonacci number of nodes that has been proposed as a network topology for parallel computing.
- A one-dimensional optimization method, called the Fibonacci search technique, uses Fibonacci numbers.
- The Fibonacci number series is used for optional lossy compression in the IFF 8SVX audio file format used on Amiga computers. The number series compands the original audio wave similar to logarithmic methods such as μ-law.
- Since the conversion factor 1.609344 for miles to kilometers is close to the golden ratio (denoted φ), the decomposition of distance in miles into a sum of Fibonacci numbers becomes nearly the kilometer sum when the Fibonacci numbers are replaced by their successors. This method amounts to a radix 2 number register in golden ratio base φ being shifted. To convert from kilometers to miles, shift the register down the Fibonacci sequence instead.

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