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RESEARCH ARTICLE

VAGUE TRANSLATIONS OF VAGUE H-IDEALS IN BCK ALGEBRAS

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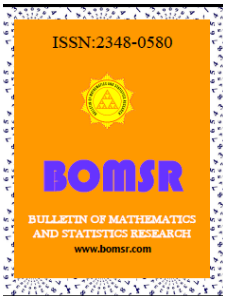

ABSTRACT

In this paper we introduce vague α-translate and vague λ-multiplication operators on vague H- ideals in BCK algebras and characterized their properties.

Keywords: vague set, vague α- translation, vague λ multiplication.

1. Introduction

BCK and BCI algebras are two classes of logical algebras. They were introduced by Imai and Iseki[7, 8] and have been extensively investigated by many researchers. BCI algebras are the generalization of BCK algebras. In 1991 Xi[15] applied the concept of fuzzy set to BCK algebra. After that Jun, Meng, Liu and several researchers investigated further properties of fuzzy BCK- algebras and fuzzy ideals[2,5,6,12]. The concept of fuzzy set was introduced by zadeh[16] in 1965. Lee et. al [10] discussed fuzzy translations in algebras. Gau. W. L and Bueher D. J[4] have initiated the study of vague sets with the hope that they form a better tool to understand, interpret and solve real life problems. RanjitBiswas[14] introduced the study of vague groups and Ramakrishna. N[13],T. Eswarlal[3], have extended the study of vague algebra. The objective of this paper is to study the concept of translate operators on H- ideals in BCK- algebra and investigate their properties.

2.Preliminaries:

Definition 2.1:[6] An algebra X with a constant 0 and a binary operation "$*$" satisfying the following axioms for all $x, y, z \in X$.:

(i) $((x*y)*(x*z))*(z*y)=0$

(ii) $(x*(x*y))*y=0$

(iii) $x*x=0$

(iv) $x*y=0$ and $y*x=0$ implies x=y.

We can define a partial ordering "≤" by x ≤ y if and only if $x * y = 0$.

If a BCI- algebra X satisfies $0 * x = 0$, for all $x \in X$, then we say that X is a BCK- algebra. Any BCK- algebra X satisfies the following axioms :

(i) $(x * y) * (x * z) \leq (z * y)$

(ii) $x * (x * y) \leq y$

(iii) $x \leq x$

(iv) $0 \leq x$

(v) $x \leq y$ and $y \leq x$ implies x=y. where x ≤ y means $x * y = 0$.

Definition 2.2:[5] A non empty subset S of X is called a subalgebra of X if $x * y \in S$ for any $x, y \in S$.

Definition 2.3:[6] A non empty subset I of X is called an ideal of X if it satisfies

(I_1) $0 \in I$ and (I_2) $x * y \in I$ and $y \in I$ imply $x \in I$.

Definition 2.4:[9] A non empty subset I of X is said to be an H- ideal of X if it satisfies(I_1) and

(I_3) $x * (y * z) \in I$ and $y \in I$ imply $x * z \in I$, for all $x, y, z \in X$.

Definition 2.5: [2] A vague set A in the universe of discourse U is characterized by two membership functions given by:

(i) A true membership function $t_A : U \to [0,1]$ and

(ii) A false membership function $f_A : U \to [0,1]$

where $t_A(x)$ is a lower bound on the grade of membership of x derived from the "evidence for x", $f_A(x)$ is a lower bound on the negation of x derived from the "evidence for x", and $t_A(x) + f_A(x) \leq 1$. Thus the grade of membership of u in the vague set A is bounded by a subinterval $[t_A(x), 1 - f_A(x)]$ of [0,1]. This indicates that if the actual grade of membership of x is μ(x), then, $t_A(x) \leq \mu(x) \leq 1 - f_A(x)$. The vague set A is written as $A = \{\langle x, [t_A(x), 1 - f_A(x)] \rangle / u \in U\}$ where the interval $[t_A(x), 1 - f_A(x)]$ is called the vague value of x in A, denoted by $V_A(x)$.

Definition 2.6:[2] Let A and B be VSs of the form $A = \{\langle x, [t_A(x), 1 - f_A(x)] \rangle / x \in X\}$ and $B = \{\langle x, [t_B(x), 1 - f_B(x)] \rangle / x \in X\}$ Then

(i) $A \subseteq B$ if and only if $t_A(x) \leq t_B(x)$ and $1 - f_A(x) \leq 1 - f_B(x)$ for all x∈X

(ii) A=B if and only if $A \subseteq B$ and $B \subseteq A$

(iii) $A^c = \{\langle x, f_A(x), 1 - t_A(x) \rangle / x \in X\}$

(iv) $A \cap B = \{\langle x, \min(t_A(x), t_B(x)), \min(1 - f_A(x), 1 - f_B(x)) \rangle / x \in X\}$

(v) $A \cup B = \{\langle x, (t_A(x) \vee t_B(x)), (1 - f_A(x) \vee 1 - f_B(x)) \rangle / x \in X\}$

For the sake of simplicity, we shall use the notation $A = \langle x, t_A, 1 - f_A \rangle$ instead of $A = \{\langle x, [t_A(x), 1 - f_A(x)] \rangle / x \in X\}$.

Definition 2.7:[11] A vague set A on X is called a vague subalgebra of x if, for any $x \in X$, we have $t_A(xy) \geq \min\{t_A(x), t_A(y)\}$ and $1 - f_A(xy) \geq \min\{1 - f_A(x), 1 - f_A(y)\}$

Definition 2.8:[11] A vague set A of a BCK- algebra X is called a vague ideal of X if the following condition is true:

(i) $(V_A(0) \geq V_A(x)), \quad (\forall\, x \in X)$

(ii) $(V_A(x) \geq i \min\{V_A(x*y), V_A(y)\}$ $(\forall\ x, y \in X)$

that is,

$t_A(0) \geq t_A(x),\ \ 1 - f_A(0) \geq 1 - f_A(x)$, and

$(t_A(x) \geq \min\{t_A(x*y), t_A(y)\}$ $(1 - f_A(x) \geq \min\{1 - f_A(x*y), 1 - f_A(y)\}$ for all $x, y \in X$

Definition 2.9:[11] Let A be a vague set of a universe X with the true- membership function t_A and the false- membership function f_A. The (α, β)- cut of the vague set A is a crisp subset $A_{(\alpha, \beta)}$of the set X given by $A_{(\alpha,\beta)} = \{x \in X / V_A(x) \geq [\alpha, \beta]\}$. Clearly $A_{(0,0)}$=X. The (α,β)- cut of the vague set A are also called vague cuts of A.

Definition 2.10:[11] The α- cut of the vague set A is a crisp subset A_α of the set X given by A_α= $A_{(\alpha,\alpha)}$ .ThusA_0=X, and if α ≥ β then $A_\beta \subseteq A_\alpha$ and A(α, β)= A_α. Equivalently, we define the α-cut as $A_\alpha = \{x : x \in X, t_A(x) \geq \alpha\}$.

3. Translation of vague H- ideal

In this paper, we take T = $1 - \sup\{t_A(x) / x \in X\}$ for any vague set V_A = [t_A,1-f_A] of X.

Definition 3.1: Let V_A = [t_A,1-f_A] be a vague subset of X and let $\alpha \in [0, T]$. An object having the form $(V_A)^T_\alpha = [(t_A)^T_\alpha, (1 - f_A)^T_\alpha]$ is called a vague α- translation of A if $(V_A)^T_\alpha(x) = V_A(x) + \alpha$ for all $x \in X$.where (i.e.,) $(t_A)^T_\alpha(x) = t_A(x) + \alpha$ and $(1 - f_A)^T_\alpha(x) = 1 - f_A(x) + \alpha$.

Example 3.3: Let X = {0, 1, 2, 3} be a BCK- algebra with the following cayley table:

*	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	3	3	0

Define a vague set V_A=[t_A,1-f_A] in X as follows:

X	0	1	2	3
V_A	[0.4,0.8]	[0.3,0.7]	[0.3,0.6]	[0.3,0.6]

Then V_A is a vague H- ideal of X and T = 0.6. If we take α = 0.12, then the vague α- translation $(V_A)^T_\alpha = [(t_A)^T_\alpha, (1 - f_A)^T_\alpha]$ of V_A is given by,

X	0	1	2	3
$(V_A)^T_\alpha$	[0.52,0.92]	[0.42,0.82]	[0.42,0.72]	[0.42,0.72]

Then $(V_A)^T_\alpha$ is also a Vague H-ideal of X.

Theorem 3.4: If V_A=[t_A,1-f_A] is a vague H- ideal of X, then the vague α- translation of $(V_A)^T_\alpha = [(t_A)^T_\alpha, (1 - f_A)^T_\alpha]$of A is a vague H- ideal of X for all $\alpha \in [0, T]$.

Proof: Let V_A=[t_A,1-f_A] be a vague H- ideal of X and $\alpha \in [0, T]$, then $(V_A)^T_\alpha(0) = V_A(0) + \alpha \geq V_A(x) + \alpha = (V_A)^T_\alpha(x)$ for all $x \in X$. Now,

$$(V_A)^T_\alpha(x*z) = V_A(x*z) + \alpha \geq \min\{V_A(x*(y*z)),\ V_A(y)\} + \alpha$$

$$= \min\{V_A(x*(y*z)) + \alpha,\ V_A(y) + \alpha\}$$

$$= \min\{(V_A)^T_\alpha(x*(y*z))\ ,\ (V_A)^T_\alpha(y)\ \} \quad \forall x, y, z \in Z$$

Hence the vague α- translation of $(V_A)^T_\alpha = [(t_A)^T_\alpha, (1 - f_A)^T_\alpha]$ of A is a vague H- ideal of X.

Theorem 3.5: If $V_A=[t_A,1-f_A]$ is a vague subset of X such that a vague α-translation $(V_A)_\alpha^T=[(t_A)_\alpha^T,(1-f_A)_\alpha^T]$ of A is a vague H-ideal of X for some $\alpha\in[0,T]$. Then $V_A=[t_A,1-f_A]$ is a vague H-ideal of X.

Proof: Assume that $(V_A)_\alpha^T=[(t_A)_\alpha^T,(1-f_A)_\alpha^T]$ is a vague H- ideal of X for some $\alpha\in[0,T]$. Let $x,y\in X$. We have $V_A(0)+\alpha=(V_A)_\alpha^T(0)\geq(V_A)_\alpha^T(x)=V_A(x)+\alpha$ which implies $V_A(0)\geq V_A(x)$. Now we have

$$\begin{aligned}V_A(x*z)+\alpha=(V_A)_\alpha^T(x*z)&\geq\min\{(V_A)_\alpha^T(x*(y*z)),\ (V_A)_\alpha^T(y)\}\\&=\min\{V_A(x*(y*z))+\alpha,\ V_A(y)+\alpha\}\\&=\min\{(V_A)(x*(y*z)),(V_A)(y)\}+\alpha\end{aligned}$$

which implies that $V_A(x*z)\geq\min\{V_A(x*(y*z)),V_A(y)\}$ for all $x,y,z\in X$. Hence $V_A=[t_A,1-f_A]$ is a vague H-ideal of X.

Theorem 3.6: If the vague α- translation $(V_A)_\alpha^T=[(t_A)_\alpha^T,(1-f_A)_\alpha^T]$ of A is a vague H-ideal of X for all $\alpha\in[0,T]$ then it must be a vague sub algebra of X.

Proof: Let the vague α- translation $(V_A)_\alpha^T=[(t_A)_\alpha^T,(1-f_A)_\alpha^T]$ of A be a vague H-ideal of X. Then we have $(V_A)_\alpha^T(x*z)\geq\min\{(V_A)_\alpha^T(x*(y*z)),(V_A)_\alpha^T(y)\}$ for all $x,y,z\in X$. Substituting y for z we get

$$\begin{aligned}(V_A)_\alpha^T(x*y)&\geq\min\{(V_A)_\alpha^T(x*(y*y)),(V_A)_\alpha^T(y)\}\\&=\min\{(V_A)_\alpha^T(x*0)),(V_A)_\alpha^T(y)\}=\min\{(V_A)_\alpha^T(x),(V_A)_\alpha^T(y)\}\end{aligned}$$

Therefore, $(V_A)_\alpha^T$ is a vague sub algebra of X.

Proposition 3.7: Let $V_A=[t_A,1-f_A]$ be a vague subset of X such that a vague α-translation $(V_A)_\alpha^T=[(t_A)_\alpha^T,(1-f_A)_\alpha^T]$ of A is a vague ideal of X for $\alpha\in[0,T]$. If $(x*a)*b=0$ for all $x,a,b\in X$, then $(V_A)_\alpha^T(x)\geq\min\{(V_A)_\alpha^T(a),(V_A)_\alpha^T(b)\}$.

Proof: Let $x,a,b\in X$ be such that $(x*a)*b=0$. Then

$$\begin{aligned}(V_A)_\alpha^T(x)&\geq\min\{(V_A)_\alpha^T(x*a),(V_A)_\alpha^T(a)\}\geq\min\{\min\{(V_A)_\alpha^T((x*a)*b),(V_A)_\alpha^T(b)\},(V_A)_\alpha^T(a)\}\\&=\min\{\min\{(V_A)_\alpha^T(0),(V_A)_\alpha^T(b)\},(V_A)_\alpha^T(a)\}\\&=\min\{(V_A)_\alpha^T(b)\},(V_A)_\alpha^T(a)\}\quad(\text{since }(V_A)_\alpha^T(0)\}\geq(V_A)_\alpha^T(b))\\&=\min\{(V_A)_\alpha^T(a)\},(V_A)_\alpha^T(b)\}\end{aligned}$$

Theorem 3.8: Let $V_A=[t_A,1-f_A]$ be a vague subset of X such that a vague α-translation $(V_A)_\alpha^T=[(t_A)_\alpha^T,(1-f_A)_\alpha^T]$ of A is a vague ideal of X for $\alpha\in[0,T]$. If it satisfies the condition $(V_A)_\alpha^T(x*y)\geq(V_A)_\alpha^T(x)$ for all $x,y\in X$,then the vague α-translation $(V_A)_\alpha^T$ of A is a vague H-ideal of x.

Proof: Let the vague α-translation $(V_A)_\alpha^T$ of A be a vague ideal of X. For any $x,y,z\in X$, we have

$$\begin{aligned}(V_A)_\alpha^T(x*z)&\geq\min\{(V_A)_\alpha^T((x*z)*(y*z)),(V_A)_\alpha^T(y*z)\}\\&=\min\{(V_A)_\alpha^T((x*(y*z))*z),(V_A)_\alpha^T(y*z)\}\geq\min\{(V_A)_\alpha^T(x*(y*z)),(V_A)_\alpha^T(y)\}\end{aligned}$$

Hence the vague α-translation $(V_A)_\alpha^T$ of A is a vague H-ideal of X for some $\alpha\in[0,T]$.

Theorem 3.9: Let V_A is a vague subset of associative BCK- algebra X such that the vague α translation $(V_A)_\alpha^T = [(t_A)_\alpha^T, (1-f_A)_\alpha^T]$ of A is a vague ideal of X for $\alpha \in [0,T]$.then the vague α translation $(V_A)_\alpha^T$ of V_A is a vague H-ideal of X.

Proof: Let the vague α translation $(V_A)_\alpha^T$ of $V_A$be a vague ideal of X. For any $x, y, z \in X$, we have

$$(V_A)_\alpha^T(x*z) \geq \min\{(V_A)_\alpha^T((x*z)*y),(V_A)_\alpha^T(y)\} = \min\{(V_A)_\alpha^T((x*y)*z),(V_A)_\alpha^T(y)\}$$
$$= \min\{(V_A)_\alpha^T(x*(y*z)),(V_A)_\alpha^T(y)\}$$

Hence the vague α translation $(V_A)_\alpha^T$ of V_A is a vague H-ideal of X.

Theorem 3.10: Let $V_A=[t_A,1-f_A]$ be a vague subset of X such that a vague α-translation $(V_A)_\alpha^T = [(t_A)_\alpha^T, (1-f_A)_\alpha^T]$ of A is a vague H- ideal of X for $\alpha \in [0,T]$, then the sets $T_{V_A} = \{x \in X \ / \ (V_A)_\alpha^T(x) = (V_A)_\alpha^T(0)\}$ are H-ideals of X.

Proof: Suppose that $(V_A)_\alpha^T = [(t_A)_\alpha^T, (1-f_A)_\alpha^T]$ is a vague ideal of X. Then $(V_A)_\alpha^T$ is a vague H-ideal of X. Obviously $0 \in T_{V_A}$. Let $x, y, z \in X$ be such that $x*(y*z) \in T_{V_A}$ *and* $y \in T_{V_A}$. Then $(V_A)_\alpha^T(x*(y*z)) = (V_A)_\alpha^T(0) = (V_A)_\alpha^T(y)$ and so $(V_A)_\alpha^T(x*z) \geq \min\{(V_A)_\alpha^T(x*(y*z)) , (V_A)_\alpha^T(y) \} = (V_A)_\alpha^T(0)$. Since $(V_A)_\alpha^T$ is a vague H- ideal of X. We conclude that $(V_A)_\alpha^T(x*z) = (V_A)_\alpha^T(0)$. This implies $(V_A)_\alpha^T(x*z) + \alpha = (V_A)_\alpha^T(0) + \alpha$ so that $x*z \in T_{V_A}$. Therefore T_{V_A} is a H- ideal of X.

Proposition 3.11: Let the vague α-translation $(V_A)_\alpha^T = [(t_A)_\alpha^T, (1-f_A)_\alpha^T]$ of A be a vague H-ideal of X for $\alpha \in [0,T]$. If $x \leq y$. Then $(V_A)_\alpha^T(x) \geq (V_A)_\alpha^T(y)$, that is $(V_A)_\alpha^T$ is order-reserving.

Proof: Let $x, y, z \in X$ be such that $x \leq y$. then $x*y = 0$ and hence

$$(V_A)_\alpha^T(x) = (V_A)_\alpha^T(x*0) \geq \min\{(V_A)_\alpha^T(x*(y*0)),(V_A)_\alpha^T(y)\}$$
$$= \min\{(V_A)_\alpha^T(x*y),(V_A)_\alpha^T(y)\} = \min\{(V_A)_\alpha^T(0),(V_A)_\alpha^T(y)\} = (V_A)_\alpha^T(y)$$

Theorem 3.12: Let $V_A = [t_A, 1-f_A]$be a vague subset of X such that the vague α-translation $(V_A)_\alpha^T = [(t_A)_\alpha^T, (1-f_A)_\alpha^T]$ of A be a vague ideal of X for $\alpha \in [0,T]$,then the following assertions are equivalent:

(i) $(V_A)_\alpha^T$ of A be a vague H-ideal of X,

(ii) $(V_A)_\alpha^T(x*y) \geq (V_A)_\alpha^T(x*(0*y))$ for all $x, y \in X$,

(iii) $(V_A)_\alpha^T((x*y)*z) \geq (V_A)_\alpha^T(x*(y*z))$ for all $x, y, z \in X$

Proof: (i)$\Rightarrow$(ii) Let $(V_A)_\alpha^T = [(t_A)_\alpha^T, (1-f_A)_\alpha^T]$ be a vague H- ideal of X. Then for all $x, y \in X$ we have $(V_A)_\alpha^T(x*y) \geq \min\{(V_A)_\alpha^T(x*(0*y)),(V_A)_\alpha^T(0)\} = (V_A)_\alpha^T(x*(0*y))$. Therefore, the inequality (ii) is satisfied.

(ii) $\Rightarrow$(iii)Assume that (ii) is satisfied. For all $x, y, z \in X$, we have

$$((x*y)*(0*z))*(x*(y*z)) = ((x*y)*(x*(y*z)))*(0*z) \leq ((y*z)*y)*(0*z)$$
$$= ((y*y)*z)*(0*z) = (0*z)*(0*z) = 0$$

It follows from proposition 3.11 that $(V_A)_\alpha^T((x*y)*(0*z))*(x*(y*z)) \geq (V_A)_\alpha^T(0)$. Since $(V_A)_\alpha^T$ are vague H- ideal of X, Therefore, we have

$(V_A)_\alpha^T((x*y)*(0*z))*(x*(y*z)) = (V_A)_\alpha^T(0)$. Using (ii)

$$(V_A)_\alpha^T((x*y)*z) \geq (V_A)_\alpha^T((x*y)*(0*z))$$
$$= \min\{(V_A)_\alpha^T(((x*y)*(0*z))*(x*(y*z))),(V_A)_\alpha^T(x*(y*z))\}$$
$$= \min\{(V_A)_\alpha^T(0),(V_A)_\alpha^T(x*(y*z))\} = (V_A)_\alpha^T(x*(y*z))$$

Therefore, inequality (iii) is also satisfied .

(iii) $\Rightarrow$ (i) Assume that (iii) is valid. For all $x, y, z \in X$, we have

$$(V_A)_\alpha^T((x*z) \geq \min\{(V_A)_\alpha^T((x*z)*y),(V_A)_\alpha^T(y)\} = \min\{(V_A)_\alpha^T((x*y)*z),(V_A)_\alpha^T(y)\}$$
$$= \min\{(V_A)_\alpha^T(x*(y*z)),(V_A)_\alpha^T(y)\}$$

Therefore, $(V_A)_\alpha^T = [(t_A)_\alpha^T,(1-f_A)_\alpha^T]$ is a vague H-ideal of X. Hence, the assertion (i) holds.

Theorem3.13: Let $V_A = [t_A, 1-f_A]$be a vague subset of X such that the vague α-translation $(V_A)_\alpha^T = [(t_A)_\alpha^T,(1-f_A)_\alpha^T]$of A be a vague ideal of X for $\alpha \in [0,T]$,then the following assertions are equivalent:

(i) $(V_A)_\alpha^T$ of A be a vague H-ideal of X,

(ii) $(V_A)_\alpha^T((x*z)*y) \geq (V_A)_\alpha^T((x*z)*(0*y))$for all $x, y, z \in X$,

(iii) $(V_A)_\alpha^T(x*y) \geq \min\{(V_A)_\alpha^T((x*z)*(0*y)),(V_A)_\alpha^T(z)\}$ for all $x, y, z \in X$

Proof: (i) $\Rightarrow$ (ii) is obvious.

(ii) $\Rightarrow$ (iii) Assume that (ii) is valid. For all $x, y, z \in X$,we have

$$(V_A)_\alpha^T((x*y) \geq \min\{(V_A)_\alpha^T((x*y)*z),(V_A)_\alpha^T(z)\} = \min\{(V_A)_\alpha^T((x*z)*y),(V_A)_\alpha^T(z)\}$$
$$\geq \min\{(V_A)_\alpha^T((x*z)*(0*y)),(V_A)_\alpha^T(z)\}.\ \text{Therefore, (iii) is satisfied.}$$

(iii) $\Rightarrow$ (i) Assume that (iii) is valid. Therefore, for all $x, y, z \in X$, we have

$(V_A)_\alpha^T(x*y) \geq \min\{(V_A)_\alpha^T((x*z)*(0*y)),(V_A)_\alpha^T(z)\}$ Putting Z=0 we get

$$(V_A)_\alpha^T(x*y) \geq \min\{(V_A)_\alpha^T((x*0)*(0*y)),(V_A)_\alpha^T(0)\} = \min\{((V_A)_\alpha^T(x*(0*y)),(V_A)_\alpha^T(0)\}$$
$$= (V_A)_\alpha^T(x*(0*y))$$

Hence the proof.

Theorem3.14: Let $V_A = [t_A, 1-f_A]$be a vague subset of X and $\alpha \in [0,T]$, then the vague α-translation $(V_A)_\alpha^T = [(t_A)_\alpha^T,(1-f_A)_\alpha^T]$of V_A be a vague H ideal of X if and only if A_α is a H- ideal of X, for all $t \in \mathrm{Im}(V_A)$ with t > α.

Proof: Suppose that $(V_A)_\alpha^T$ is a vague H- ideal of X and $t \in \mathrm{Im}(V_A)$ with t > α. Since $(V_A)_\alpha^T(0) \geq (V_A)_\alpha^T(x)$, for all $x \in X$, we have $V_A(0) + \alpha = (V_A)_\alpha^T(0) \geq (V_A)_\alpha^T(x) \geq V_A(x) + \alpha \geq t$ for $x \in A_\alpha$.Hence $0 \in A_\alpha$. Let $x, y, z \in X$ such that $x*(y*z), y \in A_\alpha$. Then $V_A(x*(y*z)) \geq t-\alpha$ and $V_A(y) \geq t-\alpha$ I.E., $(V_A)_\alpha^T(x*(y*z)) = V_A(x*(y*z)) + \alpha \geq t$ and $(V_A)_\alpha^T(y) = V_A(y) + \alpha \geq t.$ Since $(V_A)_\alpha^T$ is a vague H- ideal. So, we have $V_A(x*z) + \alpha = (V_A)_\alpha^T(x*z) \geq \min\{(V_A)_\alpha^T(x*(y*z)),(V_A)_\alpha^T(y)\} \geq t$ that is, $V_A(x*z) \geq t-\alpha$ so that $x*z \in A_\alpha$. Therefore, A_α is a H- ideal of X.

Conversely, suppose that A_α is a H- ideal of X, for all $t \in \mathrm{Im}(V_A)$ with t > α. If there exist $a \in X$ such that $(V_A)_\alpha^T(0) < \lambda \leq (V_A)_\alpha^T(x)$, then $V_A(a) \geq \lambda - \alpha$ but $V_A(0) < \lambda - \alpha$. This shows that

$a \in A_\alpha$ and $0 \notin A_\alpha$. This is a contradiction, and $(V_A)_\alpha^T(0) \geq (V_A)_\alpha^T(x)$, for all $x \in X$. Now we assume that there exist $a, b, c \in X$ such that

$(V_A)_\alpha^T(a*c) < \xi \leq \min\{(V_A)_\alpha^T(a*(b*c)), (V_A)_\alpha^T(b)\}$. Then $V_A(a*(b*c)) \geq \beta - \alpha$ and $V_A(b) \geq \beta - \alpha$ but $V_A(a*c) < \beta - \alpha$. Hence, $a*(b*c) \in A_\alpha$ and $b \in A_\alpha$ but $a*c \notin A_\alpha$ which is a contradiction. Thus, $(V_A)_\alpha^T(a*c) \geq \min\{(V_A)_\alpha^T(a*(b*c)), (V_A)_\alpha^T(b)\}$, for all $a, b, c \in X$.

Consequently, $(V_A)_\alpha^T$ is a vague H- ideal of X.

Definition 3.15: Let $V_A = [t_A, 1-f_A]$ be a vague subset of X and $\lambda \in [0,1]$. An object having the form $(V_A)_\lambda^S = [(t_A)_\lambda^S, (1-f_A)_\lambda^S]$ is called a vague λ multiplication of V_A if $(V_A)_\lambda^S(x) = V_A(x) \cdot \lambda$ for all $x \in X$.

Example 3.16: Let X = {0, 1, 2, 3} be a BCK- algebra which is given in example 3.3 and consider a fuzzy subalgebra V_A of X that is defined in example 3.3. If we take λ = 0.1, then the vague λ- multiplication $(V_A)_{0.1}^S$ of V_A is given by,

X	0	1	2	3
$(V_A)_{0.1}^S$	[0.04,0.08]	[0.03,0.07]	[0.03,0.06]	[0.03,0.06]

Therefore, clearly $(V_A)_{0.1}^S$ is a Vague H- ideal of X.

Theorem 3.17: If V_A is a vague H-ideal of X, then the vague λ- multiplication of V_A is a vague H- ideal of X, for all $\lambda \in [0,1]$.

Proof: Straightforward.

Theorem 3.18: Let V_A be a vague subset of X. Then V_A is a vague H- ideal of X if and only if the vague λ- multiplication $(V_A)_\lambda^S$ of V_A is a vague H- ideal of X, for all $\lambda \in [0,1]$.

Proof: Necessity follows from the above theorem. Let $\lambda \in [0,1]$ be such that $(V_A)_\lambda^S$ be a vague H-ideal of X. Then $V_A(0) \cdot \lambda = (V_A)_\lambda^S(0) \geq (V_A)_\lambda^S(x) = V_A(x) \cdot \lambda$ which implies that $V_A(0) \geq V_A(x)$, for all $x \in X$. Also, for $x, y, z \in X$, we have,

$$V_A(x*z) \cdot \lambda = (V_A)_\lambda^S(x*z) \geq \min\{(V_A)_\lambda^S(x*(y*z)), (V_A)_\lambda^S(y)\}$$
$$= \min\{V_A(x*(y*z)) \cdot \lambda, V_A(y) \cdot \lambda\} = \min\{V_A(x*(y*z)), V_A(y)\} \cdot \lambda$$

which implies that $V_A(x*z) \geq \min\{V_A(x*(y*z)), V_A(y)\}$, for all $x, y, z \in X$. Hence, V_A is a vague H- ideal of X.

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