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THE GENERALIZED INVERSE OF CON. SECONDARY K-NORMAL BIMATRICES

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ABSTRACT

The secondary k-generalized inverse exists for particular kind of square matrices. Which is also satiesfie the moorepenrose equation.

Keywords:Normalbimatrices, conjugate secondary k-normal matrices, secondary k- normal matricesmoorepenrosose equation.

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I. INTRODUCTION

Matrices provide a very powerful tool for dealing with linear models. Bimatrices are an advanced tool which can handle over one linear model at a time. Bimatrices will be useful when time bound comparisons are needed in the analysis of the model[6]. Unlike bimatrices can be of several types.

We have to mincle to the bimatrices and con. s-knorml matrices. The concept of con.s-k normal bimatrices are introduced [4]. In this paper we describe secondary k-normal generalized inverse of a square bimatrix, as the uique solution of a certain set of equation. This secondary k-generalized inverse exists for particular kind of square matrices. Which is also satiesfie the moore penrose equation.

II. PRELIMINARIES AND NOTATIONS

First we wish to mention that when we have a collection of **m×n** bimatrices say M_B then M_B need not be even closed with respect to addition. Further we make a defition as **m×n** zero bimatrix[6].

Thus we make the following special type of concession in case of zero and unit **m×m** bimatrices. Appropriate changes are made in case of zero **m×n**bimatrix.

We first illustrate this by the following example: that in general sum of two bimatrices is not a bimatrix.

Let $\mathbf{C_{n\times n}}$ denote the space of **n×n** complex bimatrices. We deal with secondary k-generalized inverse of con. s-k normal bimatrices [2]. Some of the condition and properties used in this paper.

III. DEFINITIONS AND THEOREMS

DEFINITION:1

A bimatrix A_B is defined as the union of two rectangular or square array of numbers A_1andA_2 arranged into rows and columns. It is written as follows A_B=$A_1 \cup A_2$ where$A_1 \neq A_2$with

$$A_1=\begin{bmatrix} a_{11}^1 & a_{12}^1 \; \dots\dots. & a_{1n}^1 \\ a_{21.}^1 & a_{22.}^1 \; \dots\dots & a_{2n.}^1 \\ \vdots & \vdots & \vdots \\ \vdots & . & . \\ a_{m1}^1 & a_{m2}^1 \cdots & a_{mn}^1 \end{bmatrix}$$

$$A_2=\begin{bmatrix} a_{11}^2 & a_{12}^2 \; \dots\dots. & a_{1n}^2 \\ a_{21.}^2 & a_{22.}^2 \; \dots\dots & a_{2n.}^2 \\ \vdots & \vdots & \vdots \\ \vdots & . & . \\ a_{m1}^2 & a_{m2}^2 \cdots & a_{mn}^2 \end{bmatrix}$$

\`∪\` the notational convenience (symbol) only.

DEFINITION 2

Let $A_B = A_1 \cup A_2$ be a bimatrix. If both A_1 and A_2 are square matrices then A_B is called the square bimatrix.

If one of the matrices in the bimatrix $A_B = A_1 \cup A_2$ is square and other is rectangular or if both A_1 and A_2 are rectangular bimatrices say $\mathbf{m_1 \times n_1}$ and $\mathbf{m_2 \times n_2}$ with $\mathbf{m_1 \neq m_2}$ or $\mathbf{n_1 \neq n_2}$ then we say A_B is a mixed bimatrix.

The following are example of a square bimatrix and the mixed bimatrix.

DEFINITION:3

A bimatrix $A_B \in \mathbf{C_{nxn}}$ is said to be con. K-normal (s-k normal)bimatrix if

$A_B(K_BV_BA_B^*V_BK_B)=\overline{(K_BV_BA_B^*V_BK_B)A_B}$

$(A_1K_1V_1A_1^*V_1K_1)\cup(A_2K_2V_2A_2^*V_2K_2)=\overline{(K_1V_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2)}$

DEFINITION:4

A bimatrix $A_B \in \mathbf{C_{nxn}}$ is said to be con.s-k-unitary if

$A_B(K_BV_BA_B^*V_BK_B)=\overline{(K_BV_BA_B^*V_BK_B)A_B}=\overline{I_B}$

$(A_1K_1VA_{1_1}^*K_1)\cup(A_2K_2V_2A_2^*V_2K)=\overline{(K_1V_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2)}$

$=\overline{I}_I \cup \overline{I}_2$

DEFINITION:5

Let $A_B \in C_{n\times n}$. The unique solution of

$A_BX_BA_B = \overline{A_B}$,

$X_BA_BX_B = \overline{X_B}$,

$K_BV_B(A_BX_B)^*V_BK_B = \overline{A_BX_B}$ and

$K_BV_B(X_BA_B)^*V_BK_B = \overline{X_BA_B}$ is called con. secondary k-generalized inverse of A_B is written as $A_B^{\dagger SK}$.

SOME OF THE THEOREMS

In this section the concept of con. Sk-normal bimatrices is introduced for complex square bimatrices as a special case of Generalised inverse of con.sk-normal bimatrices verified. Which is also satiesfie the moore penrose equation.

THEOREM 1:

If $A_B \in C_{n\times n}$, then we assume that if $A_B \neq 0$. Therefore $A_1 \cup A_2 \neq 0$ then,

$A_B(K_B V_B A_B^* V_B K_B) \neq 0$

$(A_1 \cup A_2)(K_1 \cup K_2)(V_1 \cup V_2)(A_1^* \cup A_2^*)(V_1 \cup V_2)(K_1 \cup K_2) \neq 0$

$(A_1 K_1 V_1 A_1^* V_1 K_1) \cup (A_2 K_2 V_2 A_2^* V_2 K_2) \neq 0$

PROOF:

Suppose

$A_B(K_B V_B A_B^* V_B K_B) = 0$

$A_B = 0$

$(A_1 \cup A_2) = 0$

$K_B V_B (A_B + C_B)^* V_B K_B = \overline{(K_B V_B A_B^* V_B K_B) + (K_B V_B C_B^* V_B K_B)}$

$K_B V_B (\lambda A_B)^* V_B K_B = \overline{\overline{\lambda}(K_B V_B A_B^* V_B K_B)}$

$K_B V_B (\lambda A_B)^* V_B K_B = \lambda\overline{(K_B V_B A_B^* V_B K_B)}$

$K_B V_B (C_B A_B)^* V_B K_B = \overline{(K_B V_B A_B^* V_B K_B)(K_B V_B C_B^* V_B K_B)}$

Now, if

$C_B A_B (K_B V_B A_B^* V_B K_B) = \overline{D_B A_B (K_B V_B A_B^* V_B K_B})$

$C_B A_B (K_B V_B A_B^* V_B K_B) - \overline{D_B A_B (K_B V_B A_B^* V_B K_B}) = 0$

$C_B A_B (K_B V_B A_B^* V_B K_B) - \overline{D_B A_B (K_B V_B A_B^* V_B K_B})(K_B V_B (C_B - D_B)^* V_B K_B) = 0$

$(C_B A_B - \overline{D_B A_B})(K_B V_B (C_B A_B - D_B A_B)^* V_B K_B) = 0$

$(C_B A_B - \overline{D_B A_B}) = 0$

$C_B A_B = \overline{D_B A_B}$

$(C_1 \cup C_2)(A_1 \cup A_2) = \overline{(D_1 \cup D_2)(A_1 \cup A_2)}$

$(C_1 A_1) \cup (C_2 A_2) = \overline{(D_1 A_1) \cup (D_2 A_2)}$

Therefore

$C_B A_B (K_B V_B A_B^* V_B K_B) = \overline{D_B A_B (K_B V_B A_B^* V_B K_B})$

$C_B A_B = \overline{D_B A_B}$

$(C_1 \cup C_2)(A_1 \cup A_2) = \overline{(D_1 \cup D_2)(A_1 \cup A_2)}$

$(C_1 A_1) \cup (C_2 A_2) = \overline{(D_1 A_1) \cup (D_2 A_2)}$

Similarly,

$C_B (K_B V_B A_B^* V_B K_B) A_B = \overline{D_B (K_B V_B A_B^* V_B K_B) A_B}$

$C_B (K_B V_B A_B^* V_B K_B) = \overline{D_B (K_B V_B A_B^* V_B K_B)}$

$(C_1 \cup C_2)(K_1 \cup K_2)(V_1 \cup V_2)(A_1^* \cup A_2^*)(V_1 \cup V_2)(K_1 \cup K_2)$

$= \overline{(D_1 \cup D_2)(K_1 \cup K_2)(V_1 \cup V_2)(A_1^* \cup A_2^*)(V_1 \cup V_2)(K_1 \cup K_2)}$

$(C_1 K_1 A_1^* V_1 K_1) \cup (C_2 K_2 A_2^* V_2 K_2) = \overline{(D_1 K_1 V_1 A_1^* V_1 K_1) \cup (D_2 K_2 V_2 A_2^* V_2 K_2)}$

Therefore A_B is inverse exist.

Hence proved

THEOREM 2:

For any $A_B \in C_{n\times n}$,

$A_B X_B A_B = \overline{A_B}$ ……………………………(1)

$X_B A_B X_B = \overline{X_B}$ ……………………………(2)

$K_B V_B (A_B X_B)^* V_B K_B = \overline{A_B X_B}$ ……………………………(3)

$K_B V_B (X_B A_B)^* V_B K_B = \overline{X_B A_B}$ …………………………… (4)

have a unique solution for any $A_B \in C_{n\times n}$.

PROOF:

Let

$A_1 = A_2 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \overline{A_1} = \overline{A_2}$

$X_1 = X_2 = \begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix} = \overline{X_1} = \overline{X_2}$

$K_1 = K_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \overline{K_1} = \overline{K_2}$

$V_1 = V_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \overline{V_1} = \overline{V_2}$

From **(1)**

$A_B X_B A_B = \overline{A_B}$

$(A_1 \cup A_2)(X_1 \cup X_2)(A_1 \cup A_2) = \overline{(A_1 \cup A_2)}$

$(A_1 X_1 A_1) \cup (A_2 X_2 A_2) = \overline{(A_1 \cup A_2)}$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cup \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \overline{\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cup \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cup \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cup \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Therefore $A_B X_B A_B = \overline{A_B}$

From **(2)**

$X_B A_B X_A = \overline{X_B}$

$(X_1 \cup X_2)(A_1 \cup A_2)(X_1 \cup X_2) = \overline{(X_1 \cup X_2)}$

$(X_1 A_1 X_1) \cup (X_2 A_2 X_2) = \overline{(X_1 \cup X_2)}$

$$\begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix} \cup \begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix} = \overline{\begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix} \cup \begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix}}$$

$$\begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix} \cup \begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix} \cup \begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix}$$

Therefore $X_B A_B X_A = \overline{X_B}$

From **(4)**

$K_B V_B (X_B A_B)^* V_B K_B = \overline{X_B A_B}$

$(K_1 \cup K_2)(V_1 \cup V_2)(X_1^* \cup X_2^*)(A_1^* \cup A_2^*)(V_1 \cup V_2)(K_1 \cup K_2) = \overline{(X_1 \cup X_2)(A_1 \cup A_2)}$

$(K_1 V_1 X_1^* A_1^* V_1 K_1) \cup (K_2 V_2 X_2^* A_2^* V_2 K_2) = \overline{(X_1 A_1) \cup (X_2 A_2)}$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\cup \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \overline{\begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cup \begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}}$$

$$\begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix} \cup \begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix} \cup \begin{pmatrix} 1/2 & 0 \\ 1/2 & 0 \end{pmatrix}$$

Therefore $K_B V_B (X_B A_B)^* V_B K_B = \overline{X_B A_B}$

First, we shall show that equation (2) and (3) are equivalent to the single equation.

$X_B K_B V_B (A_B X_B)^* V_B K_B = X_B$ .... **(5)**

$(X_1 \cup X_2)(K_1 \cup K_2)(V_1 \cup V_2)(A_1^* \cup A_2^*)(X_1^* \cup X_2^*)(V_1 \cup V_2)(K_1 \cup K_2) = \overline{(X_1 \cup X_2)}$

$(X_1 K_1 V_1 A_1^* X_2^* V_1 K_1) \cup (X_2 K_2 V_2 A_2^* X_2^* V_2 K_2) = \overline{(X_1 \cup X_2)}$

From equation (2) and (3),(4) follows. Since it is merely (3) substituted in (2) conversely, equation (5) implies,

$A_B X_B K_B V_B (A_B X_B)^* V_B K_B = \overline{A_B X_B}$

$(A_1 \cup A_2)(X_1 \cup X_2)(K_1 \cup K_2)(V_1 \cup V_2)(A_1^* \cup A_2^*)(X_1^* \cup X_2^*)(V_1 \cup V_2)(K_1 \cup K_2) = \overline{(A_1 \cup A_2)(X_1 \cup X_2)}$

$(A_1 X_1 K_1 V_1 A_1^* X_1^* V_1 K_1) \cup (A_2 X_2 K_2 V_2 A_2^* X_2^* V_2 K_2) = \overline{(A_1 X_1) \cup (A_2 X_2)}$

Since the left hand side is con. s-k hermitian, (3) follows. By substituting (3) in (5) we get $X_BA_BX_B = \overline{X_B}$ which is actually (2). Therefore (2) and (4) are equivalent to (5). Similarly, (1) and (4) are equivalent to the equation.

$X_BA_B(K_BV_BA_B^*V_BK_B) = \overline{(K_BV_BA_B^*V_BK_B)}$ ………………………….. **(6)**

$(X_1A_1K_1V_1A_1^*V_1K_1)\cup(X_2A_2K_2V_2A_2^*V_2K_2) = \overline{(K_1V_1A_1^*V_1K_1)\cup(K_2V_2A_2^*V_2K_2)}$ Thus to find a solution for the given set of equations, it is enough to find an x satisfying (5) and (6). Now the expressions

$((K_BV_BA_B^*V_BK_B)A_B),((K_BV_BA_B^*V_BK_B)A_B)^2,((K_BV_BA_B^*V_BK_B)A_B)^3,\ldots.$

$(K_1V_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2),((K_1V_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2))^2,$

$((K_1V_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2))^3,\ldots..$

Cannot all be linearly independent (i.e) there exists a relation.

$\lambda_1((K_BV_BA_B^*V_BK_B)A_B)+\lambda_2((K_BV_BA_B^*V_BK_B)A_B)^2+ \ \ldots. \ +\lambda_K((K_BV_BA_B^*V_BK_B)A_B)^K=0$ …………………………**(7)**

$\lambda_1((K_1V_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2))+\lambda_2((K_1V_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2))^2+\ldots\ldots+$

$\lambda_K((K_1V_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2))^K=0$

Where $\lambda_1,\lambda_2,\ldots.\lambda_k$ are not all zero. Let λ_r be the first non-zero λ. (i.e) $\lambda_1=\lambda_2=\lambda_3=\ldots\ldots\lambda_{r-1}=0$

Therefore (7) implies that

$\lambda_r((K_BV_BA_B^*V_BK_B)A_B)^r=\overline{-\{\lambda_{r+1}((K_BV_BA_B^*V_BK_B)A_B)^{r+1}+\cdots+\lambda_m((K_BV_BA_B^*V_BK_B)A_B)^m\}}$

$\lambda_r((K_1V_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2))^r=$

$\overline{-\{\lambda_{r+1}((K_1V_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2))^{r+1}+\cdots+\lambda_m((K_1V_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2))^m\}}$

If we take

$C_B=\overline{-\lambda_r^{-1}\{\lambda_{r+1}1+\lambda_{r+2}((K_BV_BA_B^*V_BK_B)A_B)+\cdots+\lambda_m((K_BV_BA_B^*V_BK_B)A_B)^{m-r-1}\}}$

$C_B=\overline{-\lambda_r^{-1}\{\lambda_{r+1}1+\lambda_{r+2}((K_1V_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2))+\ldots..+}$

$\overline{\lambda_m((K_1V_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2))^{m-r-1}\}}$

Then,

$C_B((K_BV_BA_B^*V_BK_B)A_B)^{r+1}=$

$\overline{-\lambda_r^{-1}\{\lambda_{r+1}((K_BV_BA_B^*V_BK_B)A_B)^{r+1}+\cdots+\lambda_m((K_BV_BA_B^*V_BK_B)A_B)^m\}}$

$V_BA_B^*V_BK_B)A_B)^{r+1}=\overline{((K_BV_BA_B^*V_BK_B)A_B)^r}$

$C_B((K_1V_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2))^{r+1}=\overline{((K_1V_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2))^r\}}$

By using (2) and (3) repeatedly, we get

$C_B(K_BV_BA_B^*V_BK_B)A_B(K_BV_BA_B^*V_BK_B)A_B=\overline{K_BV_BA_B^*V_BK_B}$ …………………………**8)**

Now if we take $X_B=\overline{C_B(K_BV_BA_B^*V_BK_B)}$ then (8) implies that this X_B satisfies (6) implies (4), we have

$K_BV_B(X_BA_B)^*V_BK_B)(K_BV_BA_B^*V_BK_B)=\overline{K_BV_BA_B^*V_BK_B}$

$C_B(K_BV_B(X_BA_B)^*V_BK_B)(K_BV_BA_B^*V_BK_B)=\overline{C_B(K_BV_BA_B^*V_BK_B)}$

Therefore

$X_B=\overline{C_B(K_BV_BA_B^*V_BK_B)}$ satisfies (5),

$(X_1\cup X_2)=\overline{(C_1K_1V_1A_1^*V_1K_1)\cup(C_2K_2V_2A_2^*V_2K_2)}$

Thus $X_B=\overline{C_B(K_BV_BA_B^*V_BK_B)}$ is a solution for the given set of equations.

Now let us prove that this X_B is unique. Suppose that X_B and Y_B satisfy (5) and (6). Then by substituting (4) in (2) and (3) in (1), we obtain.

$(K_BV_B(X_BA_B)^*V_BK_B)X_B=\overline{X_B}$

$(K_1V_1X_1^*A_1^*V_1K_1X_1)\cup(K_2V_2X_2^*A_2^*V_2K_2X_2)=\overline{X_1\cup X_2}$ and

$(K_BV_BA_B^*X_B^*V_BK_B)A_B=\overline{A_B}$

$(K_1V_1A_1^*X_1^*V_1K_1A_1)\cup(K_2V_2A_2^*X_2^*V_2K_2A_2)=\overline{A_1\cup A_2}$

Also,

$Y_B = \overline{(K_B V_B (Y_B A_B)^* V_B K_B) Y_B}$

$Y_1 \cup Y_2 = \overline{(K_1 V_1 Y_1^* A_1^* V_1 K_1 Y_1) \cup (K_2 V_2 Y_2^* A_2^* V_2 K_2 Y_2)}$

and,

$K_B V_B A_B^* V_B K_B = \overline{(K_B V_B A_B^* V_B K_B) A_B Y_B}$

$(K_1 V_1 A_1^* V_1 K_1) \cup (K_2 V_2 A_2^* V_2 K_2) = \overline{(K_1 V_1 A_1^* V_1 K_1 Y_1) \cup (K_2 V_2 A_2^* V_2 K_2 Y_2)}$

Now,

$X_B = \overline{X_B (K_B V_B X_B^* V_B K_B)(K_B V_B X_B^* V_B K_B)}$

$= \overline{X_B (K_B V_B X_B^* V_B K_B)(K_B V_B X_B^* V_B K_B) A_B Y_B}$

$= \overline{X_B (K_B V_B (A_B X_B)^* V_B K_B) A_B Y_B}$

$= \overline{X_B A_B Y_B}$

$= \overline{X_B A_B (K_B V_B (Y_B A_B)^* V_B K_B) Y_B}$

$= \overline{X_B A_B (K_B V_B A_B^* V_B K_B)(K_B V_B Y_B^* V_B K_B) Y_B}$

$= \overline{(K_B V_B A_B^* V_B K_B)(K_B V_B Y_B^* V_B K_B) Y_B}$

$= \overline{(K_B V_B (Y_B A_B)^* V_B K_B) Y_B}$

$X_B = \overline{Y_B}$

Therefore X_B is unique.

Hence proved

THEOREM 3:

For $A_B \in C_{n\times n}$,

(i) $(A_B^{\dagger sk})^{\dagger sk} = \overline{A_B}$

(ii) $(K_B V_B (A_B^*)^{\dagger sk} V_B K_B) = \overline{(K_B V_B (A_B^{\dagger sk})^* V_B K_B)}$

(iii) If A_B is non singular, then $A_B^{\dagger sk} = \overline{A_B^{-1}}$

(iv) $(\lambda A)^{\dagger sk} = \overline{\lambda^{\dagger sk} A_B^{\dagger sk}}$

(v) $((K_B V_B A_B^* V_B K_B) A_B)^{\dagger sk} = \overline{A_B^{\dagger sk} (K_B V_B A_B^{\dagger sk} V_B K_B)^*}$

PROOF:

Let $A_B \in C_{n\times n}$,

(i) By the definition of con. s-k-g inverse, we have

$A_B^{\dagger sk} A_B A_B^{\dagger sk} = \overline{A_B^{\dagger sk}}$ and

$A_B^{\dagger sk} (A_B^{\dagger sk})^{\dagger sk} A_B^{\dagger sk} = \overline{A_B^{\dagger sk}}$

These two equation imply that

$A_B^{\dagger sk \dagger sk} = \overline{A_B}$

$(A_1^{\dagger sk} \cup A_2^{\dagger sk})^{\dagger sk} = \overline{A_1 \cup A_2}$

(ii) From the definition of $A^{\dagger sk}$, we have

$A_B A_B^{\dagger sk} A_B = \overline{A_B}$

$(K_B V_B A_B^* V_B K_B)(K_B V_B (A_B^{\dagger sk})^* V_B K_B)(K_B V_B A_B^* V_B K_B) = \overline{(K_B V_B A_B^* V_B K_B)}$

Also,

$(K_B V_B A_B^* V_B K_B)(K_B V_B (A_B^*)^{\dagger sk} V_B K_B)(K_B V_B A_B^* V_B K_B) = \overline{(K_B V_B A_B^* V_B K_B)}$

From these two equations, we have

$(K_B V_B (A_B^{\dagger sk})^* V_B K_B) = \overline{(K_B V_B (A_B^{\dagger sk})^* V_B K_B)}$

$(K_1 V_1 (A_1^{\dagger sk})^* v_1 k_1) \cup (K_2 V_2 (A_2^{\dagger sk})^* V_2 K_2) = \overline{(K_1 V_1 (A_1^*)^{\dagger sk} V_1) \cup (K_2 V_2 (A_2^{\dagger sk})^* V_2 K_2)}$

(iii) Since A_B is non singular, A_B^{-1} exists

Dr.N.Elumalai, Prof.R.Manikandan, G.Kalpana

Now

$A_BA_B^{\dagger sk}A_B = \overline{A_B}$ (By definition of †sk)

Pre multiplying and post multiplying by A^{-1}we have

$A_B^{\dagger sk}=\overline{A_B^{-1}}$

$A_1^{\dagger sk}\cup A_2^{\dagger sk}=\overline{A_1^{-1} \cup A_2^{-1}}$

(iv) The equations,

$A_BA_B^{\dagger sk}A_B = \overline{A_B}$ and

$(\lambda A_B)(\lambda A_B)^{\dagger sk}(\lambda A_B) = \overline{\lambda A_B}$ imply that

$\lambda(\lambda A_B)^{\dagger sk}=\overline{A_B^{\dagger sk}}$

$(\lambda A_B)^{\dagger sk}= \overline{\lambda^{\dagger sk} A_B^{\dagger sk}}$ Where $\lambda^{\dagger sk}=\lambda^{-1}$

$(\lambda(A_1\cup A2))^{\dagger sk}= \overline{\lambda^{\dagger sk}(A_1^{\dagger sk} \cup A_2^{\dagger sk})}$

(v) The equation,

$A_B^{\dagger sk}(K_BV_B(A_B^{\dagger sk})^*V_BK_B)(K_BV_BA_B^*V_BK_B)= \overline{A_B^{\dagger sk}}$

$A_B^{\dagger sk}(K_BV_B(A_B^{\dagger sk})^*V_BK_B)(K_BV_BA_B^*V_BK_B)= \overline{A_B^{\dagger sk}}$

Also

$A_BA_B^{\dagger sk}A_B = \overline{A_B}$

Therefore

$A_BA_B^{\dagger sk}(K_BV_B(A_B^{\dagger sk})^*V_BK_B)(K_BV_BA_B^*V_BK_B)A_B=\overline{A_B}$

Substitute this in the right hand side of the defining relation, we get

$((K_BV_BA_B^*V_BK_B)A_B)^{\dagger sk} = \overline{A_B^{\dagger sk}(K_BV_B(A_B^*)^{\dagger sk}V_BK_B)}$

$(K_1V_1A_1^*V_1K_1A_1)^{\dagger SK} \cup (K_2V_2A_2^*V_2K_2A_2)^{\dagger SK}$

$=\overline{(A_1^{\dagger sk}K_1V_1(A_1^*)^{\dagger sk}V_1K_1) \cup (A_2^{\dagger sk}K_2V_2(A_2^*)^{\dagger sk}V_2K_2)}$

Hence proved

THEOREM 4:

A necessary and sufficient condition for the equation $A_BX_BC_B=\overline{D_B}$ to have a solution is $A_BA_B^{\dagger SK}D_BC_B^{\dagger SK}C_B =\overline{D_B}$ in which case the general solution is

$X_B=\overline{A_B^{\dagger SK}D_BC_B^{\dagger SK} + Y_B - A_B^{\dagger SK}A_BY_BC_BC_B^{\dagger SK}}$

Where Y_B is arbitrary.

PROOF:

Let us assume that X_B satisfies the equation $A_BX_BC_B=\overline{D_B}$, Then

$D_B=\overline{A_BX_BC_B}$

$=\overline{A_BA_B^{\dagger SK}A_BX_BC_BC_B^{\dagger SK}C_B}$

$=\overline{A_BA_B^{\dagger SK}D_BC_B^{\dagger SK}C_B}$ (By the definition of †sk)

Conversely if $D_B = \overline{A_BA_B^{\dagger SK}D_BC_B^{\dagger SK}C_B}$

Then,

$X_B = \overline{A_B^{\dagger SK}D_BC_B^{\dagger SK}}$

Then it is a particular solution of $A_BX_BC_B =\overline{D_B}$

Since, $A_BX_BC_B =\overline{A_BA_B^{\dagger SK}D_BC_B^{\dagger SK}C_B} =\overline{D_B}$

If $Y_B \in C_{n\times n}$, then any expression of the from $X_B =\overline{A_B^{\dagger SK}D_BC_B^{\dagger SK} + Y_B - A_B^{\dagger SK}A_BY_BC_BC_B^{\dagger SK}}$ is a solution of $A_BX_BC_B=\overline{D_B}$ and conversely, if X_B is a solution $A_BX_BC_B=\overline{D_B}$, then

$X_B=\overline{A_B^{\dagger SK}D_B C_B^{\dagger SK}+X_B-A_B^{\dagger SK}A_B X_B C_B^{\dagger SK}C_B}$ satisfies $A_B X_B C_B=\overline{D_B}$.

$(A_1X_1C_1)\cup(A_2X_2C_2)=\overline{(D_1\cup D_2)}$

Hence proved

THEOREM 5:

The bimatrix equation $A_BX_B=\overline{C_B}C_B$ and $X_BD_B=\overline{E_B}$ have a common solution if and only if each equation has a solution and $A_BE_B=\overline{C_B D_B}$

PROOF:

It is easy to see that the conditions is necessary, conversely $A_B^{\dagger SK}C_B$ and $E_BD_B^{\dagger SK}$ are solution of $A_BX_B=\overline{C_B}$ and $X_BD_B=\overline{E_B}$ and hence,

$A_BA_B^{\dagger SK}C_B=\overline{C_B}$ and

$E_BD_B^{\dagger SK}D_B=\overline{E_B}$

Also,

$A_BE_B=\overline{C_B D_B}$

By using these facts it can be prove that

$X_B=\overline{A_B^{\dagger SK}C_B+E_BD_B^{\dagger SK}-A_B^{\dagger SK}A_BE_BD_B^{\dagger SK}}$

$X_1\cup X_2=\overline{\left(A_1^{\dagger SK}C_1\right)\cup\left(A_2^{\dagger SK}C_2\right)+\left(E_1D_1^{\dagger SK}\right)\cup\left(E_2D_2^{\dagger SK}\right)-\left(A_1^{\dagger SK}A_1E_1D_1^{\dagger SK}\right)\cup\left(A_1^{\dagger SK}A_2E_2D_2^{\dagger SK}\right)}$

is a common solution of the given equation.

Hence proved

CONCLUSION: Some of the characterization and propertices of con.secondary k- normal bimatrices can be verified. The secondary k-generalized inverse exists for particular kind of square matrices and also satiesfie the moore penrose equation.

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