Vol.5.Issue.1.2017 (Jan.-Mar)

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com

Email:editorbomsr@gmail.com

ISSN:2348-0580

RESEARCH ARTICLE

SOME INEQUALITIES ON D-DISTANCES

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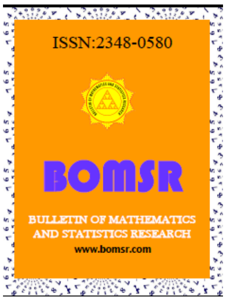

ABSTRACT

The D-distance between vertices of a graph is obtained by considering the path lengths and as well as the degrees of vertices present on the path. In this article, we study some relations between edge-to-vertex and edge-edge D-distances.

2000 Mathematics Subject Classification: 05C12.

Key words: D-distance, edge-to-vertex, edge-to-edge D-distance.

1. Introduction

In the study of graph theory the concept of distance is one of the important concepts in study of graphs. Distance is the basis of many concepts of symmetry in graphs. We refer [1] for basics.

In an earlier article authors have introduced the concept of D-distance between vertices (see [2]), by considering path length between vertices as well as the degrees of all vertices present in a path while defining the D-distance. This concept has been extended to D-distance between vertices and edges etc. in a natural way (see [3]).

In this article, we study inequalities similar to triangular inequalities.

2. Preliminaries

Throughout this article, by a graph G(V, E) or simply G, we mean a non-trivial, finite, undirected graph, connected graph without multiple edges and loops.

We begin with some definitions.

Definition 2.1: If u, v are vertices of a connected graph G the D-length of a u–v path s is defined as $l^D(s) = l(s) + \deg(u) + \deg(v) + \Sigma \deg(w)$ where sum runs over all intermediate vertices w of the path s.

Definition 2.2: (D-distance). The $D-$ *distance,* $d^D(u,v)$, between two vertices u,v of a connected graph G is defined as $d^D(u,v)=\min\{l^D(s)\}$ if u,v are distinct and $d^D(u,v)=0$ if $u=v$, where the minimum is taken over all $u-v$ paths s in G (see[2]).

Definition 2.3: If v is a vertex in a graph G and $e=xy$ is any edge we define the D-distance between vertex-to-edge as $d^D(v,e)=\min\{d^D(v,x),d^D(v,y)\}$. (See[3])

Similarly we can defin

Definition 2.4:If $e=xy$ is an edge in a graph G and v is any vertex we define the edge-to-vertex D-distance as $d^D(e,v)=\min\{d^D(x,v),d^D(y,v)\}$.

Definition 2.5: Let G be a connected graph and $e=xy, f=uv$ be two edges of G. Then the edge-to-edge D-distance is defined as $d^D(e,f)=\min\{d^D(x,u),d^D(x,v),d^D(y,u),d^D(y,v)\}$.

3. Inequalities

In this section we prove some inequalities between vertex-to-vertex, vertex-to-edge, edge-to-vertex and edge-edge D-distances. These are similar to triangular inequality.

Theorem 3.1: *For any two edges* e, f *in a connected graph* G *we have*

$d^D(e,f)\le d^D(e,g)+d^D(g,f)$ *for any edge* $g\in G$.

Proof: Let e be any edge with end vertices a and b, let f be any edge with end vertices c and d. Then by definition of edge-to-edge D-distance $d^D(e,f)=$

$\min\{d^D(a,c),d^D(a,d),d^D(b,c),d^D(b,d)\}$. Further for any arbitrary edge $g=xy\in G$ we have $d^D(e,g)=\min\{d^D(a,x),d^D(a,y),d^D(b,x),d^D(b,y)\}$ and $d^D(g,f)=$

$\min\{d^D(x,c),d^D(x,d),d^D(y,c),d^D(y,d)\}$.

To prove the required inequality, we need to consider sixty four cases. Below we prove one case. The remaining cases can be proved similarly. Without lose of generality let us assume that $d^D(e,f)=d^D(a,d), d^D(e,g)=d^D(a,x)$ and $d^D(g,f)=d^D(x,d)$. Then

$$
\begin{aligned}
d^D(e,g)+d^D(g,f) &= d^D(a,x)+d^D(x,d)\\
&\ge d^D(a,d) \text{ (bytriangular inequality)}\\
&= d^D(e,f)
\end{aligned}
$$

Thus $d^D(e,f)\le d^D(e,g)+d^D(g,f)$.

Theorem 3.2: *For any two edges* e, f *in a connected graph* G *we have* $d^D(e,f)\le$ $d^D(e,v)+d^D(v,f)$ *for any vertex* $v\in V(G)$.

Proof: Let e be any edge with end vertices x and y and f f be any edge with end vertices a and b. Then by definition of vertex-to-edge D-distance we have $d^D(v,f)=\min\{d^D(v,a),d^D(v,b)\}$ and edge-to-vertex D-distance $d^D(e,v)=\min\{d^D(x,v),d^D(y,v)\}$. Further, by definition of edge-to-edge D-distance we have $d^D(e,f)=\min\{d^D(x,a),d^D(x,b),d^D(y,a),d^D(y,b)\}$.

To prove the required inequality, we need to consider sixteen cases. Below we prove one case. The remaining cases can be proved in a similar fashion.Without loss of generality, let us assume that $d^D(e,v)=d^D(x,v), d^D(v,f)=d^D(v,a)$ and $d^D(e,f)=d^D(x,a)$.

$$d^D(e,v)+d^D(v,f)=d^D(x,v)+d^D(v,a)$$
$$\geq d^D(x,a) \text{ (by triangular inequality)}$$
$$=d^D(e,f)$$

Thus $d^D(e,f)\leq d^D(e,v)+d^D(v,f)$.

Theorem 3.3*: Let $v\in V(G)$ and $e\in E(G)$ of a connected graph G then $d^D(e,v)\leq$*
$d^D(e,f)+d^D(f,v)$ for any edge $f\in E(G)$.

Proof: Assume that $e=xy$ be any edge with end vertices a and b. Then by definition of edge-to-vertex D-distance we have $d^D(e,v)=\min\left\{d^D(x,v),d^D(y,v)\right\}$ and $d^D(f,v)=$ $\min\left\{d^D(a,v),d^D(b,v)\right\}$. Further, by definition of edge-to-edge D-distance $d^D(e,f)=\min\left\{d^D(x,a),d^D(x,b),d^D(y,a),d^D(y,b)\right\}$. To prove the required inequality, we need to consider sixteen cases. Below we prove in one case. The remaining cases can be proved similarly.

Without loss of generality, let us assume that
$d^D(e,v)=d^D(x,v), d^D(f,v)=d^D(a,v)$ and $d^D(e,f)=d^D(x,a)$. Then
$d^D(x,v)\leq d^D(y,v)$ and $d^D(a,v)\leq d^D(b,v)$.

$$d^D(e,f)+d^D(f,v)=d^D(x,a)+d^D(a,v)$$
$$\geq d^D(x,v) \text{ (by triangular inequality)}$$
$$=d^D(e,v)$$

Thus $d^D(e,v)\leq d^D(e,f)+d^D(f,v)$.

References

[1]. F. Buckley, F. Harary, Distance in Graphs, Addison-Wesley, Longman, 1990.

[2]. D. Reddy Babu, L.N. Varma, P., D-distance in graphs, *Golden Research Thoughts*, Vol. 2, (2013) pp. 53-58.

[3]. D. Reddy Babu, L. N. Varma, P. , Vertex-to-edge centers w.r.t. D-distance, *Italian journal of pure and applied Mathematics*, Volume 35, (2015), pp. 101-108