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RESEARCH ARTICLE

ON THE CONSTRUCTION OF PARTIALLY BALANCED n-ARY BLOCK DESIGNS USING BSP METHODOLOGY ON PBIBD(2)

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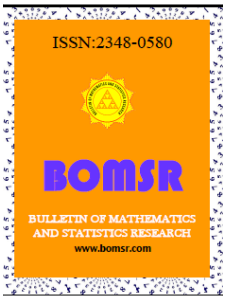

ABSTRACT

We construct partially balanced n-ary block designs using a tool Block Sum and Product (BSP) Methodology on a simple partially balanced incomplete block design with two associate classes, abbreviated PBIBD(2)'s whose incidence matrix takes only binary values. We use the simple PBIBD(2) design with parameters V=10, B=5, R=2, K=4, $\lambda_1=1$, $\lambda_2=0$ for our construction. This gives four new partially balanced n-ary block designs with the same association scheme. By applying some operations on incidence matrices of newly constructed, partially balanced n-ary design, we get some other new incidence matrices of PBIBD(2) with the same association scheme.

Keywords : Incidence matrix, BIBD, PBIBD(m), Balanced n-ary design, Association scheme, Partially Balanced n-ary design, BSP Methodology.

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1 INTRODUCTION

In Randomized Block Design, block size must equal to number of treatments. The problem arises when block size is less than the number of treatments. In such cases Incomplete Block Designs (IBD) are used. Generally Incomplete Block Designs are used when the number of treatments are very large and blocking is necessary. The most popular incomplete block designs are Balanced Incomplete Block Design (BIBD) introduced by Yates (1936) and Partially Balanced Incomplete Block Design (PBIBD) introduced by Bose and Nair (1939). Tocher (1952) introduced balanced n-ary block design.

Murthy and Das (1967), Saha and Dey (1973), Nigam (1974), Sharma and Agarwal (1976), Agarwal and Das (1987, 1990) and many others gave the construction of balanced n-ary designs through BIBD. Saha (1975) has given two methods of construction of balanced ternary (3-ary) designs. Mehta et al. (1975) generalized the concept of balanced n-ary block designs to partially balanced n-ary block designs. Kumar (2007) has given the construction of PBIBD through unreduced BIBDs. Garg and Farooq (2014) constructed PBIBD through chosen lines and triangles of graphs. Phad and Pawar (2016) constructed balanced and partially balanced n-ary t-designs by BSP Methodology on 2-design. In this paper we construct partially balanced n-ary block designs by using Block Sum and Product (BSP) Methodology on PBIBD(2) design.

Definition 1.1 : The incidence matrix of an incomplete block design is $N=((n_{ij}))$, where n_{ij} is the number of times i^{th} treatment occurs in the j^{th} block.

Definition 1.2 : A balanced incomplete block design (BIBD) is an arrangement of V (≥ 2) treatments in B blocks, such that:

i) Each block contains exactly K treatments (K<V).

ii) Each treatment appears in exactly R blocks.

iii) Each pair of treatments appear simultaneously in exactly λ blocks.

V, B, R, K and λ are called the parameters of the BIBD. The parameters of BIBD must satisfy the following relations:

$$VR = BK \quad ; \quad \lambda(V-1) = R(K-1) \quad ; \quad B \geq V$$

In BIBD the entries of incidence matrix (n_{ij}) are either 0 or 1.

Definition 1.3 : Given V treatment {1, 2,....,V} a relation satisfies the following conditions is said to form an association scheme with m associate classed if the following conditions are satisfied:

i) Any two treatments are either 1^{st} or 2^{nd}, ..., or m^{th} associates, the relation of association being symmetrical; that is if the treatment α is the i^{th} associate of the treatment β, then β is the i^{th} associate of α.

ii) Each treatment α has N_i-i^{th} associates, the number N_i being independent of α.

iii) If α and β are i^{th} associates, then the number of symbols that are j^{th} associates of α, and k^{th} associates of β, is P^{i}_{jk} and is independent of the pair of i^{th} associates of α and β.

Definition 1.4 : Consider a set of treatments {1, 2,....,V}, we have a partially balanced incomplete block design with m associate classes, abbreviated PBIBD(m)'s if V treatments are arranged in B blocks such that:

i) Each block contains exactly K treatments.

ii) Each treatment appears in exactly R blocks.

iii) Any two i^{th} associates occur together in λ_i (i=1,2,.....,m) blocks.

The parameters V, B, R, K, λ_1, λ_2,.... λ_m are called the parameters of the first kind and N_1, N_2,.... N_m, P^{i}_{jk} is called parameters of the second kind. The parameters of PBIBD(m) must satisfy the following relations:

$$VR = BK \quad ; \quad \sum_{i=1}^{m} N_i = V-1$$

$$\sum_{i=1}^{m} N_i\lambda_i = R(K-1) \quad ; \quad N_k p^{k}_{ij} = N_i p^{i}_{jk} = N_j p^{j}_{ki}$$

$$\sum_{k=1}^{m} p^{i}_{jk} = \begin{cases} N_j - 1 & if \quad i = j \\ N_j & if \quad i \neq j \end{cases}$$

In PBIBD(m) the entries of incidence matrix (n_{ij}) are either 0 or 1.

Definition 1.5 : Balanced n-ary block design is an arrangement of V treatments in B blocks such that:

i) i^{th} treatment occurs in the j^{th} block n_{ij} times, i=1,2,….,V; j=1,2,….,B.

ii) n_{ij} can take 0 or 1 or 2…..or (n-1) value.

iii) $\sum_{i=1}^{V} n_{ij} = K, \quad \sum_{j=1}^{B} n_{ij} = R, \quad \sum_{j=1}^{B} n_{lj} n_{mj} = \begin{cases} \Delta & if \quad l = m \\ \lambda & if \quad l \neq m \end{cases}$

The parameters of balanced n-ary block design must satisfy the following relations:

$$VR = BK \quad ; \qquad \lambda(V-1) = RK - \Delta; \qquad B \geq V$$

The balanced n-ary block design for n=2 becomes BIBD with R = Δ.

Definition 1.6 : Consider a set of treatments {1, 2,....,V}, we have a partially balanced n-ary block designs with m-associate classes if V treatments are arranged in B blocks such that:

i) i^{th} treatment occurs in the j^{th} block n_{ij} times, i=1,2,….,V; j=1,2,….,B.

ii) n_{ij} can take 0 or 1 or 2…..or (n-1) value.

iii) $\sum_{i=1}^{V} n_{ij} = K, \quad \sum_{j=1}^{B} n_{ij} = R, \quad \sum_{j=1}^{B} n_{ij}^2 = \Delta$

iv) Any two i^{th} associates occur together in λ_i (i=1,2,.....,m) blocks

The parameters of partially balanced n-ary block designs must satisfy the following relations:

$$VR = BK \quad ; \qquad \sum_{i=1}^{m} N_i = V - 1$$

$$\sum_{i=1}^{m} N_i \lambda_i = RK - \Delta \quad ; \quad N_k p_{ij}^{k} = N_i p_{jk}^{i} = N_j p_{ki}^{j}$$

$$\sum_{k=1}^{m} p_{jk}^{i} = \begin{cases} N_j - 1 & if \quad i = j \\ N_j & if \quad i \neq j \end{cases}$$

The partially balanced n-ary block design with m associate classes for n=2 becomes PBIBD(m) with R = Δ.

We use a tool Block Sum and Product (BSP) Methodology (2007) on a simple partially balanced incomplete block designs with two associate classes whose incidence matrix takes only binary values to construct new partially balanced n-ary block designs. By applying some operations on incidence matrices of newly constructed design, we get some other new incidence matrices of partially balanced incomplete block designs with two associate classes.

2 Construction of partially balanced n-ary block designs by BSP Methodology on PBIBD(2)

Consider a simple PBIBD(2) design as a parent design as shown in table 1 with parameters V=10, B=5, R=2, K=4, λ_1=1, λ_2=0 for the construction.

Table 1 The arrangement of 10 treatments in 5 blocks

Blocks	Treatments
1	1 2 3 4
2	1 5 6 7
3	2 5 8 9
4	3 6 8 10
5	4 7 9 10

The treatments are said to be:

- First associates if both occurs in the same block 1 times.
- Second associates if both occurs in the same block 0 times.

Table 2 The first, second associates of all the 10 treatments

Treatment Number	First Associates	Second Associates
1	2, 3, 4, 5, 6, 7	8, 9, 10
2	1, 3, 4, 5, 8, 9	6, 7, 10
3	1, 2, 4, 6, 8, 10	5, 7, 9
4	1, 2, 3, 7, 9, 10	5, 6, 8
5	1, 2, 6, 7, 8, 9	3, 4, 10
6	1, 3, 5, 7, 8, 10	2, 4, 9
7	1, 4, 5, 6, 9, 10	2, 3, 8
8	2, 3, 5, 6, 9, 10	1, 4, 7
9	2, 4, 5, 7, 8, 10	1, 3, 6
10	3, 4, 6, 7, 8, 9	1, 2, 5

The parameters of the second kind are: N_1=6, N_2=3,

$$p_{jk}^{1}=\begin{bmatrix} p_{11}^{1} & p_{12}^{1} \\ p_{21}^{1} & p_{22}^{1} \end{bmatrix}=\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix};\ p_{jk}^{2}=\begin{bmatrix} p_{11}^{2} & p_{12}^{2} \\ p_{21}^{2} & p_{22}^{2} \end{bmatrix}=\begin{bmatrix} 4 & 2 \\ 2 & 0 \end{bmatrix}$$

To apply BSP Methodology, replace 1, 2,...,10 treatments of table 1 by X_1, X_2,..., X_{10} and consider the product of block sums B_i. The notations for BSP Methodology are shown below:

Table 3 The notation for BSP Methodology

Block Number (i)	Treatment content in block i	Treatment replaced for BSP	Block sum (B_i) for BSP
1	1 2 3 4	X_1 X_2 X_3 X_4	$B_1=X_1+X_2+X_3+X_4$
2	1 5 6 7	X_1 X_5 X_6 X_7	$B_2=X_1+X_5+X_6+X_7$
3	2 5 8 9	X_2 X_5 X_8 X_9	$B_3=X_2+X_5+X_8+X_9$
4	3 6 8 10	X_3 X_6 X_8 X_{10}	$B_4=X_3+X_6+X_8+X_{10}$
5	4 7 9 10	X_4 X_7 X_9 X_{10}	$B_5=X_4+X_7+X_9+X_{10}$

Consider,

$$D=\prod_{i=1}^{5}B_i=B_1.B_2.B_3.B_4.B_5=1\left[500\quad\text{terms of the type}\quad X_i^2.X_j^1\,X_k^1.X_l^1.X_m^0.X_n^0.X_o^0.X_p^0.X_q^0.X_r^0\right]$$

$$+\ 2\left[222\quad\text{terms of the type}\quad X_i^1.X_j^1\,X_k^1.X_l^1.X_m^1.X_n^0.X_o^0.X_p^0.X_q^0.X_r^0\right]$$

$$+\ 1\left[60\quad\text{terms of the type}\quad X_i^2.X_j^2\,X_k^1.X_l^0.X_m^0.X_n^0.X_o^0.X_p^0.X_q^0.X_r^0\right]$$

$$+\ 2\left[10\quad\text{terms of the type}\quad X_i^2.X_j^1\,X_k^1.X_l^1.X_m^0.X_n^0.X_o^0.X_p^0.X_q^0.X_r^0\right]$$

The polynomial D is of degree 5 in variables X_1, X_2, …, X_{10}. This polynomial contains $K^B=4^5$ (=1024) terms. We used C# language program to generate this type of polynomial. Similar types of terms of this polynomial are classified according to powers and new 4 designs are constructed.

In the polynomial D, there are 500 terms of the type of the power 2111000000 and each term is repeated 1 time, the powers of these 500 terms give columns of the incidence matrix N_1 of design no.1 with V=10 and B=500; 222 terms of the type of the power 1111100000 and each term is repeated 2 times, the powers of these 222 terms give columns of the incidence matrix N_2 of design no.2 with V=10 and B=222; 60 terms of the type of the power 2210000000 and each term is repeated 1 time, the powers of these 60 terms give columns of the incidence matrix N_3 of design no.3 with V=10 and B=60; 10 terms of the type of the power 2111000000 and each term is repeated 2 times, the powers of these 10 terms give columns of the incidence matrix N_4 of design no.4 with V=10 and B=10.

The association scheme of newly constructed design coincides with parent PBIBD(2) design. According to incidence matrix and parameters of newly constructed design it classified into balanced or partially balanced n-ary block design. Type of design and the parameters of newly constructed design are given below:

Table 4 Type of design and the parameters of newly constructed design

Design No.	Type of Design	V	B	R	K	Δ	λ_1	λ_2	N_1	N_2
1	Partially balanced 3-ary	10	500	250	5	350	90	120	6	3
2	Partially balanced 2-ary (PBIBD(2))	10	222	111	5	111	48	52	6	3
3	Partially balanced 3-ary	10	60	30	5	54	4	24	6	3
4	Partially balanced 3-ary	10	10	5	5	7	1	4	6	3

From the incidence matrices $N=((n_{ij}))$ of design no. 1, 3 & 4 of table 4, we get the new PBIBD(2) designs with incidence matrices $M=((m_{ij}))$, Where,

$$m_{ij} = \begin{cases} 1 & if \quad n_{ij} = 2 \\ n_{ij} & ; \quad otherwise \end{cases}$$

Since design no.2 is already PBIBD(2), therefore above operation is not applicable for design no.2. The parameters of new PBIBD(2) designs after operation are given below:

Table 5 The parameters of new PBIBD(2) designs after operation

Type of Design	V	B	R	K	Δ	λ_1	λ_2	N_1	N_2
PBIBD(2)	10	500	200	4	200	60	80	6	3
PBIBD(2)	10	60	18	3	18	2	8	6	3
PBIBD(2)	10	10	4	4	4	1	2	6	3

From the incidence matrices $N=((n_{ij}))$ of design no. 1 & 4 of table 4, we get the new PBIBD(2) designs with incidence matrices $M=((m_{ij}))$, Where,

$$m_{ij} = \begin{cases} 0 & if \quad n_{ij} = 2 \\ n_{ij} & ; \quad otherwise \end{cases}$$

Since design no. 2 is already PBIBD(2) and in design no. 3 if we use this operation then block size becomes 1 (i.e. K=1) therefore above operation is not applicable for design no. 2 & 3. The parameters of new PBIBD(2) design after operation are given below:

Table 6 The parameters of new PBIBD(2) design after operation

Type of Design	V	B	R	K	Δ	λ_1	λ_2	N_1	N_2
PBIBD(2)	10	500	150	3	150	30	40	6	3
PBIBD(2)	10	10	3	3	3	1	0	6	3

3 Discussion

In this paper, we constructed four partially balanced n-ary block designs by using Block Sum and Product (BSP) Methodology tool on Partially Balanced Incomplete Block Design with 2 associate classes whose incidence matrix takes only binary values. The association scheme of newly constructed design coincides with parent PBIBD(2) design. By applying some operations on incidence matrices of newly constructed design, we get some new incidence matrices of PBIBD(2) designs with same association scheme. Block Sum and Product (BSP) Methodology tool can be applied on any simple PBIBD(2) design whose incidence matrix takes only binary values. According to incidence matrix and parameters of newly constructed design it can classified into balanced or partially balanced n-ary block designs.

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