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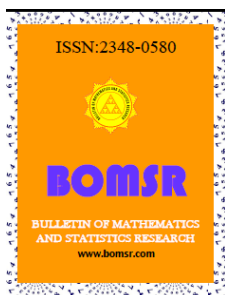


## MATHEMATICAL CALCULATION OF FUZZY STATISTICS FOR QUALITY EVALUTION

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### ABSTRACT

Quality evaluation is an essential need for accepting cotton towel products having desired quality specification. Human perceived sensation depends on physiological and physical properties of quality as well as demographic characters of consumers. In this paper, a mathematical computational method of fuzzy statistics for quality evaluation is proposed. It illustrates the cotton towel products of quality evaluation such as colour, designs, quality and price etc. of a product with consumer's demographic characters on a hedonic scale.

Keywords: Fuzzy set, fuzzification, de-fuzzification, performance measures, hedonic scale.

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### 1. INTRODUCTION

Fuzzy sets were introduced by Lotfi A Zadeh in 1965 to represent/manipulate data and information possessing non statistical uncertainties. It was specifically designed to mathematically represent uncertainty and vagueness and to provide formalized tools for dealing with the imprecision intrinsic to many problems. Fuzzy number is an extension of the interval of confidence on uncertainty.  $A$  is a fuzzy set and  $x$  is a relevant object, the proposition  $x$  is a member of  $A$  is not necessarily either true or false, as required by two – valued logic, but it may be true only to some degree, the degree to which  $x$  is actually a member of  $A$ . It most common, but not required to express degrees of membership in fuzzy sets as well as degrees of truth of the associated propositions by numbers in the closed unit interval  $[0,1]$ . The extreme values in this interval, 0 and 1, then represent, respectively. The fuzzy evaluation considers the maximum presumption to be at zero. However, one must not confuse fuzzy numbers with random numbers. For example, instead of describing the weather today in terms of the exact percentage of cloud cover, we can just say that is

sunny. While the latter description is vague and less specific, it is often more useful. In order for a term such as sunny to accomplish the desired introduction of vagueness, however, we cannot use it to mean precisely 0% cloud cover. Its meaning is not totally arbitrary, however; a cloud cover of 100% is not sunny, and neither, in fact, is a cloud cover of 80%. We can accept certain intermediate states, such as 10% or 20% of cloud cover, as sunny. But where do we draw the line? If, for example, any cloud cover of 25% or less is considered sunny, does this mean that a cloud cover of 20% is not? This is clearly unacceptable, since 1% of cloud cover hardly seems like a distinguishing characteristic between sunny and not sunny. Randomness and uncertainty are two very different and important concepts which can be used together but should not be confused. Fuzzy sets support a flexible sense of membership of elements to a set, while in set theory only an element either belongs to a set. In the fuzzy set theory, many degrees of membership between 0 and 1 are obtained. Let  $X$  be a nonempty set. A fuzzy set  $A$  in  $X$  is characterized by its membership function  $A : X \rightarrow [0,1]$  and  $A(x)$  is interpreted as the degree of membership of element  $x$  in fuzzy set  $A$  for each  $x \in X$ . If people can use the membership function to express the degree of their feeling based on their own concept, then the result will be closer to their real thought.

Table 1: Response Comparison between Fuzzy Logic and Traditional Method

Panel member	Fuzzy Logic Evaluation		Traditional Method	
	Good	Poor	Good	Poor
$F_1$	80%	20%	✓	–
$F_2$	99%	1%	✓	–
$F_3$	46%	54%	–	✓
$F_4$	32%	68%	–	✓
$F_5$	45%	55%	–	✓
$F_6$	35%	65%	–	✓
Score	75.4%	24.6%	45%	55%

Consider a fuzzy set as well as traditional method used to evaluate the taste of grapes juice from six evaluator's sensory evaluation of quality with the two methods (Table1).Based on traditional method, taste of grapes juice is poor while based on fuzzy set taste is good. The fuzzy sensory evaluation of quality method is more reasonable as compared to traditional method. Fuzzy linguistic descriptions are formal representations of a system made through fuzzy. In the IF-THEN rules system, one encodes knowledge about a system in a statement from if a set of conditions is satisfied and then a set of consequents can be inferred. A collection of rules referring to a particular system is known as a fuzzy rule base for a system as a single scalar quantity. This conversion of a fuzzy set to single value is called de-fuzzification.

Fuzzy reasoning is an effective analysing method for sensory evaluation. There are many methods for fuzzification. Helledoorn and Thomas (1993) have specified five criteria which to measure the methods. These criteria will be repeated here for the benefit of the reader who also ponders the question just given in terms of the advantages and disadvantages of the various methods. The first criterion is *continuity*. Second, a criterion is called *dis-ambiguity*. The third criterion is called plausibility. To be plausible,  $Z^*$  should lie approximately in the middle of the support region of  $\tilde{C}_k$  and have a high degree of membership in  $\tilde{C}_k$ . The centroid method equation  $Z^*$  does not exhibit plausibility in some situations when it lies in a region of the output that has a low degree of membership. The fourth criterion is that of *computational simplicity* and the fifth criterion is called the *weighting* method. Sargunamary (2009) et al. has studied fuzzy statistical measures for

linguistic variables associated with traditional ranks through rectangular membership function. Shu-Meei and Berlin Wu (2008) introduce a fuzzification method for fuzzy statistical measures. They propose an integrated fuzzy evaluation procedure to measure the intellectual capital. In this method linguistic variables are uniformly used for fuzzification. Lee et al. have used IF-THEN rules on fuzzy reasoning application for sensory evaluation of sausages. Evaluators are asked to rate the contribution level of each attribute like texture, taste, order and appearance of a product to describe the overall preference as one out of five alternatives i.e., very important, important, moderate, slight and very slight. The preference level of each attribute for sausage samples is also simultaneously evaluated on a hedonic scale going from excellent, good, fair, poor and very poor. The results of the sensory evaluations are converted in to fuzzy sets. The fuzzy sets for the contribution of weight and preference of the attributes are composed to infer the overall preference of sausage by fuzzy reasoning. Martinez has proposed linguistic decision analysis to sensory evaluation. He uses 2-tuple representation model for managing the uncertainty and vagueness of the information in sensory evaluation of human senses (sight, taste, touch, smell and hearing). His frame work is based on a decision analysis scheme. Hough et al. have determined the consumer acceptance limits of ultra-heat the sensory properties such as taste, flavour and consumer acceptance limits of ultra-heat treated milk using survival analysis. Jaeger et al. develop model systems for testing the sensory properties such as taste, flavour and consumer acceptance of new fruit cultivators. Sinija (2011) et al has studied ranking of the quality of instant green tea powder and granule samples using triangular fuzzy membership distribution. The ranking method cannot be helpful for comparing similar qualities involving large sample size. Sune (2002) et al. makes a comparative study of sensory attribute used by children and experts to evaluate e chocolate. They have surprisingly found that some of the attributes most cited by children are not those better explained by experts. Their study finds the semantic gaps and differences between children and adult experts in rating sensory attributes. It is essential to understand these differences which may be explained by demographic characters like age, environment, social status, etc. In this paper, mathematical modelling of fuzzy statistics for quality evaluation using a hedonic scale is proposed and thereby a desired consistency table is constructed for obtaining coefficient of association between physical intensity and evaluator's demographic character like age, gender, social status or environment. An example of cotton towel is illustrated the computation feasibility of the fuzzy statistics quality measures on the hedonic scale.

## 2. Mathematical Models for Quality Evaluation

Huda Habib (2015) has carried out the Bamboo textile is one of the oldest materials, which have been came under the spot light and has become greatly available selection over the last few years to be used in terry towels, due to its good properties of absorption. It has been studied the effect of some production parameters on the properties of terry towels such as the quality of terry fabrics to meet the required properties at the lowest cost. The author research has nine experimental samples of terry towels fabrics, which differ in the levels of the pile length and pile density were used. The selection of these parameters is because of their effect on the absorption properties of the towel terry fabric. Results were compared according to standard specification ASTM D5433 to evaluate the quality of terry towel fabrics which produced by bamboo yarns with the previous variables. Results indicated that all the tested samples not only meet the requirements of the standard values of specifications for the properties of durability but also better than them, such as tensile strength in the warp and weft direction and pile withdraw. Also the results achieved acceptable values of the of absorption properties such as vertical wicking of water, spreading of water in horizontal direction, and absorption rate. In addition to that, the hand properties, such as

softness, smoothness and drape ability, were compatible with standard specification and the end use. This confirms that every sample which produced in this search achieved all requirements of the standard specification of terry towel and will be suitable for the end use and aftercare processing.

Luis Martinez (2007) evaluation is a process that analyses elements in order to achieve different objectives such as quality inspection, marketing and other fields in industrial companies. This paper focuses on sensory evaluation where the evaluated items are assessed by a panel of experts according to the knowledge acquired via human senses. In these evaluation processes the information provided by the experts implies uncertainty, vagueness and imprecision. The use of the Fuzzy Linguistic Approach (32) has provided successful results modelling such a type of information. In sensory evaluation it may happen that the panel of experts have more or less degree knowledge of about the evaluated items or indicators. So, it seems suitable that each expert could express their preferences in different linguistic term sets based on their own knowledge. In this paper, we present a sensory evaluation model that manages multi-granular linguistic evaluation framework based on a decision analysis scheme. This model will be applied to the sensory evaluation process of Olive Oil.

Renato Coppi et al. (2005) have been developed in which a coalition of Fuzzy Sets Theory and Statistics has been established with different purposes. These namely are: (i) To introduce new data analysis problems in which the objective involves either fuzzy relationships or fuzzy terms; (ii) To establish well-formalized models for elements combining randomness and fuzziness; (iii) To develop *uni-variate* and *multivariate* statistical methodologies to handle fuzzy-valued data; and (iv) To incorporate fuzzy sets to help in solving traditional statistical problems with non-fuzzy data. In spite of a growing literature concerning the development and application of fuzzy techniques in statistical analysis, the need is felt for a more systematic insight into the potentialities of cross fertilization between Statistics and Fuzzy Logic. In line with the synergistic spirit of Soft Computing, some instances of the existing research activities on the topic are recalled. Particular attention is paid to summarize the papers gathered in this Special issue, ranging from the position paper on the theoretical management of uncertainty by the *father* of Fuzzy Logic to a wide diversity of topics concerning foundational/methodological/applied aspects of the integration of Fuzzy Sets and Statistics.

Hrehova Stella-Vagaska Alena (2012) has carried out the possibility of using artificial intelligence elements in order to evaluate the quality of a manufacturing process. There are described selected indexes of a production process quality evaluation based on statistical process control (SPC), their interpretation and evaluation by means of fuzzy sets, which enable us to work with inaccurate, incomplete or vague information about a monitored and reviewed phenomenon. There are described possibilities of using program system *MATLAB* and its toolboxes *SIMULINK* and Fuzzy Logic to evaluate quality of the manufacturing process based on fuzzy principles.

Cotton towel evaluation is based on quality. Quality can't be directly measured in ratio scales. After the measurement of nominal or ordinal scales, it can be converted into quantitative measure using fuzzy logic, since traditional method helps to select only one option. In fuzzy logic, many possible values are considered for reducing the uncertainty of human thoughts. Further, quality evaluation of Cotton towel depends not only on physical and physiological properties of Cotton towel but also consumers' demographic characters. Differential equations are of great importance in science and engineering as many physical laws and relations appear mathematically in the form of differential equations. Assessment of the quality of a product (Cotton towel) can be classified into two categories viz. acceptability and non-acceptability depending on several attributes. This kind of classification is realistic since perceived sensation is a fuzzy concept. In the fuzzy concept of sensory evaluation of Cotton towel, judgment fuzzy membership function is

required to determine the appropriate measures. This following theorem is proposed for the sensory evaluation of Cotton towel.

*Theorem 1:* The perceived sensory evaluation of Cotton towel quality attributes of a product is a power curve.

$$y = \begin{cases} ax^b & \text{for } y_0 = 0 \text{ when } x \geq 0, -1 < r < 0 \text{ and } 0 < r < 1 \\ 0 & \text{otherwise} \end{cases} \quad (2.1)$$

The evaluation of the quality attributes is a power function of assessment with an exponent of  $b$  where  $a = [k(1-r)]^{\frac{1}{(1-r)}} > 0$  and  $b = \frac{1}{(1-r)} > 0$   $r$  is the Yule's coefficient of association between

physical intensity of the quality attributes and demographic character of the evaluators.

*Proof:* Assume that the rate of perceived sensation of cotton towel ( $y$ ) of a product's quality attributes with respect to assessment ( $x$ ) is proportional to power ( $r$ ) of the perceived sensation of cotton towel that is  $y^r$  where  $r$  is the measure of coefficient of association between physical intensity and demographic character of the evaluators on quality attributes.

$$\frac{dy}{dx} = ky^r \quad -1 \leq r \leq 1 \quad (2.2)$$

Where,  $k$  is constant. If the coefficient of association between physical intensity of quality attributes and evaluators' demographic characters is completely associated, completely disassociated and independent, then  $r = 1, 0$  and  $-1$  respectively. If  $r = 1$  then the quality equation becomes  $\frac{dy}{dx} = ky$

and the solution is an exponential curve

$$y = \begin{cases} y_0 e^{kx} & \text{for } y_0 \geq 0 \text{ and } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (2.3)$$

If  $r = 0$  then  $\frac{dy}{dx} = k$  and the solution is straight line.

The mathematical model for the quality evaluation is

$$y = \begin{cases} kx + y_0 & \text{for } y_0 \geq 0 \text{ and } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (2.4)$$

If  $-1 \leq r \leq 1$ , then the analytical solution of the differential equation is  $\frac{dy}{dx} = ky^r$  with initial

boundary condition  $y(0) = y_0 = 0$   $\frac{dy}{dx} = k$

$$y^{1-r} = \begin{cases} k(1-r)x & -1 \leq r \leq 1, \quad x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$y = \begin{cases} ax^b & -1 \leq r \leq 1, \quad x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Where,  $a = [k(1-r)]^{\frac{1}{(1-r)}} > 0$ . and  $b = \frac{1}{(1-r)} > 0$

In the paper, hedonic scale and coefficient of association between physical intensity of a product and demographic characters of evaluators were considered for finding the fuzzy sensory evaluation of cotton towel measures. The assigned scores of the scale are 1,2,3,4 and 5 for poor, sufficient,

good, very good and excellent, respectively. In Yule’s coefficient of association to form the dichotomous classification of the attributes  $A$  and  $B$ ,  $A$  is the physical intensity scores of quality attributes on the hedonic scale greater than the median of the hedonic and  $B$  is the evaluators’ age which is greater than the desired age group. The classification is a  $(2 \times 2)$  consistency table with cell frequencies  $(AB)$ ,  $(B\alpha)$ ,  $(A\beta)$  and  $(\alpha\beta)$  where  $\alpha$  and  $\beta$  are complimentary attributes of  $A$  and  $B$ , respectively. These represent the frequencies of all the four combinations of  $A, B, \alpha$  and  $\beta$ . For the five point hedonic scale, the best physical intensity scores are greater than or equal to 3 (median) which are the desirable physical intensity scores. The scores are uniform in nature. Significance of the fuzzy response of linear sensory evaluation of cotton towel on five point scale is 75% (Median  $\times$  relative growth rate =  $3 \times 0.25 = 0.75$ ) whereas in nine point scale, it is 55% ( $5 \times 0.11$ ). When  $r = 0$ , then there is no association between physical intensity of the quality attributes and demographic characters of evaluators. The mathematical model for the quality evaluation is:

$$y = \begin{cases} x & \text{for } k=1, y_0=0 \text{ and } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (2.6)$$

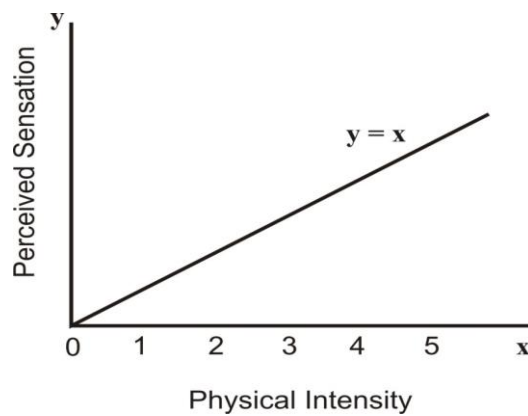


Figure 1: Linear quality evaluation of cotton towel

Figure 1 represents the evaluation of the quality attributes in the five point hedonic scale scores. The relative growth rate of the evaluation on the scale is 0.20 at  $x=1,2,3,4$  and 5. The linear sensory evaluation has uniform relative growth rate because of the independence between physical intensity and demographic characters of the evaluators. Where  $r=-1$ , the association between physical intensity of the quality attributes and demographic characters of the evaluators is completely disassociated, then the mathematical model is:

$$y = \begin{cases} 2x & \text{for } y_0=0, k=1, y_0=0 \text{ and } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (2.7)$$

Figure 2 is the parabola curve for  $y_0=0$  and the five point hedonic scale scores are 1,2,3,4, and 5 such that  $k=1$ . Here, the relative growth rates for the evaluation are 0.71, 0.50, 0.41, 0.35 and 0.32 at  $x=1, 2, 3, 4$  and 5 respectively. For the parabolic sensory evaluation of cotton towel, the relative growth rates at each score of the hedonic scale are distinct. The score increases with the decrease in the rate of physical intensity because of the negative association between physical intensity and demographic characters of the evaluator’s are completely associated, then the mathematical model is

$$y = \begin{cases} y_0 e^{kx} & \text{for } x \geq 0 \text{ and } y_0 \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (2.8)$$

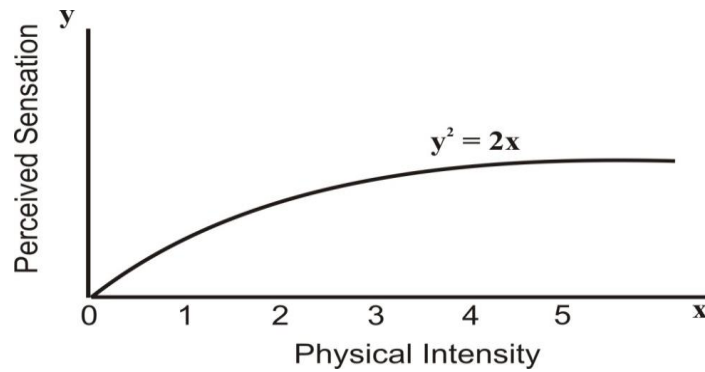


Figure 2: Parabolic quality evaluation

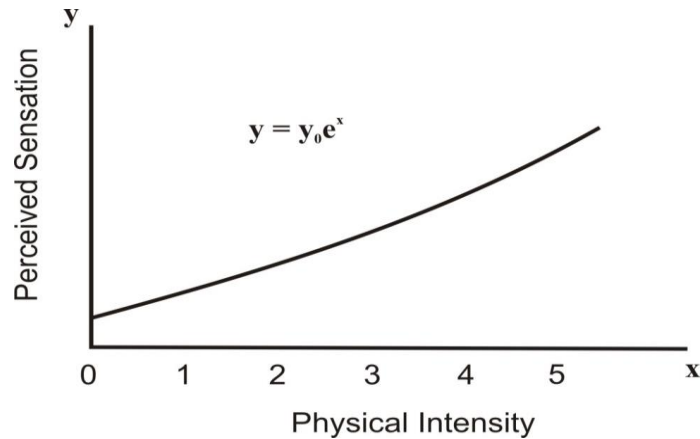


Figure 3: Parabolic quality evaluation

Table 2: Quality evaluation of the Exponential model

Physical intensity (x)	0	1	2	3	4	5
Perceived sensation (y)	0.50	1.36	3.69	10.04	27.30	74.20
Relative growth rates when k = 0.1	0.050	0.055	0.061	0.067	0.075	0.082

Figure 3 is the exponential curve with  $y_0 = 0.5$  for  $x = 0$  and the five point hedonic Scale scores are 1,2,3,4 and 5 such that  $k = 0.1$  Table 2 shows the quality evaluation relative growth rates at each score of the hedonic scale increases with the increase in scores because of the positive association between the physical intensity of the quality attributes and demographic characters of the evaluators. If  $k = 1$ , then the relative growth rate at each hedonic score is equal to the quality evaluation value, since

$$\frac{dy}{dx} = y_0 e^x \quad x = 0,1,2,3,4,5 \text{ at } k = 1$$

**3. Fuzzy Statistics Analysis on Quality Evaluation of Cotton Towels**

In this section soft computing method is proposed for calculating fuzzy response of quality attributes.

Step 1: Let  $U$  be the universal set,  $L = \{L_1, L_2, \dots, L_k\}$  be a set of  $k$  linguistic variables on  $U$  and  $U_{ij}$  be the degrees of membership of  $i^{th}$  evaluator in  $j^{th}$  linguistic variable.

Step 2: Determine the evaluators and linguistic variables of the hedonic scale. Contact the evaluators and get their traditional hedonic scores for the linguistic variables. Let  $L_j, j=1$  to  $k$  be the linguistic variables with the traditional score  $R_j$  on the universal set. Form a desired consistency table with the evaluators' scores of the attributes and physiological differences like age, gender, social status or

environment which is considered for finding the coefficient of association between them. Let the evaluators' score for the linguistic variables of n panel members be  $X = \{R_k, R_2, R_5, \dots, R_1, R_6\}$ . Rewrite the traditional scores in the increasing order, i.e.,  $X = \{R_1, R_2, \dots, R_k\}$ .

Step 3: The degrees of membership functions of the linguistic variables are:

$$\mu_{L_j}(x_j) = [k(1-r)x_j + y_0]^{1/(1-r)} \quad -1 \leq r \leq 1 \text{ and } x_j \geq 0$$

and  $\mu_{L_j}(x_j) = y_0 e^{kx}$  where  $L_j = \{(L_j, \mu_{L_j}(x_j)), x_j \in X\}$

Step 4: For converting the traditional scores into degrees of membership function values, one can divide the interval  $[0, R_k + 1]$  into desired partitions with homogenous intervals based on the linguistic variables scores, say  $U = \{[0, R_1], [R_1, R_2], \dots, [R_k, R_k + 1]\}$ . The medians of each interval are  $m_1, m_2, \dots, m_{k+1}$  respectively.

Step 5: For  $k=5$ , the fuzzy entity of the linguistic variables  $L_1, L_2, L_3, L_4$  and  $L_5$  of memberships are:

$L_1$	Poor	1
$L_2$	Sufficient	2
$L_3$	Good	3
$L_4$	Very Good	4
$L_5$	Excellent	5

Step 6: when the evaluators' traditional score of  $L_j$  falls in the middle of medians of the intervals, then the fuzzy entity values of scores are  $m_{ij} = \int_{m_j}^j \mu_{L_j}(x) dx$  and  $m_{ij+1} = \int_j^{m_{j+1}} \mu_{L_j}(x) dx$ . Thus the fuzzy quality evaluator entity values of the score1 are:  $F_1 = \{m_{11}, m_{12}, 0, 0, 0\}$

Step 7: For obtaining the total degree of membership sensory score equal to 1, individual fuzzy sensory scores are divided by total score  $\sum_{j=1}^5 m_{ij}$  for  $i = 1, 2, 3, \dots, n$

$$\frac{m_{ij}}{\sum_{j=1}^5 m_{ij}} = M_{ij} \text{ and } \sum_{j=1}^5 M_{ij} = 1, i = 1, 2, \dots, n$$

Table 3: Fuzzy Response for an Attribute

Panel member	Poor $L_1=1$	Sufficient $L_2=2$	Good $L_3=3$	Very Good $L_4=4$	Excellent $L_5=5$
$F_1$	$M_{11}$	$M_{12}$	0	0	0
$F_2$	0	$M_{22}$	$M_{23}$	0	0
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
$F_n$	0	0	0	$M_{n4}$	$M_{n5}$

Step 8: The fuzzy quality responses of the evaluators for an attribute are obtained in Table 3. If fuzzy response is the highest, then  $m_{i4} = m_{i5}$  and  $m_{i5} = m_{i6}$  for all  $i = 1, 2, \dots, n$ . when the maximum score of the hedonic scale are 5.



Step 9: The fuzzy statistics quality measure such as mean  $E[X]$  and variance  $V[X]$  can be obtained by the method proposed below:

$$E[X] = \frac{(M_{11} + M_{21} + \dots + M_{n1})/n}{1} + \frac{(M_{12} + M_{22} + \dots + M_{n2})/n}{2} + \dots + \frac{(M_{15} + M_{25} + \dots + M_{n5})/n}{5}$$

$$E[X^2] = \frac{(\sum_{i=1}^n M_{i1}^2)/n}{1} + \frac{(\sum_{i=1}^n M_{i2}^2)/n}{2} + \dots + \frac{(\sum_{i=1}^n M_{i5}^2)/n}{5}$$

$$V[X] = (EX^2 - (EX)^2) \text{ and Standard Deviation } \sigma = \sqrt{V[X]}$$

The Standard Error (SE) of the fuzzy statistics mean =  $\frac{\sigma}{\sqrt{n}}$  where  $n$  = number of respondents. It

reveals the uncertainty of the fuzzy statistical mean. The estimate of the fuzzy statistical mean varies

in the limits  $E[X] \pm \frac{\sigma}{\sqrt{n}}$ .

#### 4. Fuzzy Evaluation of Quality of Cotton Towels

In this section, the planned method is applied for the fuzzy evaluation of cotton towel. Luxury towels were of the utmost importance. With all this creative drive being thrust upon the weavers the birth of the first looped towel happened sometime in the **18th century**. The new invention was called *havly* and it sported rows of loops making up little rectangular clusters. There are different types and sizes are given below;

**Towel:** A towel is a piece of absorbent fabric or paper used for drying or wiping. It draws moisture through direct contact, often using a blotting or a rubbing motion. Common household textile towels are made from **cotton**, rayon, bamboo, **nonwoven** fibers or a few other materials.

**Types of Towel:** There are many types of towel. Some are them given below:

1. Baby Towels
2. Bath Towels
3. Beach Towels
4. Golf Towels
5. Hand Towels
6. Hotel Towels

**Baby Towel:** Baby towel is made with fine quality 100% cotton yarn to give it that soft texture and smooth feel. This is very essential for a baby, because a baby's skin is very sensitive in nature. The baby requires a baby towel which is mild, and not harsh on the skin. A *Hooded Towel* is a variety of a baby towel, which is extremely popular and used by almost all mothers. When babies are slightly wet from being bathed, they lose heat very quickly, especially on the head. *Hooded Towels* eliminate this problem by covering the head. A hooded towel is usually square in shape and has a triangle shaped hood in one corner. It is usually made out of soft unbleached 100% cotton yarn. The reason being, it has to be very soft and mild, since it comes in direct contact with the baby's tender skin.

**Bath Towel:** A Bath Towel is typically rectangular in shape and is normally available in a size of approximately  $75 \times 150$  cms. Bath towels in any form are one of the most basic needs of a human being. We tend to ignore the importance of a bath towel. A point of great interest and one to note is that towel day is celebrated every year on May 25<sup>th</sup>. The standard *bath towel*, understand the need

for high quality bath towels, and it is our endeavour to supply 100% cotton yarn towels to all our esteemed clients, at prices which are highly economical.

**Beach Towel:** Beach towels as the name suggests, are towels carried outdoors in general, and to the beach in particular. They cater to all and sundry, right from babies and little toddlers, to adults and older people. Due to this reason, they come in all types of sizes and weights. Beach towels are available in many varieties. Two of the more popular ones are the *Terry Velour Beach Towel* and the *Printed Beach Towel*. Both these types are made with 100% cotton yarn. The standard beach towels are soft, absorbent and generously size and also provide the buyer with a lot of flexibility, in terms of size, content and material used. Beach towels are also made with *polyester cotton yarn* and *blended yarn*. This form of blended yarn is used essentially as per customer requirements.

**Golf Towel:** Golf Towels have been conceived for the sport of golf. Today these towels still cater to the sport, but the range and quality of Golf Towels have grown by leaps and bounds. They are available in many sizes and their uses are varied. To begin with, there is *The Original Mini Golf Towel*. This is a towel which is  $7.5 \times 7.5$  inches in size, with an attached ball marker.

**Hand Towel:** A hand towel is significantly smaller than a bath towel. The basic aim of any towel is to have good water absorbency, and the hand towels supplied by us, boast of the same. Today, a hand towel has taken on a different meaning and it can be used for various purposes. There are different sizes of towels are available and starting to  $30 \times 30$  cm small one, to a  $50 \times 80$  cm self-coloured large one. The weight of a hand towel is also variable and could oscillate between 15g/pc to about 50g/pc. The range of hand towels and the colours in which they can be made available are enormous. This ranges from small to big in terms of size, and from the bright reds to the more sober shades like lemon, sky blue etc in terms of colour. Due consideration can also be given to the weight factor, keeping in mind the specific needs of a client. Another innovative feature that can be incorporated within the hand towel is the use of satin strips, to give it that different look.

**Hotel Towel:** Hotel Towels are a very big part of the towel manufacturing and supplying industry. They are generally white in colour and made of 100% cotton. These towels can also be of the polyester/cotton blended variety. Today, some hotel towels are made with, twisted loop ring spun, long cotton threads. These loop terry towels are some of the most absorbent. Hotel Towels consist of bath towels, hand towels, wash cloths etc, which vary in size and weight. They can be broadly classified into four categories, in terms of size:

1. Large or Jumbo size bath towels.(Approximate size of  $67 \times 140$  cm)
2. Medium size bath towels.(Approximate size of  $60 \times 120$ cm)
3. Small size hand towels.(Approximate size of  $30 \times 50$ cm)
4. Smallest size towels, which are suitable for the face.

In our study, we are taken the standard bath towels come in different dimensions, depending on the manufacturer, but they typically range in size from 27 inches by 52 inches to 30 inches by 58 inches. A sample survey method is used to collect the physical intensity form the five point hedonic scale for quality evaluation. The College students aged are 24 or less than 24 and Public aged more than 24 are considered from a college and a public having a total strength of 490 and 230 respectively. There are 270 male and 220 females in the selected college and 120 males and 110 females in the selected public peoples. The total population size is 720 and there are four stratum sizes  $N_1=270$ ,  $N_2=220$ ,  $N_3=120$ ,  $N_4=110$ . A sample size of  $n=27$  is selected on the basis of square root of the population size ( $N$ ). Using the method of proportion allocation of sample size to stratum, that is  $n_i \propto N_i$  or  $n_i = (n/N) \times N_i = 1, 2, 3$  and 4 the stratum sample size are  $n_1=10$ ,  $n_2=8$ ,  $n_3=5$ , and  $n_4=4$ , Such that  $n_1 + n_2 + n_3 + n_4 = 27$ . In this method a sample of 27 evaluators of which 10 males and 8

females aged less than are equal to 24; and 5 males and 4 females aged more than 24 are selected by simple random sampling method for the mathematical computation of fuzzy statistics on quality evaluation of towel. They are selected based on good health, non-smokers, non-beetle leaf chewers, interested in quality evaluation, ability to concentrate, learn and familiarity with bath towel. They are trained quality evaluation procedure in terms of identification of quality attributes, score sheet and method of scoring before the actual test. Evaluation are asked to rate the physical intensity of quality attributes of cotton on the five point hedonic scale running from excellent, very good, good, satisfied and poor. The assigned scores of the attributes on hedonic scale are 5,4,3,2 and 1 for excellent, very good, good, satisfied and poor respectively.  $A$  and  $B$  be the attributes, Where  $A$  students for the physical intensity scores greater than median as the scale and  $B$  stands for the panellists age which may be greater than 24.  $\alpha$  and  $\beta$  are the complimentary attributes of  $A$  and  $B$  respectively. The  $2 \times 2$  consistency table each attributes i.e., colour, design, quality, and price of the product is formed and then Yule’s Co-efficient of associations calculated. Table 4 shows the mathematical models of perceived sensation of the quality attributes based on the Yule’s Co-efficient of association.

Table 4: Co-efficient of association with the mathematical models

Attributes	Yule’s co-efficient	Model
Colour	0.48	$y = 0.28x^{1.92}$
Design	0.31	$y = 0.58x^{1.45}$
Quality	0.53	$y = 0.20x^{1.13}$
Price	0.39	$y = 0.44x^{1.64}$
Over all	0.62	$y = 0.08x^{1.63}$

The interval  $[0, 6]$  is chosen since the range of the scale is 1 to 5. The  $[0, 6]$  interval is further divided into different equal length sub-intervals such as  $[1,1]$ ,  $[1,2]$ ,  $[2,3]$ ,  $[4,5]$ , and  $[5,6]$ . The median of these intervals are found as 0.5, 1.5, 2.5, 3.5, 4.5, and 5.5 respectively. If the score of hedonic scale is 1, then 1 is included between 0.5 and 1.5. The fuzzy response is very low that is  $L_1 = 1$  for color which

is  $m_{11} = \int_{0.5}^1 0.28x^{1.92} dx = 0.0832$  and  $m_{12} = \int_1^{1.5} 0.28x^{1.92} dx = 0.2174$  for other score i.e., 2, 3, 4 and 5.

Table 5: Fuzzy Response for Traditional Scores

Traditional score	Poor	Satisfaction	Good	Very Good	Excellent	Fuzzy Response
	$L_1 = 1$	$L_2 = 2$	$L_3 = 3$	$L_4 = 4$	$L_5 = 5$	
1	0.0832	0.2174	0	0	0	0.3006
	0.2768	0.7232	0	0	0	1
2	0	0.4124	0.6666	0	0	1.079
	0	0.3822	0.6178	0	0	1
3	0	0	0.9783	1.3484	0	2.3272
	0	0	0.4206	0.5794	0	1
4	0	0	0	1.7735	2.2546	4.0281
	0	0	0	0.4403	0.5597	1
5	0	0	0	2.7908	3.3816	6.1724
	0	0	0	0.4521	0.5479	1

Table 6: Fuzzy Response for Color of the Cotton Towel

Panel Member	Traditional score	$L_1=1$	$L_2=2$	$L_3=3$	$L_4=4$	$L_5=5$
$F_1$	2	0	0.3822	0.6178	0	0
$F_2$	4	0	0	0	0.4403	0.5597
$F_3$	1	0.2768	0.7232	0	0	0
$F_4$	5	0	0	0	0.4521	0.5479
$F_5$	3	0	0	0.4206	0.5797	0

Table 7: Fuzzy Statistics Quality Measures

Attributes	Mean	SD	SE	Mean Limits	$r$
Color	0.0654	0.1709	0.0329	0.0654±0.0329	0.48
Design	0.0661	0.1688	0.0325	0.0661±0.0325	0.31
Quality	0.0649	0.1715	0.0330	0.0649±0.0330	0.53
Price	0.0657	0.1702	0.0328	0.0657±0.0328	0.39
Over all	0.0637	0.1736	0.0334	0.0637±0.0334	0.62

The attributes expect for design which implies that there is a disassociation between the age groups and physical intensity for color quality and price whereas in case of design there is almost zero positive association. The overall estimated mean of the perceived sensation 6.52 % with 5% level of significance which is less than the desired level of 75 % as defined earlier. Hence, the positive association of the quality attributes of cotton towel is adversely influenced by the age of evaluators.

## 5. CONCLUSION

Fuzzy statistics quality evaluation is based on structure of non-digital set theory and rules. The evaluation of a product lies in its method to handle human thought and recognition. The advantage is its ability to deal with unclear systems and its use of linguistic variables. An accurate quantitative model is required to determine appropriate statistical performance measures for quality evaluation. The discussed procedure differs from the traditional assessment method and establishes the membership functions of evaluator's index to capture better results which will be closer to their real thoughts about the quality attributes. Further, this method of quality evaluation depends on the mathematical and statistics principles and does not involve complicated iterations. The fuzzy statistical measures obtained from the mathematical computations methods using fuzzy logic should be more reasonable than the traditional method of analysis. The mathematical calculation methods of fuzzy statistics for quality evaluation increases the amount of information from the evaluators and discusses the quality of factors such as color, design, quality, price etc. of a product with consumers demographic characters on a hedonic scale.

## 6. Relative Merits of the Present Method

A. Santhakumaran and C.V Kavitha Abirani (2016) have discussed the mathematical computation of fuzzy statistics for sensory evaluation.

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