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RESEARCH ARTICLE

CONNECTED TWO-OUT DEGREE EQUITABLE DOMINATION NUMBER FOR TREES

M.S.MAHESH[1], P.NAMASIVAYAM[2]
[1]Department of Mathematics,Francis Xavier Engineering College,
Tirunelveli, Tamil Nadu
Email: maheshthulasimuthu@gmail.com
PG and Research Department of Mathematics, The M.D.T. Hindu College, Tirunelveli, India

M.S.MAHESH

ABSTRACT

Let G be a simple graph. Let D be dominating set in a graph G is called connected two out degree equitable dominating set if for any two vertices u,v∈ D,such that $|od_D(u) - od_D(v)| \leq 2$, and the induced sub graph <D> is connected The minimum cardinality of a connected two-out degree equitable dominating set is called connected two-out degree equitable domination number, and it is denoted by $\gamma_{c2oe}(G)$. In this paper we obtain some bounds of connected two-out degree equitable domination number for trees.

Key words: connected, two-out degree, equitable, domination number, trees
AMS Classification: 05C9

1. INTRODUCTION

By a graph G=(V,E). we mean a finite, undirected with neither loops nor multiple edges. The order and size of G are denoted by p and q respectively. For graph theoretic terminology we refer to Chartand and Lenisk[4].

Let G=(V,E) be a graph For any vertex v∈ V then open neighborhood of v is the set N(v)= { u∈ V; uv ∈ $E(G)$} and closed neighborhood of v is the set N[v]= N(v) ∪ v. A set D⊆ V of vertices in a graph G is a dominating set if for every vertex v ∈ $V - D$, there exists a vertex u ∈ Dsuch that v is adjacent to u. . The minimum cardinality of a dominating set is called domination number is denoted by $\gamma(G)$. An excellent treatment of the fundamentals of domination is given in the book by Haynes et al [7]. Various types of domination have defined and studied by several authors and more than 75 models of domination are listed in the appendix of Haynes et at [6]. Sampath Kumar and Waliker [3] introduced the concept of connected domination in graph. A dominating set D of G is called a

connected dominating set if the induced sub graph <D> is connected. The minimum cardinality of connected dominating set is called connected domination number and it is denoted γ_{c2oe}(G)
The out degree of v with respect to D denoted by $od_D(v)$, and is defined as $od_D(v) = |N(\text{v}) \cap (V - D)|$. Ali Sahal and Veena Mathad [2] are define two out degree equitable dominating set. A dominating set D in a graph G is called a two-out degree equitable dominating set if for any two vertices u,v∈ D,such that $|od_D(u) - od_D(v)| \leq 2$. The minimum cardinality of a two-out degree equitable dominating set is called the two-out degree equitable domination number of G, and is denoted by γ_{2oe}(G). A graph is acyclic if it has no cycles. A tree is a connected acyclic graph. A graph G is a tree if and only if every two distinct vertices of G are joined by a unique path of G.A caterpillar is a tree T for which the removal of the end vertices leaves a path, which called a spine of G.A Wonderedspider is the graph formed by subdividing at most n-1 of the edges of a star K_{1p} for p≥ 0.Any path with a pendant edge attached at each vertex is called Hoffmantree and is denoted by. ${P_p}^+$

Result [1.1] $\gamma_{c2oe}(k_{1,p}) = p - 2$

2. CONNECTED TWO OUT DEGREE EQUITABLE DOMINATION NUMBER

Definition: 2.1

Let G be a connected graph. A non empty set D of G is called connected two-out degree equitable dominating set if D is dominating set, then for any two vertices $u, v \in D \textit{ such that } |od_D(u) - od_D(v)| \leq 2$ the and induced sub graph <D> is connected . The minimum cardinality of a connected two-out degree equitable domination number of G and is denoted by γ_{c2oe}(G)

Example: 2.2

From the below figure 1, we can find connected two out degree equitable domination number.

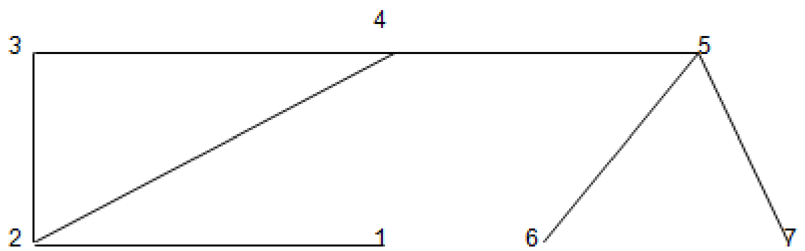

Figure 1: Graph to find connected two out degree equitable domination number

From the above figure {2, 3, 5}and {2,3,4,5} is connected two-out degree equitable dominating set and {2,3,5} is connected two-out degree equitable dominating set with minimum cardinality so $\gamma_{c2oe}(G)$ =3.

3. CONNECTED TWO- OUT DEGREE EQUITABLE DOMINATION NUMBER OF TREES

Theorem: 3.1

Let T be a tree with two pendent vertices and two support vertices then $\gamma_{c2oe}(T) = p - 2$.

Proof:

Let T be any tree of order at least two.

Let V(T)= $\{v_1, v_2 - - - - - - v_p\}$

If there exists two pendent vertices $\{v_1, v_p\}$ which is adjacent to two support vertices $\{v_2, v_{p-1}\}$ respectively

Let D= $\{v_2, v_3 - - - - - - v_{p-1}\}$ and V-D= $\{v_1, v_p\}$

Clam D is connected two out degree equitable dominating set.

Since G is a tree and V-D= $\{v_1, v_p\}$ are pendent vertices then D is connected

Since D is connected and $\{v_2, v_{p-1}\}$ is support vertex to $\{v_1, v_p\}$ then D is dominating set

Let $v_i \in$ D such that v_i is support vertex and each support vertex adjust vertex then $od_D(v_i) = |\mathrm{N}(v_i) \cap V - D| = 1$

Let $v_i \in$ D such that v_i is not support vertex so

$od_D(v_i) = |\mathrm{N}(v_i) \cap V - D| = 0$

$|od_D(u) - od_D(v)| \leq 2$

So D is connected two out degree equitable dominating set and D is minimal connected dominating set

Hence $\gamma_{c2oe}(T) = p - 2$.

Theorem: 3.2

For any tree T, $\gamma_{c2oe}(T) = p - 3$, such that almost two pendent vertex which is adjacent to a support vertex of degree three.

Proof:

Let V=$\{v_1, v_2 - - - - - - - - v_p\}$ be set of vertices in G.

Since T is a tree so $\{v_i, v_{i+1}\}$ be a support vertices and it is adjacent to some pendent vertices. Let $\{v_{p-2}, v_{p-1}, v_p\}$ be the pendent vertices.

Given degree of any support vertices is 3. Let deg(v_i)=3

So $\{v_{p-2}, v_{p-1}\}$ are adjacent to v_i and v_p is adjacent to v_{i+1}

Let D= $\{v_1, v_2 - - - v_i, v_{i+1} - - - - v_{p-3}\}$

Claim D is connected two out degree equitable dominating set

Since T is tree and D doesn't contains pendent vertices

So D is connected and dominating set

$od_D(v_i) = |\mathrm{N}(v_i) \cap V - D|$=2

$od_D(v_{i+1}) = |\mathrm{N}(v_{i+1}) \cap V - D| = 1$

$|od_D(u) - od_D(v)| = 1$

So D is connected two out degree equitable dominating set and D is connected minimal dominating set

D is minimal connected two out degree equitable dominating set

$\gamma_{c2oe}(T) = p - 3$

Corollary : 3.3

For any tree T, $\gamma_{c2oe}(T) = p - \Delta(T)$, such that almost two pendent vertex which is adjacent to a support vertex of degree three.

Proof:

Since T is a tree so $\{v_i, v_{i+1}\}$ be a support vertices and it is adjacent to some pendent vertices. Let $\{v_{p-2}, v_{p-1}, v_p\}$ be the pendent vertices.

Given degree of any support vertices is 3. Let deg(v_i)=3

By theorem 3.2 $\gamma_{c2oe}(T) = p - 3$

Here $\Delta(T) = 3$

$\gamma_{c2oe}(T) = p - \Delta(T)$

Theorem: 3.4

Let T be a tree in which every non-pendent vertex is either a support or adjacent to a support and every non-pendent vertex is support is adjacent to two pendent vertices. Then $\gamma_{c2oe}(T) = p - e$, where e is number of pendent vertex.

Proof:

Let T be any tree of order p.

Let V(T)= $\{v_1, v_2 ------v_p\}$
Let D=$\{v_i, v_{i+1} ------v_k\}$ are set all non pendant vertices and |D|=$p-e$ where e is the number of pendant vertices. $V-D$= $\{v_j, v_{j+1} ------v_t\}$ are set of pendent vertices
Clearly D is a connected set.
Clam D is two out degree equitable dominating set
Let u ∈D ,v is non pendant vertex and u is support vertex
Since every support vertex is adjacent to two pendent vertex then N (u) =2
So $od_D(u) = |N(u) \cap V-D| = 2$
Let v ∈D ,v is non pendant vertex and v is adjacent support vertex then N(v)⊂ D
So $od_D(v) = |N(v) \cap V-D| = 0$
$|od_D(u) - od_D(v)| = 2$
Hence $\gamma_{c2oe}(T) = p-e$.
Theorem 3.5
For any tree T, $\gamma_{c2oe}(T) = p-e$ if and only if T=P_p;or every non pendent vertices of T is either a support or adjacent to a support and each support vertex is adjacent to at most two pendent vertices.
Proof:
Let T be any tree of order p.
Let V (T) = $\{v_1, v_2 ------v_p\}$ and each support vertex is adjacent to at least two pendent vertices
Let assume $\gamma_{c2oe}(T) = p-e$
Then D = $\{v_1, v_2 ------v_{p-e}\}$ be $\gamma_{c2oe}(T)$ set.
To prove T=P_p;or every non pendent vertices of T is either a support or adjacent to a support and each support vertex is adjacent to at most two pendent vertices.
By the definition it is clear that T=P_p
Suppose each support vertex is adjacent to at least two pendent vertices
Let u,v∈ D,if u is not support vertex the $od_D(u)$ =0
v is support vertex then $od_D(v)$>2
$|od_D(u) - od_D(v)| > 2$
Then D is not two out degree equitable dominating set.
This is contradiction so support vertex is adjacent to at most two pendent vertices.
Conversely
Suppose T=P_p;or every non pendent vertices of T is either a support or adjacent to a support and each support vertex is adjacent to at most two pendent vertices.
By theorem 3.4 $\gamma_{c2oe}(T) = p-e$
Let T be any tree of order at least two.
Let D= $\{v_1, v_2 ------v_{p-e}\}$ be non pendent vertices
Clam D is connected two out degree equitable dominating set.
Every non pendent vertices of T is either a support or adjacent to a support and each support vertex is adjacent to at most two pendent vertices.
Let v_i ∈ D, is non pendent vertex and which is adjacent to support vertex then
$od_D(v_i)$ =0,
Let v_i ∈ D, is support vertex and it is adjacent to almost two vertex
$od_D(v_i)$ ≤2,
Then $|od_D(u) - od_D(v)| \leq 2$

D is two out degree equitable domination set and D is connected

So D connected two out degree equitable domination set

$\gamma_{c2oe}(T) = p - e.$

Theorem: 3.6

Let T be a tree it contains one support vertex and it is adjacent to m pendent vertices then $\gamma_{c2oe}(T) = p - 2.$

Proof:

Let V (T) = $\{v_1, v_2 ------v_p, v_i\}$ be vertex of a tree

Since T be a tree it contains one support vertex and it is adjacent to m pendent vertices it forms a star $k_{1,p}$

By result [1.1] $\gamma_{c2oe}(k_{1,p}) = p - 2$

Theorem: 3.7

Let T be a wounded spider obtained from the star $k_{1,p-1}$; n≥ 5 by subdividing m edges exactly once. Then $\gamma_{c2oe}(T) = \begin{cases} p & if\ q = p-1; \\ p-1 & if\ q = p-2\ ; \\ p-2 & if\ \ q \le p-3 \end{cases}$

Proof:

Let V=$\{v_i, v_1, v_2 ----v_p\}$ be the vertices of $k_{1,p-1}$

Case-1 let q=$p - 1$

$k_{1,p-1}$is sub divided into $p - 1$ edges.

$\{v_i, v_1\}, \{v_i, v_2\}, \{v_i, v_3\}, ----------\{v_i, v_{p-1}\}$ Be the subdivision of $k_{1,p-1}$. Clearly each subdivision are connected, and each subdivision is two out degree equitable dominating set.

Therefore V=$\{v_i, v_1, v_2 ----v_{p-1}\}$ of $k_{1,p-1}$ is connected two out degree equitable dominating set

Then $\gamma_{c2oe}(T)$=p.

Case-2 let q=$p - 2$

$k_{1,p-1}$is sub divided into $p - 2$ edges.

$\{v_i, v_1, v_2\}, \{v_i, v_3\}, \{v_i, v_4\}, ----------\{v_i, v_{p-1}\}$ Be the subdivision of $k_{1,p-1}$. Since by definition star $\{v_i, v_1\}$or $\{v_i, v_2\}$ are connected two out degree equitable domination set, and $\{v_i, v_3\}, \{v_i, v_4\}, ----------\{v_i, v_{p-1}\}$ clearly subdivisions are connected, and each subdivision is two out degree equitable dominating set.

Therefore V=$\{v_i, v_2, v_3 ----v_{p-1}\}$ or V=$\{v_i, v_1, v_3 ----v_{p-1}$of $k_{1,p-1}$ is minimum connected two out degree equitable dominating set

Then $\gamma_{c2oe}(T)$=$p - 1$.

Case-3 let $q \le p - 3$

$k_{1,p-1}$is sub divided into at most $p - 3$ edges.

Each sub division forms a star and connected two out degree equitable domination number for star is $p - 2$

$\gamma_{c2oe}(T) = p - 2$

Theorem: 3.8

Let T be a centre pillar with 'p' vertices in central path, and vertices of center path is adjacent to at most two pendent vertices then $\gamma_{c2oe}(T) = p$

Proof:

Let V=$\{v_1, v_2, v_3 ----v_p, v_{p+1}, v_{p+2}, -----v_r\}$ and

$\{v_1, v_2, v_3 ----v_p\}$ are the vertices of central path.

Let us take D=$\{v_1, v_2, v_3 - - - - v_p\}$. By the definition of centre pillar D is connected
Now we want to prove D are two out degree equitable dominating set
If vertices of center path is u and v adjacent to at most two pendent vertices then the out degree of u, $od_D(u) \leq 2$ and $od_D(v) \leq 2$
$|od_D(u) - od_D(v)| \leq 0$
If vertices of center path is u and v adjacent to at most two pendent vertices then the out degree of u, $od_D(u) \leq 2$ other wise $od_D(v) \leq 0$
$|od_D(u) - od_D(v)| \leq 2$
Therefore D is connected two out degree equitable dominating set
$\gamma_{c2oe}(T) = p$

Theorem: 3.9

For any Hoffman tree$\gamma_{c2oe}(P_p{}^+) = p$

Proof:

Let V$(P_p{}^+) = \{v_1, v_2 v_3 - - - - - - v_p, v_{p1}, v_{p2}, v_{p3}, - - - - - v_{pp}\}$
Here $\{v_1, v_2 v_3 - - - - - - v_p\}$ be the vertices of the path, $\{ v_{p1}, v_{p2}, v_{p3}, - - - - - v_{pp}\}$ be the pendant edge attached at each vertex of the path.
Let D= $\{v_1, v_2 v_3 - - - - - - v_p\}$ be minimal dominating set and $V - D$= $\{v_{p1}, v_{p2}, v_{p3}, - - - - - v_{pp}\}$
Each vertices of path vi have a neighborhood in D and other $V - D$
$od_D(v_i) = |N(v_1) \cap V - D|$=1 for all i=1,2,3------n
$|od_D(v_i) - od_D(v_j)| \leq 2$
Then D is two out degree equitable dominating set and<D> form a path , D is minimal dominating set
Then D is minimal connected two out degree equitable dominating set
Then $\gamma_{c2oe}(P_p{}^+) = |D| = p$

Theorem: 3.10

For any tree T with p vertices and maximum degree $\Delta(T)$ then $\gamma_{c2oe}(T) = p - \Delta(T)$ if and if only if T is a spider

Proof:

Let v be a vertex with maximum degree $\Delta(T)$ in a tree T.
If T is a spider with v as a root, then we see that the tree T has exactly $\delta(T)$ branches from v. [since vertices in each of these branches has a degree less than 3 and T is tree]
Thus no of leaves in the tree is $\delta(T)$.
Hence connected two out equitable domination number of spider is $p - \Delta(T)$

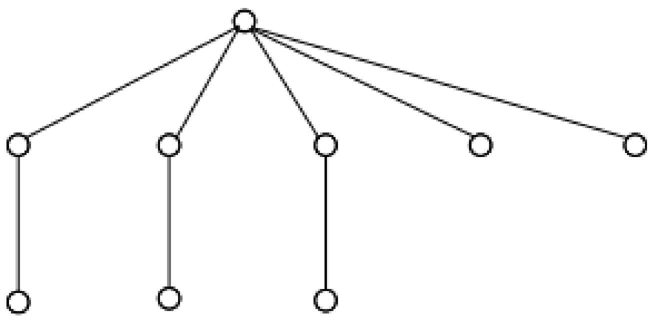

Figure 2: Spider

Conversely

Suppose T is not a spider

There exists a vertex other than v with degree not less than 3 in T

Therefore the tree T has a branch with more than one leaf in it.

This shows that the tree T has more than $\Delta(T)$ leaves which is contradiction

Then T is not a spider

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