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RESEARCH ARTICLE

Solving Fully Fuzzy Linear Systems For ST Decomposition Method Using Gauss Jordan for Triangular Fuzzy Matrices

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ABSTRACT

In this article, we introduce ST decomposition procedure to solve fully fuzzy linear systems. This paper mainly to discuss a decomposition of non singular fuzzy matrix, Symmetric times triangular (ST)decomposition. Every non singular fuzzy matrix can be represented as a product of symmetric matrix S and triangular matrix T in the form of triangular fuzzy number matrices. By this method, we obtain the solution of (either positive or negative) of fully fuzzy linear systems of equations in the case of triangular fuzzy number matrices. A numerical example has been illustrated.

Keywords: Triangular decomposition, Symmetric Matrix, Triangular fuzzy number matrices.

1 INTRODUCTION

Zadeh introduced the concepts of fuzzy numbers and fuzzy arithmetic. Fuzzy systems are used to solve fuzzy metric spaces, fuzzy differential equations, fuzzy linear and non linear system etc. A major application of fuzzy number arithmetic is to solve fully fuzzy linear systems. Problems under Engineering, Physics and Economics should be represented by fuzzy rather than crisp numbers. We develop numerical procedures that would treat fully fuzzy linear system and solve them.

Solving fuzzy $n\times n$ linear system whose coefficient matrix is crisp and the right hand side column is an arbitrary fuzzy number vector was introduced by Friedman et al [6].

Some authors Mosleh [8] introduced LU decomposition method for solving fuzzy linear systems. M. Mosleh [8] introduced solving general fuzzy linear systems using ST decomposition procedure. Muzziloi et al. [9] developed fully fuzzy linear system of the form $A1x + b1 = A2x + b2$ where $A1$, $A2$ are square matrices of fuzzy coefficients, $b1$, $b2$ are fuzzy numbers. Dehgan et al.[2]

considered fully fuzzy linear system of the form *Ax = b* where *A* is a fuzzy matrix, *x* is a fuzzy vector, and the constant *b* are vectors. Vijayalakshmi et al. [13] introduced the concept of solving fully fuzzy linear system for triangular fuzzy number matrices.

The structure of this paper is organized as follows. In section 2, preliminary concepts of triangular fuzzy number matrices have been discussed. In section 3, a new algorithm to solve fuzzy linear system in the form of triangular fuzzy number matrices of order3. In section 4, numerical example have been discussed to solve a solution using Gauss elimination method. In section 5, conclusion about the results have been discussed.

2 Preliminary Definitions

1. A fuzzy set is characterized by a membership function mapping the element of a domain, space or universe of discourse X to the unit interval [0,1]. A fuzzy set A in a universe of discourse X is defined as the following set of pairs

$$A = \{(x, \mu_A(x))\ ;\ x \in X\}$$

Here $\mu_A: X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and μ_A (x) is called the membership value of $x \in X$ on the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

2. A triangular fuzzy number denoted by M = <m,, α, β> has the membership function

$$\mu_{\bar{\mathrm{A}}_{LR}}(x) = \begin{pmatrix} 1 - \dfrac{\overset{0}{m} - x}{\alpha}; -\infty < x \le m\alpha \\ ; m - d \ \le x < m \\ 1 - \dfrac{\overset{\alpha}{x} \ - x}{\beta}; m \le d < x + \beta \\ 0 \qquad ; m + \beta \le x < \infty \end{pmatrix}$$

The basic operations on TFNs. Here we introduce the definition of arithmetic operation.

Let M= <m, α, β> and N = <x,y,γ> be two TFNs.

a) **Addition:** *M + N =< m+ x, α + y , β + γ >*

b) **Scalar Multiplication:** Let *λ* be scalar then *λM =< λm, λα, λβ >* when *λ ≥* 0, *λM =< λm, λα- λβ, >* when *λ ≤* 0. In particular−*M =< −m,−α, β>*

c) **Subtraction:** *M − N =< m,α,β > − < x,y,γ >=< m−x, α−y, β +γ >*. For any 2 TFNs *M* and *N* their addition, subtraction, and scalar multiplication *M + N*, *M − N*, *λM* are all TFNs.

d) **Multiplication:**

Case 1 When *M ≥* 0, *N ≥* 0

M.N =< m,α,β > . < x,y,γ >

Case 2 When *M ≤* 0, *N ≤* 0

M.N =< mx, αy, xβ − mγ, yα − n β >

Case 3 When *M ≤* 0, *N ≤* 0

M.N =< mx, αy,−mγ − xm,−yα− nβ>

e) **Exponentiation:** Using the definition of multiplication it can be shown that M^n is given by

M^n=$<\ m, \alpha, \beta\ >^n$≅<m^n,α^n, $-\alpha m^{n-1}\beta, -n n^{n-1}$> when *n* is negative.

$<\ m, \alpha, \beta\ >^n$≅<m^n,α^n, $-n\alpha^{n-1}\beta, -n n^{n-1}$> when *n is positive.*

3. A matrix $\bar{A}$= (*aij*) is called a fuzzy matrix if each element of $\bar{A}$ is a fuzzy number. A fuzzy matrix $\bar{A}$ will be positive denoted by $\bar{A}$> 0 if each element of $\bar{A}$be positive. To represent *n × n* fuzzy matrix $\bar{A} = (a_{ij})_{n\times n}$ such that matrix *aij* = (*aij, bij, αij*) with the new notation
$\bar{A}$= (*A,B,M*) where *A* = (*aij*), *B* = (*bij*), *M* = (*αij*) are three *n × n* crisp matrices.
4. Consider the *n × n* fuzzy linear systems of equations
$(\bar{a}_{11} \otimes \bar{x}_1) \oplus (\bar{a}_{21} \otimes \bar{x}_2) \oplus \ldots (\bar{a}_{1n} \otimes \bar{x}_n) = \bar{b}_1$
$(\bar{a}_{21} \otimes \bar{x}_1) \oplus (\bar{a}_{22} \otimes \bar{x}_2) \oplus \ldots (\bar{a}_{2n} \otimes \bar{x}_n) = \bar{b}_2$
. . . .
$(\bar{a}_{n1} \otimes \bar{x}_1) \oplus (\bar{a}_{n2} \otimes \bar{x}_2) \oplus \ldots (\bar{a}_{nn} \otimes \bar{x}_n) = \bar{b}_n$
The matrix of the above equation is ¯ *A* ⊗ ¯*x* = ¯*b* where the coefficient matrix $\bar{A}$ = (*aij*), 1 ≤ *i, j* ≤ *n* is a *n × n* fuzzy matrix and $\bar{x}, \bar{b}, \in F(R)$. This system is called fully fuzzy linear system.

Proposed Method

In this method, for solving the crisp linear system of equation *Ax* = *b* is reduced to diagonal matrix by gauss elimination method and applying the back substitution to get the corresponding unknown values in the form of triangular fuzzy numbers. Given any fuzzy linear system of equations in the form of triangular fuzzy matrices that can be decomposed into the form such that *A* = *ST* decomposition method whereas *S* is the symmetric matrix and *T* is upper triangular matrix. An algorithm has been introduced to rewritten A as the product of symmetric matrix and upper triangular matrix.

3 An Algorithm for Solving Triangular Fuzzy Number Matrices of order 3 by ST Decomposition Method

Step : 1 Consider the non singular triangular fuzzy number matrices *A*.

$$\text{set} A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

STEP:2: Decompose the matrix $A, A = ST$ where S is symmetric matrix and T is upper triangular matrix.

$$S = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{21} & s_{22} & s_{23} \\ a_{31} & s_{32} & s_{33} \end{pmatrix}, T = \begin{pmatrix} 1 & t_{12} & t_{13} \\ 0 & 1 & t_{23} \\ 0 & 0 & 1 \end{pmatrix}$$

Where $t_{12} = \frac{a_{12} - a_{21}}{a_{11}}$,

$$s_{22} = \frac{a_{11}a_{22} - a_{12}a_{21} + a_{21}^2}{a_{11}}$$

$$s_{23} = \frac{a_{32}a_{11} - a_{31}a_{12} + a_{21}a_{31}}{a_{11}}$$

$$t_{23} = \frac{(a_{13-}a_{31})a_{21} + (a_{23-}s_{23})a_{11}}{a_{12}a_{21} - a_{11}a_{22}}$$

$$t_{13} = \frac{a_{13-}a_{21}a_{12} - a_{13}a_{11}a_{22} - a_{31}a_{21}a_{12} + a_{31}a_{11}a_{22} - a_{13}a_{21}^2 + a_{31}a_{21}^2 + a_{23}a_{11}a_{21} - s_{23}a_{11}a_{21}}{a_{11}a_{12}a_{21} - a_{11}^2a_{22}}$$

$$s_{33} = \frac{\begin{array}{c} a_{33}a_{11}(a_{21}a_{12} - a_{11}a_{22}) + a_{31}a_{13}a_{11}a_{22} - a_{31}^2a_{11}a_{22} - a_{32}a_{11}a_{13}a_{21} + a_{32}a_{11}a_{31}a_{21} - a_{13}a_{21}^2 + a_{32}a_{23}a_{11}^2 \\ -a_{32}a_{23}a_{11}^2 - a_{31}a_{12}a_{23}a_{11} - a_{31}a_{12}a_{23}a_{11} \end{array}}{a_{11}a_{12}a_{21} - a_{11}^2a_{22}}$$

STEP:3: On solving $A \otimes x = b$ we have

$$(A, B, M) \otimes (x, y, z) = (b, g, h)$$

$$Ax = b \Longrightarrow x = A^{-1}b.$$

$$By = g \Longrightarrow y = B^{-1}g.$$

$$Az + Mx = h \Longrightarrow z = A^{-1}(h - Mx)$$

$$Bz + Ny = h \Longrightarrow z = B^{-1}(h - My)$$

STEP:4: Replace $A = ST, B = S_1T_1$ we have $x = T^{-1}S^{-1}b$.

$y = T_1^{-1}S_1^{-1}g,\ \ z == T^{-1}S^{-1}(h - Mx)$

Using this formula we are finding the solution of x, y, z using ST decomposition.

4.Numerical Example: Consider the following fully fuzzy linear system in the form of 3x3 triangular fuzzy matrices.

$$(1,2,3)\overline{x_1} + (2,3,4)\overline{x_2} + (5,6,7)\overline{x_3} = (1,2,3)$$

$$(3,4,7)\overline{x_1} + (4,5,6)\overline{x_2} + (8,9,10)\overline{x_3} = (2,4,6)$$

$$(9,10,11)\overline{x_1} + (2,4,6)\overline{x_2} + (3,4,6)\overline{x_3} = (3,5,7)$$

Let $A = \begin{pmatrix} 1 & 2 & 5 \\ 3 & 4 & 8 \\ 9 & 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 & 6 \\ 4 & 5 & 9 \\ 10 & 4 & 4 \end{pmatrix}\begin{pmatrix} 3 & 4 & 7 \\ 7 & 6 & 10 \\ 11 & 6 & 6 \end{pmatrix}$, $M = \begin{pmatrix} 5 & 5 & 8 \\ 8 & 8 & 11 \\ 12 & 7 & 7 \end{pmatrix}$,

$$b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix},\quad g = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}\quad h = \begin{pmatrix} 3 \\ 6 \\ 7 \end{pmatrix}$$

Applying the above algorithm by ST decomposition we have $A = ST$ where

$$S = \begin{pmatrix} 1 & 3 & 9 \\ 3 & 7 & 11 \\ 9 & 11 & -33 \end{pmatrix},\quad T = \begin{pmatrix} 1 & -1 & -\frac{19}{2} \\ 0 & 1 & -\frac{9}{2} \\ 0 & 0 & 1 \end{pmatrix}$$

$$t_{12} = -1,\ \ s_{22} = 7,\ \ s_{23} = 11,\ \ t_{23} = -\frac{19}{2},\ \ s_{33} = -33,\ \ t_{13} = \frac{19}{2}$$

Applying gauss jordan method to find inverse, $\left(\frac{A}{I}\right) = (\frac{I}{A^{-1}})$

$$S^{-1} = \begin{pmatrix} \frac{15003}{500} & -\frac{3336}{469} & \frac{2179}{1876} \\ -\frac{3336}{469} & \frac{3837}{938} & -\frac{575}{938} \\ \frac{2179}{1876} & -\frac{575}{938} & \frac{303}{1876} \end{pmatrix},\quad T^{-1} = \begin{pmatrix} 1 & 1 & -5 \\ 0 & 1 & 9/2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$x = T^{-1}S^{-1}b = \begin{pmatrix} -\frac{481}{10} \\ -\frac{6210}{469} \\ \frac{29}{67} \end{pmatrix}, h - Mx = \begin{pmatrix} \frac{1528}{25} \\ \frac{2046}{5} \\ \frac{15149}{25} \end{pmatrix}$$

$$z = T^{-1}S^{-1}(h - Mx) \begin{pmatrix} \frac{694209}{100} \\ \frac{53201}{100} \\ \frac{8826}{25} \end{pmatrix}$$

Consider $B = \begin{pmatrix} 2 & 3 & 6 \\ 4 & 5 & 9 \\ 10 & 4 & 4 \end{pmatrix}$by above algorithm.

$B = T_1S_1$ where $T_1 = \begin{pmatrix} 1 & -\frac{1}{2} & 14 \\ 0 & 1 & -8 \\ 0 & 0 & 1 \end{pmatrix}$, $S_1 = \begin{pmatrix} 2 & 4 & 10 \\ 4 & 7 & 9 \\ 10 & 9 & -64 \end{pmatrix}$

Finding the inverse using gauss Jordan method

$$S_1^{-1} = \begin{pmatrix} \frac{6943}{224} & -\frac{575}{28} & \frac{155}{112} \\ -\frac{575}{28} & \frac{96}{7} & -\frac{13}{14} \\ \frac{155}{112} & -\frac{13}{14} & -\frac{1}{56} \end{pmatrix}, \quad T_1^{-1} = \begin{pmatrix} 1 & \frac{1}{2} & -10 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{pmatrix}$$

$$y = T_1^{-1}S_1^{-1}g = \begin{pmatrix} \frac{34151}{100} \\ \frac{225}{14} \\ \frac{89}{1000} \end{pmatrix}$$

Conclusion

In this article, a new methodology is applied to find the solution of fully fuzzy linear systems in the form of triangular fuzzy matrices. We obtain both positive and negative solution of FFLS. An algorithm is introduced to solve triangular fuzzy matrices by ST decomposition procedure method. Gauss Jordon method has been applied to find the corresponding inverses. We have discussed by taking an example of order 4 to solve fully fuzzy linear system of triangular fuzzy matrices using ST decomposition method.

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