Vol.5.Issue.2.2017 (April-June)
©KY PUBLICATIONS

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com
Email:editorbomsr@gmail.com

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

RESEARCH ARTICLE

IMPROVED EXPONENTIAL RATIO AND PRODUCT TYPE ESTIMATORS FOR POPULATION MEAN IN THE PRESENCE OF NON-RESPONSE

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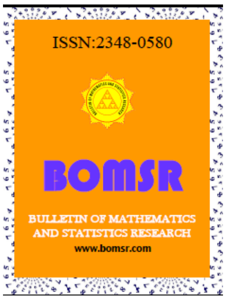

ABSTRACT

This paper present the improved exponential ratio and product type estimators for population mean using known coefficient of variation of study variable and known population mean of auxiliary variable in the presence of non-response. The properties of the proposed estimators have been studied. The proposed estimators are compared theoretically with that of the other existing estimators. An empirical study is also given to show the performance of the proposed estimators.

Keywords: Study variable, auxiliary variable, non-response, known coefficient of variation, mean square error.

1. INTRODUCTION

Estimation of population mean by using classical ratio and product estimators is widely used when the linear relationship between study variable and auxiliary variable is very strong. But when the linear relationship between study variable and auxiliary variable is not very strong, in this situation, the exponential ratio and product type estimators were proposed by Bahl and Tuteja (1991), is very useful for estimating the population mean. The problem of non-response often occurs during conducting the sample surveys. For estimating the population mean in the presence of non-response, Hansen and Hurwitz (1946) first suggested the estimator for the population mean. Further by using the known population mean of auxiliary variable, various estimators for population mean of study variable in the presence of non-response, have been proposed by Rao (1986, 87), Khare and Srivastava (1996, 97), Singh et al. (2010), Kumar and Bhougal (2011) and Kumar (2013).

For estimating the population mean, the value of known coefficient of variation of study variable is also useful. Searls (1964, 1967) first suggested the estimator for population mean by using the known value of coefficient of variation of study variable. Further, Khare and Kumar (2009) proposed the ratio, product and regression type estimators for population mean by using the known value of coefficient of variation of study variable in the presence of non-response.

In this paper, we have proposed improved exponential ratio and product type estimators for population mean using known value of coefficient of variation of study variable and known value of population mean of auxiliary variable in the presence of non-response. The properties of the proposed estimators are studied. The conditions, in which the proposed estimators are more efficient in comparison to the relevant estimators, are obtained. An empirical study is also given in support of the problem under study.

2. THE ESTIMATORS

Let (Y_i , X_i): *i=1,2…N*. be the values of *N* units of the population for study variable y and auxiliary variable x having population means ($\bar{Y}$, $\bar{X}$) respectively. Suppose the whole population is divided into responding group having N_1 units and non-responding group having N_2 units. Using the technique of Hansen and Hurwitz (1946), a sample of size $n(< N)$ is drawn from the population of size *N* by using simple random sampling without replacement (SRSWOR) method and find that n_1 units respond and n_2 units do not respond in the sample of size n. Further, a subsample of size $m(= n_2 / k, k > 1)$ is drawn from n_2 non-responding units by SRSWOR method of sampling and the information on m units is collected by personal interview. Using the information of study variable y on n_1 and m units, the estimator for population mean $\bar{Y}$ of study variable y in the presence of non-response was defined by Hansen and Hurwitz (1946) is given as follows:

$$\bar{y}^* = \frac{n_1}{n}\bar{y}_1 + \frac{n_2}{n}\bar{y}_2' , \tag{2.1}$$

where $\bar{y}_1$ and $\bar{y}_2'$ denote the means of study variable y based on n_1 and m units respectively.

Similarly, the estimator for population mean $\bar{X}$ of auxiliary variable x is defined as:

$$\bar{x}^* = \frac{n_1}{n}\bar{x}_1 + \frac{n_2}{n}\bar{x}_2' , \tag{2.2}$$

where $\bar{x}_1$ and $\bar{x}_2'$ denote the means of auxiliary variable x based on n_1 and m units respectively.

The mean square error (MSE) of the estimators $\bar{y}^*$ and $\bar{x}^*$ are given as:

$$MSE(\bar{y}^*) \cong \frac{f}{n}S_y^2 + \frac{W_2(k-1)}{n}S_{y2}^2 \tag{2.3}$$

and

$$MSE(\bar{x}^*) \cong \frac{f}{n}S_x^2 + \frac{W_2(k-1)}{n}S_{x2}^2 , \tag{2.4}$$

where $f = (1 - n/N)$, $W_2 = N_2 / N$, (S_y^2, S_{y2}^2) and (S_x^2, S_{x2}^2) are the population mean squares of the study variable y and auxiliary variable x for the entire population and for the non-responding part of the population.

Using the known value of coefficient of variation of study variable, Searls (1964) first defined an estimator for population mean which is given as follows:

$$\bar{y}_s = D_0\bar{y}, \tag{2.5}$$

where D_0 is constant and $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$.

By minimizing the MSE of the estimator $\bar{y}_s$, the optimum value of D_0 is obtained as follows:

$$D_{0(opt)} = \left(1 + \frac{f}{n} C_y^2\right)^{-1}, \tag{2.6}$$

where C_y^2 is the known coefficient of variation of study variable y.

Following Searls (1964) estimator, an estimator for population mean $\bar{Y}$ of study variable y in the presence of non-response was defined by Khare and Kumar (2009) as follows:

$$\bar{y}^{**} = D_1 \bar{y}^*, \tag{2.7}$$

where D_1 is constant.

By minimizing the *MSE* of the estimator $\bar{y}^{**}$, the optimum value of D_1 is obtained as follows:

$$D_{1(opt)} = \left(1 + \frac{f}{n}\frac{S_y^2}{\bar{Y}^2} + \frac{W_2(k-1)}{n}\frac{S_{y2}^2}{\bar{Y}^2}\right)^{-1}. \tag{2.8}$$

Since $\frac{S_y^2}{\bar{Y}^2}$ and $\frac{S_{y2}^2}{\bar{Y}^2}$ do not differ significantly, so we may approximate $V_y^2 = \frac{S_y^2}{\bar{Y}^2} = \frac{S_{y2}^2}{\bar{Y}^2}$ and neglecting the terms of order 1/N, we get

$$D_{1(opt)} = \left[1 + \frac{V_y^2}{n}\{1 + W_2(k-1)\}\right]^{-1}, \tag{2.9}$$

where V_y is the known coefficient of variation of study variable y.

Hence, the *MSE* of the estimator $\bar{y}^{**}$ is obtained as:

$$MSE(\bar{y}^{**}) = (1-2M)\left\{\frac{f}{n} S_y^2 + \frac{W_2(k-1)}{n} S_{y2}^2\right\}, \tag{2.10}$$

where
$$M = \frac{V_y^2}{n}\{1 + W_2(k-1)\} \tag{2.11}$$

Bahl and Tuteja (1991) first proposed an exponential ratio $(\bar{y}'_{er})$ and product $(\bar{y}'_{ep})$ type estimators for population mean $\bar{Y}$ of study variable y using known population mean $\bar{X}$ of auxiliary variable x which are given as:

$$\bar{y}'_{er} = \bar{y}\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \tag{2.12}$$

and

$$\bar{y}'_{ep} = \bar{y}\exp\left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}}\right), \tag{2.13}$$

where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$.

When non-response occurs both on study and auxiliary variables, Following the Bahl and Tuteja (1991) estimators, Singh et al. (2010) proposed the conventional exponential ratio (T'_{1r}) and product (T'_{1p}) type estimators for population mean $\bar{Y}$ of study variable y using known population mean $\bar{X}$ of auxiliary variable x in the presence of non-response, which are given as follows:

$$T'_{1r} = \bar{y}^* \exp\left(\frac{\bar{X} - \bar{x}^*}{\bar{X} + \bar{x}^*}\right) \tag{2.14}$$

and

$$T'_{2p} = \bar{y}^* \exp\left(\frac{\bar{x}^* - \bar{X}}{\bar{x}^* + \bar{X}}\right) \tag{2.15}$$

When non-response occurs only on study variable, Following the Bahl and Tuteja (1991) estimators, Singh et al. (2010) proposed the alternate exponential ratio (T'_{3r}) and product (T'_{4p}) type estimators for population mean $\bar{Y}$ of study variable y using known population mean $\bar{X}$ of auxiliary variable x in the presence of non-response, which are given as follows :

$$T'_{3r} = \bar{y}^* \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \tag{2.16}$$

and

$$T'_{4p} = \bar{y}^* \exp\left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}}\right) \tag{2.17}$$

3. THE PROPOSED ESTIMATORS

When non-response occurs both on study and auxiliary variables, Following the Singh et al. (2010) estimators, we propose the conventional exponential ratio (T'_{5r}) and product (T'_{6p}) type estimators for population mean $\bar{Y}$ of study variable y using known coefficient of variation of study variable y and known population mean $\bar{X}$ of auxiliary variable x in the presence of non-response, which are given as:

$$T'_{5r} = \bar{y}^{**} \exp\left(\frac{\bar{X} - \bar{x}^*}{\bar{X} + \bar{x}^*}\right) \tag{3.1}$$

and

$$T'_{6p} = \bar{y}^{**} \exp\left(\frac{\bar{x}^* - \bar{X}}{\bar{x}^* + \bar{X}}\right), \tag{3.2}$$

where $\bar{y}^{**} = D_1 \bar{y}^*$.

When non-response occurs only on study variable, Following the Singh et al. (2010) estimators, we propose the alternate exponential ratio (T'_{7r}) and product (T'_{8p}) type estimators for population mean $\bar{Y}$ of study variable y using known coefficient of variation of study variable y and known population mean $\bar{X}$ of auxiliary variable x in the presence of non-response, which are given as:

$$T'_{7r} = \bar{y}^{**} \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \tag{3.3}$$

and

$$T'_{8p} = \bar{y}^{**} \exp\left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}}\right). \tag{3.4}$$

4. THE MEAN SQUARE ERROR (MSE) OF THE PROPOSED ESTIMATORS

In order to derive the expressions for the mean square error of the proposed estimators. Let $\bar{y}^* = \bar{Y}(1+\varepsilon_0)$, $\bar{x}^* = \bar{X}(1+\varepsilon_1)$, $\bar{x} = \bar{X}(1+\varepsilon_2)$ such that $E(\varepsilon_0) = E(\varepsilon_1) = E(\varepsilon_2) = 0$.

By using simple random sampling without replacement method of sampling, we have

$$E(\varepsilon_0^2)=\frac{1}{\bar{Y}^2}V(\bar{y}^*)=\frac{1}{\bar{Y}^2}\left[\frac{f}{n}S_y^2+\frac{W_2(k-1)}{n}S_{y2}^2\right],$$

$$E(\varepsilon_1^2)=\frac{1}{\bar{X}^2}V(\bar{x}^*)=\frac{1}{\bar{X}^2}\left[\frac{f}{n}S_x^2+\frac{W_2(k-1)}{n}S_{x2}^2\right],$$

$$E(\varepsilon_2^2)=\frac{1}{\bar{X}^2}V(\bar{x})=\frac{f}{n}\frac{S_x^2}{\bar{X}^2},\ E(\varepsilon_0\varepsilon_1)=\frac{1}{\bar{Y}\bar{X}}COV(\bar{y}^*,\bar{x}^*)=\frac{1}{\bar{Y}\bar{X}}\left[\frac{f}{n}S_{yx}+\frac{W_2(k-1)}{n}S_{yx2}\right]$$

and $$E(\varepsilon_0\varepsilon_2)=\frac{1}{\bar{Y}\bar{X}}COV(\bar{y}^*,\bar{x})=\frac{1}{\bar{Y}\bar{X}}\frac{f}{n}S_{yx}, \tag{4.1}$$

where $S_{yx}=\rho_{yx}S_yS_x$, $S_{yx2}=\rho_{yx2}S_{y2}S_{x2}$ and (ρ_{yx},ρ_{yx2}) are correlation coefficients between study variable y and auxiliary variable x for responding and not responding units of the population.

The expressions for MSE of the proposed estimators T'_{5r} ,T'_{6p}, T'_{7r} and T'_{8p} up to the terms of order n^{-1} are given as follows:

$$MSE(T'_{5r})=\bar{Y}^2\left[\frac{f}{n}\left\{(1-2M)C_y^2+\frac{(1-4M)}{4}C_x^2-(1-3M)C_{yx}\right\}+\frac{W_2(k-1)}{n}\left\{(1-2M)C_{y2}^2+\frac{(1-4M)}{4}C_{x2}^2-(1-3M)C_{yx2}\right\}\right], \tag{4.2}$$

$$MSE(T'_{6p})=\bar{Y}^2\left[\frac{f}{n}\left\{(1-2M)C_y^2+\frac{1}{4}C_x^2+(1-3M)C_{yx}\right\}+\frac{W_2(k-1)}{n}\left\{(1-2M)C_{y2}^2+\frac{1}{4}C_{x2}^2+(1-3M)C_{yx2}\right\}\right], \tag{4.3}$$

$$MSE(T'_{7r})=\bar{Y}^2\left[\frac{f}{n}\left\{(1-2M)C_y^2+\frac{(1-4M)}{4}C_x^2-(1-3M)C_{yx}\right\}+\frac{W_2(k-1)}{n}(1-2M)C_{y2}^2\right] \tag{4.4}$$

and

$$MSE(T'_{8p})=\bar{Y}^2\left[\frac{f}{n}\left\{(1-2M)C_y^2+\frac{1}{4}C_x^2+(1-3M)C_{yx}\right\}+\frac{W_2(k-1)}{n}(1-2M)C_{y2}^2\right], \tag{4.5}$$

where $C_{yx}=\rho_{yx}C_yC_x$, $C_{yx2}=\rho_{yx2}C_{y2}C_{x2}$, $C_y=\frac{S_y}{\bar{Y}}$, $C_x=\frac{S_x}{\bar{X}}$, $C_{y(2)}=\frac{S_{y(2)}}{\bar{Y}}$, $C_{x(2)}=\frac{S_{x(2)}}{\bar{X}}$,

$S_y^2=\frac{1}{N-1}\sum_{i=1}^{N}(Y_i-\bar{Y})^2$, $S_x^2=\frac{1}{N-1}\sum_{i=1}^{N}(X_i-\bar{X})^2$, $S_{y2}^2=\frac{1}{N_2-1}\sum_{i=1}^{N_2}(Y_i-\bar{Y}_2)^2$,

$S_{x2}^2=\frac{1}{N_2-1}\sum_{i=1}^{N_2}(X_i-\bar{X}_2)^2$ and $(\bar{Y}_2,\bar{X}_2)$ are population means of study variable y and auxiliary variable x for the non-responding units (N_2) of the population.

The expressions for the MSE of the estimators T'_{1r}, T'_{2p}, T'_{3r} and T'_{4p} up to the terms of order n^{-1} are given as follows:

$$MSE(T'_{1r})=\bar{Y}^2\left[\frac{f}{n}\left\{C_y^2+\frac{1}{4}C_x^2-C_{yx}\right\}+\frac{W_2(k-1)}{n}\left\{C_{y2}^2+\frac{1}{4}C_{x2}^2-C_{yx2}\right\}\right], \tag{4.6}$$

$$MSE(T'_{2p})=\bar{Y}^2\left[\frac{f}{n}\left\{C_y^2+\frac{1}{4}C_x^2+C_{yx}\right\}+\frac{W_2(k-1)}{n}\left\{C_{y2}^2+\frac{1}{4}C_{x2}^2+C_{yx2}\right\}\right], \tag{4.7}$$

$$MSE(T'_{3r}) = \bar{Y}^2\left[\frac{f}{n}\left\{C_y^2 + \frac{1}{4}C_x^2 - C_{yx}\right\} + \frac{W_2(k-1)}{n}C_{y2}^2\right] \tag{4.8}$$

and
$$MSE(T'_{4p}) = \bar{Y}^2\left[\frac{f}{n}\left\{C_y^2 + \frac{1}{4}C_x^2 + C_{yx}\right\} + \frac{W_2(k-1)}{n}C_{y2}^2\right]. \tag{4.9}$$

5. COMPARISON OF THE PROPOSED ESTIMATORS WITH RELEVANT ESTIMATORS

$MSE(T'_{5r}) < MSE(\bar{y}^*)$ If $\rho_{yx} > \frac{1}{4}\frac{C_x}{C_y} - \frac{1}{4}\left[\frac{M(8C_y^2 + C_x^2)}{(1-3M)C_xC_y}\right]$ and $\rho_{yx2} > \frac{1}{4}\frac{C_{x2}}{C_{y2}} - \frac{1}{4}\left[\frac{M(8C_{y2}^2 + C_{x2}^2)}{(1-3M)C_{x2}C_{y2}}\right]$

$MSE(T'_{6p}) < MSE(\bar{y}^*)$ If $\rho_{yx} < -\frac{1}{4}\frac{C_x}{C_y} + \frac{1}{4}\left[\frac{M(8C_y^2 + 3C_x^2)}{(1-3M)C_xC_y}\right]$ and

$\rho_{yx2} < -\frac{1}{4}\frac{C_{x2}}{C_{y2}} + \frac{1}{4}\left[\frac{M(8C_{y2}^2 + 3C_{x2}^2)}{(1-3M)C_{x2}C_{y2}}\right]$,

$MSE(T'_{7r}) < MSE(\bar{y}^*)$ If $\rho_{yx} > \frac{1}{4}\frac{C_x}{C_y} - \frac{1}{4}\left[\frac{M(8C_y^2 + C_x^2)}{(1-3M)C_xC_y}\right]$,

$MSE(T'_{8p}) < MSE(\bar{y}^*)$ If $\rho_{yx} < -\frac{1}{4}\frac{C_x}{C_y} + \frac{1}{4}\left[\frac{M(8C_y^2 + 3C_x^2)}{(1-3M)C_xC_y}\right]$,

$MSE(T'_{5r}) < MSE(\bar{y}^{**})$ If $\rho_{yx} > \frac{1}{4}\frac{C_x}{C_y}\frac{(1-4M)}{(1-3M)}$ and $\rho_{yx2} > \frac{1}{4}\frac{C_{x2}}{C_{y2}}\frac{(1-4M)}{(1-3M)}$,

$MSE(T'_{6p}) < MSE(\bar{y}^{**})$ If $\rho_{yx} - < \frac{1}{4(1-3M)}\frac{C_x}{C_y}$ and $\rho_{yx2} - < \frac{1}{4(1-3M)}\frac{C_{x2}}{C_{y2}}$,

$MSE(T'_{7r}) < MSE(\bar{y}^{**})$ If $\rho_{yx} > \frac{1}{4}\frac{C_x}{C_y}\frac{(1-4M)}{(1-3M)}$,

$MSE(T'_{8p}) < MSE(\bar{y}^{**})$ If $\rho_{yx} - < \frac{1}{4(1-3M)}\frac{C_x}{C_y}$,

$MSE(T'_{5r}) < MSE(T'_{1r})$ If $\rho_{yx} < \frac{1}{3}\left[\frac{C_x}{C_y} + \frac{2C_y}{C_x}\right]$ and $\rho_{yx2} < \frac{1}{3}\left[\frac{C_{x2}}{C_{y2}} + \frac{2C_{y2}}{C_{x2}}\right]$,

$MSE(T'_{6p}) < MSE(T'_{2p})$ If $\rho_{yx} > -\frac{2}{3}\frac{C_y}{C_x}$ and $\rho_{yx2} > -\frac{2}{3}\frac{C_{y2}}{C_{x2}}$,

$MSE(T'_{7r}) < MSE(T'_{3r})$ If $\rho_{yx} < \frac{1}{3}\left[\frac{C_x}{C_y} + \frac{2C_y}{C_x}\right]$

and

$MSE(T'_{8p}) < MSE(T'_{4p})$ If $\rho_{yx} > -\frac{2}{3}\frac{C_y}{C_x}$.

6. EMPIRICAL STUDY

6.1. Data set-I [Khare and Kumar(2009)]

For the population of 96 villages of rural areas under polish station, Singur, District Hooghly from 'District Census Handbook, 1981, published by government of India, the data on the number of cultivators, as a study variable (y) and the population of villages, as an auxiliary variable (x) have been taken. The values of the parameters of the population are given as follows:

$\bar{X}$ =1807.23, $\bar{Y}$ =185.22, S_x =1921.77, S_y =195.03, ρ_{yx} = 0.904, n=25, $V_y = 1$.

The non-response rate in the population is considered to be 25%. So, the values of the population parameters based on the non-responding parts, which are taken as the last 25% units of the population are given as follows:

$\bar{X}_2$ =1571.71, $\bar{Y}_2$ =128.46, S_{x2} = 68.44, S_{y2} =97.82, ρ_{yx2} =0.895.

Table 1: Relative efficiency (in %) of the proposed estimators and relevant estimators with respect to $\bar{y}^*$ for fixed values of N, n and different values of k (N=96, n=25)

Estimators	1/k		
	1/4	1/3	1/2
$\bar{y}^*$	100.00 (1412.32)	100.00 (1316.63)	100.00 (1220.94)
$\bar{y}^{**}$	116.28 (1214.59)	113.64 (1158.63)	111.11 (1098.85)
T'_{1r}	215.99 (0653.88)	233.36 (0564.21)	257.29 (0474.54)
T'_{3r}	210.19 (0671.93)	228.48 (0576.24)	254.07 (0480.56)
T'_{5r}	237.24 (0595.32)	251.06 (0524.42)	271.29 (0450.05)
T'_{7r}	231.68 (0609.60)	246.42 (0534.30)	268.24 (0455.17)

Numbers in the parenthesis give the MSE.

From table-1, It has been observed that using the known value of coefficient of variation (V_y) of study variable y and the known value of population mean $\bar{X}$ of auxiliary variable x, the proposed exponential ratio type estimators (T'_{5r} , T'_{7r} **)** are more efficient in comparison to the estimators ($\bar{y}^*$, $\bar{y}^{**}$) and the corresponding estimators (T'_{1r} , T'_{3r}) for different values of k. The values of the *MSE* of all the estimators $\bar{y}^*$, $\bar{y}^{**}$, T'_{1r} , T'_{3r} , T'_{5r} and T'_{7r} decrease as the values of k decrease.

6.2. Data set II [Khare and Kumar (2009)]

In the case of negative correlation, for the 56 countries from U.S. Bureau of the census, city and country Data Book, 1986, the data on the average size of farms (hundreds of acres) is considered as an auxiliary variable x and the average value of products sold ($ thousand) is considered as a study variable (y). The values of parameters of the population are given as follows:

$\bar{X}$ =75.79, $\bar{Y}$ =61.59, S_x =12.47, S_y =24.03, ρ_{yx} = -0.508, n=15, $V_y = 0.5$.

The non-response rate in the population is considered to be 20%. So, the values of the population parameters based on the non-responding parts, which are 53 taken as the last 20% units (from 43th unit- 53th unit) of the population are given as follows:

$\bar{X}_2$ =57.60, $\bar{Y}_2$ =51.02, S_{x2} = 10.50, S_{y2} =13.91, ρ_{yx2} =-.379.

Table 2: Relative efficiency of the proposed estimators and relevant estimators with respect to $\bar{y}^*$ for fixed values of N, n and different values of k (N=56, n=15)

Estimators	1/k		
	1/4	1/3	1/2
$\bar{y}^*$	100.00 (35.79)	100.00 (33.25)	100.00 (30.72)

$\bar{y}^{**}$	105.59 (33.89)	104.87(31.71)	104.15 (29.49)
T'_{2p}	119.49 (29.95)	119.76 (27.77)	120.07 (25.58)
T'_{4p}	115.43 (31.00)	116.81 (28.47)	118.45(25.93)
T'_{6p}	124.81 (28.67)	124.43 (26.72)	124.09 (24.75)
T'_{8p}	120.96 (29.59)	121.61 (27.34)	122.53 (25.07)

Numbers in the parenthesis give the *MSE.*

From table-2, It has been observed that using the known value of coefficient of variation (V_y) of study variable y and the known value of population mean $\bar{X}$ of auxiliary variable x, the proposed exponential product type estimators (T'_{6p}**,** T'_{8p}**)** are more efficient in comparison to the estimators ($\bar{y}^*$, $\bar{y}^{**}$) and the corresponding estimators (T'_{2p}, T'_{4p}) for different values of k. The values of *MSE* of all the estimators $\bar{y}^*$, $\bar{y}^{**}$, $T'_{2p}, T'_{4p}, T'_{6p}$ and T'_{8p} decrease as the values of k decrease.

7. CONCLUSION

Since it has been observed that using the known value of coefficient of variation (V_y) of study variable, the proposed exponential ratio (T'_{5r}, T'_{7r}**)** and product (T'_{6p}**,** T'_{8p}**)** estimators are more efficient in comparison to the estimators ($\bar{y}^*$, $\bar{y}^{**}$) and corresponding estimators (T'_{1r}, T'_{3r}) and (T'_{2p}, T'_{4p}) in the presence of non-response. Hence we conclude that the coefficient of variation (V_y) has an important contribution in increasing the efficiency of the estimators.

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