Vol.5.Issue.2.2017 (April-June)
©KY PUBLICATIONS

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com
Email:editorbomsr@gmail.com

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

RESEARCH ARTICLE

THE GENERALIZED WEIBULL-GUMBEL DISTRIBUTION

Innocent Boyle ERAIKHUEMEN[1], Terna Godfrey IEREN[2]
[1]Department of Mathematics and Computer Science, Benson Idahosa University Benin-city, Nigeria
ieraikhuemen@biu.edu.ng
[2]Department of Statistics, Ahmadu Bello University Zaria, Nigeria
ternagodfrey@gmail.com

ABSTRACT

This paper studies another generalization of the Gumbel distribution known as the Generalized Weibull-Gumbel distribution useful for modeling lifetime data. The mathematical properties of the new distribution are provided. The explicit expressions for its quantile, survival, and hazard functions were studied. Some plots of the distribution indicate that the Generalized Weibull-Gumbel distribution can take various shapes such as symmetrical, left-skewed, right-skewed etc. This fact implies that the Generalized Weibull-Gumbel distribution can be very useful for data sets under various shapes. The implications of the plots for the survival and hazard functions means that the Generalized Weibull-Gumbel distribution (*GWGD)* will be appropriate in modeling time or age-dependent events, where survival rate decreases with time and failure increases with time or age. The method of maximum likelihood is used for estimating the model parameters and the density function for the minimum and maximum order statistics are also derived. We finally illustrate the usefulness of the proposed model.

Keywords: Gumbel Distribution, Reliability analysis, Maximum likelihood estimation, order statistics and Generalized Weibull-Gumbel distribution.

1 Background

The Gumbel distribution is a very popular statistical model due to its wide applicability. An extensive list of the Gumbel model applications can be obtained in Kotz and Nadarajah (2000). Some applications of the Gumbel model include: climate modeling, global warming problems, offshore modeling, rainfall and wind speed modeling Nadarajah (2006). The applications of this model can also be found in various areas of engineering, such as flood frequency analysis, network, nuclear, risk-based, space, software reliability, structural and wind engineering Cordeiro *et al*. (2012). Due to its wide applicability, several works aimed at extending the Gumbel model have been carried out by many researchers among who are : Nadarajah and Kotz (2004), Nadarajah (2006), Cordeiro *et al*. (2012) and Andrade *et al*. (2015).

The cumulative distribution function *(cdf) G(x)* and probability density function (*pdf) g(x*) of the Gumbel distribution are given by.

$$g(x)=\frac{1}{\sigma}\exp\left(-\frac{x-\mu}{\sigma}\right)\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\} \tag{1}$$

and

$$G(x)=\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\} \tag{2}$$

with location parameter $-\infty < \mu < \infty$ and scale parameter $\sigma > 0$ respectively.

There are several generalized families of probability distributions proposed by different researchers which have been used by others to extend so many standard or classical distributions to produce compound distributions found to be better than the classical ones. Some of these among others are the Kumaraswamy-G by Cordeiro and de Castro (2011), Gamma-G (type 1) by Zografos and Balakrishnan (2009), ßeta-G by Eugene *et al*. (2002), McDonald-G by Alexander *et al*. (2012), Gamma-G (type 2) by Ristic *et al*. (2012), Gamma-G (type 3) by Torabi and Montazari (2012), Log-gamma-G by Amini *et al*. (2012), Exponentiated T-X by Alzaghal *et al*. (2013), Transmuted family of distributions by Shaw and Buckley (2007), Logistic-G by Torabi and Montazari (2014), Gamma-X by Alzaatreh *et al*., (2013), Logistic-X by Tahir *et al*. (2015), Weibull-X by Alzaatreh *et al*. (2013), Weibull-G by Bourguignon *et al*. (2014), Exponentiated-G (EG) by Cordeiro *et al*. (2013) and Beta Marshall-Olkin family of distributions by Alizadeh *et al*. (2015) e.t.c. Cordeiro *et al*. (2015) defined the generalized Weibull family of distributions (*GW-G*) with two extra parameters $\alpha > 0$ and $\beta > 0$, who's *pdf f(x)* and *cdf F(x) is* given by;

$$f(x)=\alpha\beta\frac{g(x)}{G'(x)}\left(-\log[1-G(x)]\right)^{\beta-1}\exp\left\{-\alpha\left(-\log[1-G(x)]\right)^{\beta}\right\} \tag{3}$$

and

$$F(x)=1-\exp\left\{-\alpha\left(-\log[1-G(x)]\right)^{\beta}\right\} \tag{4}$$

respectively.

Where $g(x)$ and $G(x)$ are the *pdf* and the *cdf* of any baseline continuous distribution respectively while $G'(x)=1-G(x)$ and $\alpha > 0$ and $\beta > 0$ are the scale and shape parameters respectively.

The aim of this article is to formulate the Generalized Weibull-Gumbel distribution (*GWGD)* from the proposed family by Cordeiro *et al*. (2015).

The rest of this article is organized as follows: In section 2, we defined the proposed distribution and provide a plot for its *pdf* and *cdf*. In section 3, we obtain some properties of the new distribution. In section 4, we provide the *pdf* of its smallest and largest order statistics. The maximum likelihood estimates (MLEs) of the unknown model parameters are provided in section 5. Finally, in section 6, we provide some useful conclusions.

2. The *GWGD*

Using the *pdf* (1) and the *cdf* (2) of the Gumbel distribution with location parameter $\mu\in\mathbb{R}$ and dispersion parameter $\sigma>0$. The *pdf* and *cdf* of the *GWGD* are obtained from equation (3) and (4) as

$$f(x)=\alpha\beta\frac{\exp\left(-\frac{x-\mu}{\sigma}\right)\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}}{\sigma\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]}\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta-1}\exp\left\{-\alpha\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta}\right\} \quad (5)$$

And the corresponding *cdf* is given by;

$$F(x)=1-\exp\left\{-\alpha\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta}\right\} \quad (6)$$

respectively.

Graphical analysis of the *pdf* and *cdf*.

Given some values for the parameters *α, β, μ* and *σ*, we provide some possible shapes for the *pdf* and the *cdf* of the *GWGD* as shown in figure 1 and 2 below:

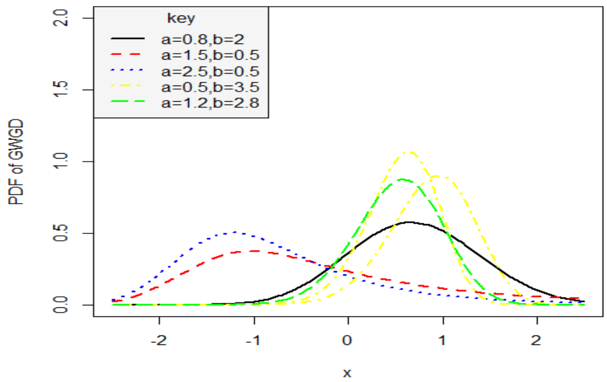

Fig. **1**: PDF of the GWGD for different values of α and β and some values of the parameters

$$a=\alpha, b=\beta, \mu=0 and \sigma=1.$$

From figure **(1)** it can be seen that the *GWGD* distribution has various shapes such as symmetrical, left-skewed, right-skewed shapes. This means that distribution can be very useful for data sets with different shapes.

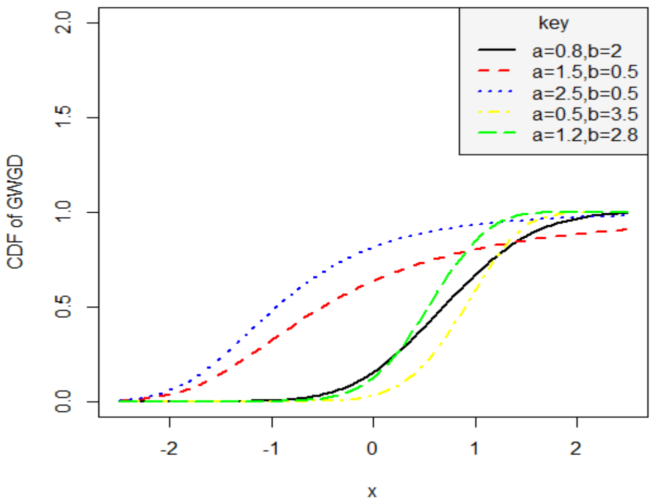

Fig. **2**: CDF of the GWGD for different values of α and β and some values of the parameters

$$a=\alpha, b=\beta, \mu=0 and \sigma=1.$$

From the above *cdf* plot, the *cdf* increases when *X* increases, and approaches 1 when *X* becomes large, as expected.

3. Properties

In this section, we study some properties of the *GWGD* distribution.

3.1 Quantile function of the *GWGD*.

It is used for the generation of some moments of random variables such as skewness and kurtosis. It is also used to obtain the median and for generation of random numbers. It is derived by inverting the *cdf* of the distribution in question.

The quantile function, say *X=Q(u)*, of the *GWGD* can be obtained as the inverse of Equation (6) as;

$$F(x) = 1 - \exp\left\{-\alpha\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta}\right\}$$

$$X = Q(u) = \mu + \sigma \ln\left\{\left[\ln\left(\frac{1}{1-\exp\left\{-\left(\frac{1}{\alpha}\ln\left(\frac{1}{1-u}\right)\right)^{\frac{1}{\beta}}\right\}}\right)^{-1}\right]\right\} \tag{7}$$

Hence, the median of *X* from the *GWGD* is simply $X_{\frac{1}{2}} = Q(1/2)$ is derived by setting *u*=0.5 in equation (7). Furthermore, it is possible to generate *GWGD* variates by setting *X=Q(u)*, where *u* is a uniform variate on the unit interval *(0,1).* The lower and the upper quartile can also be derived from (7) by setting *u*=0.25 and *u*=0.75 respectively.

3.2 Skewness and kurtosis

The classical measures of skewness and kurtosis which are well-known are given in this section. There are many heavy tailed distributions for which these measures are infinite. So, it becomes uninformative precisely when it needs to be. The Bowley's Skewness Kenney and Keeping, (1962) based on quartile is defined as;

$$B = \frac{Q\left(\frac{3}{4}\right) + Q\left(\frac{1}{4}\right) - 2Q\left(\frac{1}{4}\right)}{Q\left(\frac{3}{4}\right) - Q\left(\frac{1}{4}\right)} \tag{8}$$

And the Moor's Kurtosis Moors (1998) based on octiles is given by;

$$M = \frac{Q\left(\frac{7}{8}\right) - Q\left(\frac{5}{8}\right) + Q\left(\frac{3}{8}\right) - Q\left(\frac{1}{8}\right)}{Q\left(\frac{6}{8}\right) - Q\left(\frac{2}{8}\right)} \tag{9}$$

where $Q(.)$ represents the quantile function.

3.3 Reliability analysis of the *GWGD*.

The reliability of a system tells about its probability of survival or failure. The survival function gives the probability that the system will survive beyond a specified time. Mathematically, the survival function is given by;

$$S(X) = P(X > x) = 1 - F(x) \tag{10}$$

Therefore, the survival function for the *GWGD* can be simplified to give;

$$S_{OGEGD}(X) = \exp\left\{-\alpha\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta}\right\} \tag{11}$$

For *x>0*, where *α, β, μ* and *σ are the parameters.*

A plot for the survival function of the *GWGD* at different parameter values is as shown in figure **3** below

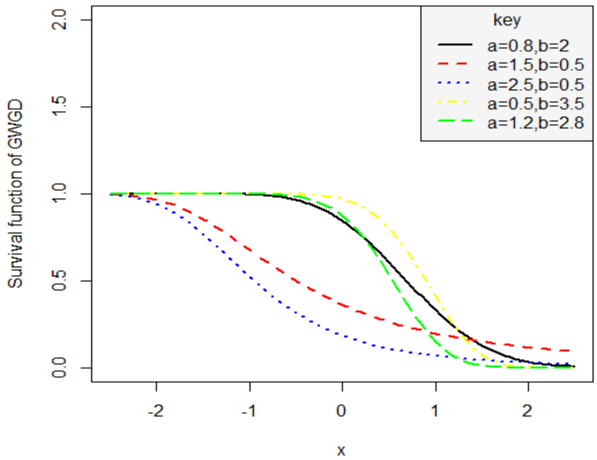

Figure. 3: Survival function of the *GWGD* for different values of α and β and some values of the parameters $a=\alpha, b=\beta, \mu=0 and \sigma=1.$

From the graph in figure **3**, we can see that the value of the survival function equals one at an initial time or early age and it decreases as *X* increases and remains constant as *X* equals zero. The implication of this behavior explains that the *GWGD* may be appropriate in modeling time or age-dependent events, where the probability of life or success decreases with time or age, that is, it gets smaller as time goes on till it reaches zero.

The hazard function which is the probability of death or failure of an individual or a system is obtained mathematically as the ratio of the *pdf*, *f(x)* to the survival function *S(X)*. It is given by;

$$h(x)=\frac{f(x)}{S(x)}=\frac{f(x)}{1-F(x)} \tag{12}$$

Hence, the expression for the hazard rate of the *GWGD* is given by

$$h_{GWGD}(x)=\frac{\alpha\beta\exp\left(-\frac{x-\mu}{\sigma}\right)\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta-1}}{\sigma\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]} \tag{13}$$

where $\alpha,\beta,\sigma>0$ and $-\infty\le\mu\le\infty$.

The following are some possible plots for the hazard rate at various values of the parameters $\alpha,\beta,\sigma>0$ and $-\infty\le\mu\le\infty$ as shown in F

figure **4** below:

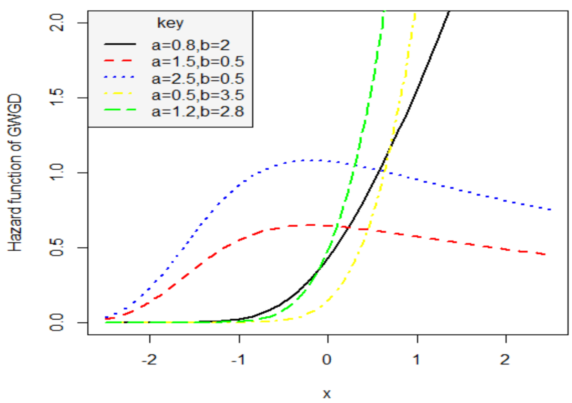

Figure. 4: Hazard function of the *GWGD* for different values of α and β and some values of the parameters $a=\alpha, b=\beta, \mu=0 and \sigma=1.$

From the graph, we can see that the value of the hazard function decreases when *X* increases. It gets higher as the value of *X* decreases. This means that the *GWGD* may be appropriate for modeling events where risk or hazard decreases as time goes on.

4 Order Statistics

In this section, we derive closed form expressions for the *pdf* of the a^{th} order statistics of the *GWGD*. Suppose $X_1, \dots, X_n$ is a random sample from the *GWGD* and let $X_{1:n} < \cdots < X_{a:n}$ denote the corresponding order statistic obtained from this sample. The *pdf*, $f_{a:n}(x)$ of the a^{th} order statistic can be obtained by

$$f_{i:n}(x) = \frac{n!}{(a-1)!(n-a)!}\sum_{k=0}^{n-a}(-1)^k\binom{n-a}{k}f(x)F(x)^{k+a-1} \tag{14}$$

Using (5) and (6), the *pdf* of the a^{th} order statistics $X_{a:n}$, can be expressed from (14) as;

$$f_{a:n}(x) = \frac{n!}{(a-1)!(n-a)!}\sum_{k=0}^{n-a}(-1)^k\binom{n-a}{k}\left[\alpha\beta\frac{\exp\left(-\frac{x-\mu}{\sigma}\right)\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}}{\sigma\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]}\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta-1}\exp\left\{-\alpha\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta}\right\}\right]$$
$$*\left[1-\exp\left\{-\alpha\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta}\right\}\right]^{a+k-1} \tag{15}$$

Hence, the *pdf* of the minimum order statistic $X_{(1)}$ and maximum order statistic $X_{(n)}$ of the *GWGD* are given by;

$$f_{1:n}(x) = n\sum_{k=0}^{n-1}(-1)^k\binom{n-1}{k}\left[\alpha\beta\frac{\exp\left(-\frac{x-\mu}{\sigma}\right)\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}}{\sigma\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]}\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta-1}\exp\left\{-\alpha\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta}\right\}\right]$$
$$*\left[1-\exp\left\{-\alpha\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta}\right\}\right]^{k} \tag{16}$$

and

$$f_{n:n}(x) = n\left[\alpha\beta\frac{\exp\left(-\frac{x-\mu}{\sigma}\right)\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}}{\sigma\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]}\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta-1}\exp\left\{-\alpha\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta}\right\}\right]$$
$$\left[1-\exp\left\{-\alpha\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta}\right\}\right]^{n-1} \tag{17}$$

respectively.

5 Estimation of Parameters of the *GWGD*.

In this section, the estimation of the parameters of the *GWGD* is done by using the method of maximum likelihood estimation. Let $X_1, \dots, X_n$ be a sample of size 'n' independently and identically distributed random variables from the *GWGD* with unknown parameters *α, β, μ,* and *σ* defined previously. The *pdf* of the *GWGD* is given as

$$f(x)=\alpha\beta\frac{\exp\left(-\frac{x-\mu}{\sigma}\right)\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}}{\sigma\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]}\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta-1}\exp\left\{-\alpha\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta}\right\}$$

The likelihood function is given by;

$$L(X_1,X_2,.....,X_n/\alpha,\beta,\mu,\sigma)=\frac{(\alpha\beta)^n}{\sigma^n\sum_{i=1}^{n}\left[1-\exp\left\{-\exp\left(-\frac{x_i-\mu}{\sigma}\right)\right\}\right]}\exp\sum_{i=1}^{n}\left(-\frac{x_i-\mu}{\sigma}\right)\exp\sum_{i=1}^{n}\left\{-\exp\left(-\frac{x_i-\mu}{\sigma}\right)\right\}$$

$$*\sum_{i=1}^{n}\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta-1}\exp\sum_{i=1}^{n}\left\{-\alpha\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta}\right\} \quad (18)$$

Let the log-likelihood function, $l=\log L(X_1,X_2,.....,X_n/\alpha,\beta,\mu,\sigma)$, therefore

$$l=n\log\alpha+n\log\beta-n\log\sigma-\sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma}\right)-\sum_{i=1}^{n}\left\{\exp\left(-\frac{x_i-\mu}{\sigma}\right)\right\}-\sum_{i=1}^{n}\log\left(1-\exp\left\{-\exp\left(-\frac{x_i-\mu}{\sigma}\right)\right\}\right)$$

$$-(\beta-1)\sum_{i=1}^{n}\log\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)-\alpha\sum_{i-1}^{n}\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta} \quad (19)$$

Differentiating l partially with respect to *α, β, μ* and *σ* respectively gives;

$$\frac{\partial l}{\partial\alpha}=\frac{n}{\alpha}-\sum_{i=1}^{n}\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta} \quad (20)$$

$$\frac{\partial l}{\partial\beta}=\frac{n}{\beta}+\sum_{i=1}^{n}\log\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta}$$

$$-\alpha\sum_{i=1}^{n}\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta}\log\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right) \quad (21)$$

$$\frac{\partial l}{\partial\mu}=\frac{n}{\sigma}-\frac{1}{\sigma}\sum_{i=1}^{n}\exp\left(-\frac{x_i-\mu}{\sigma}\right)-\frac{1}{\sigma}\sum_{i=1}^{n}\left\{\frac{\exp\left(-\frac{x_i-\mu}{\sigma}\right)\exp\left\{-\exp\left(-\frac{x_i-\mu}{\sigma}\right)\right\}}{\left[1-\exp\left\{-\exp\left(-\frac{x_i-\mu}{\sigma}\right)\right\}\right]}\right\}$$

$$+\frac{(\beta-1)}{\sigma}\sum_{i=1}^{n}\left\{\frac{\exp\left(-\frac{x_i-\mu}{\sigma}\right)\exp\left\{-\exp\left(-\frac{x_i-\mu}{\sigma}\right)\right\}}{\left(\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)\left[1-\exp\left\{-\exp\left(-\frac{x_i-\mu}{\sigma}\right)\right\}\right]\right)}\right\}$$

$$+\frac{\alpha\beta}{\sigma}\sum_{i=1}^{n}\left\{\frac{\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta-1}\exp\left(-\frac{x_i-\mu}{\sigma}\right)\exp\left\{-\exp\left(-\frac{x_i-\mu}{\sigma}\right)\right\}}{\left[1-\exp\left\{-\exp\left(-\frac{x_i-\mu}{\sigma}\right)\right\}\right]}\right\} \quad (22)$$

$$\frac{\partial l}{\partial\sigma}=-\frac{n}{\sigma}-\frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu)\exp\left(-\frac{x_i-\mu}{\sigma}\right)-\frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu)\exp\left(-\frac{x_i-\mu}{\sigma}\right)-\frac{1}{\sigma^2}\sum_{i=1}^{n}\frac{(x_i-\mu)\exp\left(-\frac{x_i-\mu}{\sigma}\right)\exp\left\{-\exp\left(-\frac{x_i-\mu}{\sigma}\right)\right\}}{\left[1-\exp\left\{-\exp\left(-\frac{x_i-\mu}{\sigma}\right)\right\}\right]}$$

$$-\frac{(\beta-1)}{\sigma^2}\sum_{i=1}^{n}\left[\frac{(x_i-\mu)\exp\left(-\frac{x_i-\mu}{\sigma}\right)\exp\left\{-\exp\left(-\frac{x_i-\mu}{\sigma}\right)\right\}}{\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)\left[1-\exp\left\{-\exp\left(-\frac{x_i-\mu}{\sigma}\right)\right\}\right]}\right]$$

$$+\frac{\alpha\beta}{\sigma}\sum_{i=1}^{n}\left\{\frac{(x_i-\mu)\exp\left(-\frac{x_i-\mu}{\sigma}\right)\exp\left\{-\exp\left(-\frac{x_i-\mu}{\sigma}\right)\right\}\left(-\log\left[1-\exp\left\{-\exp\left(-\frac{x-\mu}{\sigma}\right)\right\}\right]\right)^{\beta-1}}{\left[1-\exp\left\{-\exp\left(-\frac{x_i-\mu}{\sigma}\right)\right\}\right]}\right\} \quad (23)$$

The solution of the non-linear system of equations of $\frac{dl}{d\alpha}=0,\ \frac{\partial l}{\partial\beta}=0,\ \frac{\partial l}{\partial\mu}=0$ and $\frac{\partial l}{\partial\sigma}=0$ will give us the maximum likelihood estimates of parameters α,β,μ and σ. However, the solution cannot be gotten analytically except numerically with the aid of suitable statistical software like R, SAS, e.t.c when data sets are available.

6 Conclusion

This article studied a new four-parameter model named the *GWGD*. Some mathematical and statistical properties of the distribution have been studied appropriately. We have derived explicit expressions for its survival, hazard and quantile functions. Some plots of the distribution showed that the *GWGD* distribution has various shapes such as symmetrical, left-skewed and right-skewed. This means that the *GWGD* can be useful in modeling data sets with various shapes. The significance of the plots for the survival function is that the *GWGD* can be appropriate in modeling time or age-dependent events, where survival decreases with time while failure increases with time or age. We also obtained the *pdf* of its minimum and maximum order statistics. We estimated the model parameters using the method of maximum likelihood estimation.

References

[1]. Alexander, C., Cordeiro, G. M., Ortega, E. M. M. and Sarabia, J. M. (2012). Generalized beta-generated distributions. *Computational Statistics and Data Analysis,* 56:1880–1897

[2]. Alizadeh, M., Cordeiro, G. M., de Brito, E., and Demetrio, C. G. B., (2015). The beta Marshall-Olkin family of distributions. *Journal of Statistical Distributions and Applications,* 2(4):1–18

[3]. Alzaatreh, A., Famoye F. and Lee, C. (2013). Weibull–Pareto distribution and its applications. *Communication in Statistics: Theory and Methods*, 42(9):1673–1691

[4]. Alzaghal, A., Lee, C.and Famoye, F. (2013). Exponentiated T-X family of distributions with some applications. *International Journal of Probability and Statistics,* 2:31–49

[5]. Amini, M., MirMostafaee, S. M. T. K. and Ahmadi, J. (2012). Log-gamma-generated families of distributions. *Statistics.* doi:10.1008/02331888.2012.748775

[6]. Andrade, T., Rodrigues, H., Bourguignon, M and Cordeiro, G. (2015). The Exponentiated Generalized Gumbel Distribution. *Revista Colombiana de Estadística, 38(1): 123 - 143DOI: http://dx.doi.org/10.15446/rce.v38n1.48806*

[7]. Bourguignon, M., Silva R. B. and Cordeiro, G. M. (2014). The Weibull-G Family of Probability Distributions. *Journal of Data Science,* 12: 53-68.

[8]. Cordeiro, G. M., and de Castro, M. (2011). A New family of generalized distributions. *Journal of Statistical Computation and Simulation*, 81:883–898

[9]. Cordeiro, G. M., Nadarajah, S. and Ortega, E. M. M. (2012). The Kumaraswamy Gumbel distribution. *Statistical Methods and Applications,* 2: 139–168.

[10]. Cordeiro, G. M., Ortega, E. M. M., and da Cunha, D. C. C. (2013). The exponentiated generalized class of distribution. *Journal of Data Science,* 11:1–27

[11]. Eugene, N., Lee, C., and Famoye, F. (2002). Beta-normal distribution and its applications. *Communication in Statistics: Theory and Methods*, 31:497–512

[12]. Kenney, J. F. & Keeping, E. S. (1962). Mathematics of Statistics, 3 edn, Chapman & Hall Ltd, New Jersey.

[13]. Kotz, S. & Nadarajah, S. (2000). Extreme Value Distributions: Theory and Applications. *Imperial College Press, London*.

[14]. Lawless, J. F. (2003). Statistical models and methods for lifetime data. *2nd Edn., John Willey, New jersey.*

[15]. Moors, J. J. (1988). A quantile alternative for kurtosis. *Journal of the Royal Statistical Society: Series* D 37, 25–32.

[16]. Nadarajah, S. & Kotz, S. (2004). The beta Gumbel distribution. *Mathematical Problems in Engineering,* 4: 323–332.

[17]. Nadarajah, S. (2006). The exponentiated Gumbel distribution with climate application. *Environmetrics*, 17:13–23.

[18]. Risti´c, M, M. and Balakrishnan, N. (2012). The gamma-exponentiated exponential distribution. *Journal of Statistical Computation and Simulation*, 82:1191–1206

[19]. Shaw, W. T. and Buckley, I. R. (2007). The alchemy of probability distributions: beyond Gram-Charlier expansions and a skew-kurtotic-normal distribution from a rank transmutation map. *Research report*

[20]. Tahir, M. H., Cordeiro, G. M., Alzaatreh, A., Mansoor, M. and Zubair, M. (2015). The logistic-X family of distributions and its applications. Commun Stat-Theory Methods (forthcoming)

[21]. Torabi, H. and Montazari, N. H. (2012). The gamma-uniform distribution and its application. *Kybernetika* 48:16–30

[22]. Torabi, H. and Montazari, N. H. (2014). The logistic-uniform distribution and its application. *Commun Stat-Simul Comput* 43: 2551–2569

[23]. Zografos, K. and Balakrishnan, N. (2009). On families of beta- and generalized gamma-generated distributions and associated inference. *Statistical Methodology*, 6, 344-362.