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RESEARCH ARTICLE

On Bipolar Vague Baire Spaces

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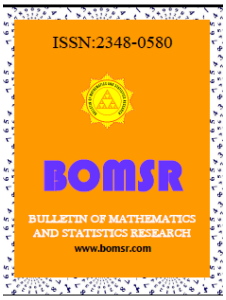

ABSTRACT

The aim of this is to explore the notion of vague sets in bipolar space. Also we define a new class of space called bipolar vague Baire space and characterized some of its properties.

Keywords: Bipolar vague first category, Bipolar vague second category, Bipolar vague Baire space, Bipolar vague D-Baire space.

1. INTRODUCTION

The concepts of Fuzzy sets and Fuzzy set operations were first introduced by L.A. Zadeh in his classical paper [12] in 1965. The theory of Fuzzy topological space was introduced and developed by C.L. Chang [3] and since then various notions in classical topology have been extended to fuzzy topological space. The concepts of Baire spaces have been studied extensively in classical topology in [5,7,9]. The concept of Baire spaces in Fuzzy setting was introduced and studied by the authors in [10].Vague set theory is actually an extension of fuzzy set theory. The basic concepts of vague set theory and its extensions defined by W.L Gau and Buehrer [4]. The theory of vague sets started with of interpreting the real life problems in a better way than the fuzzy sets do. Several authors work under the vague sets in different fields [8,11]. Bipolar valued fuzzy sets, which was introduced by Lee [6] is an extension of fuzzy sets whose membership degree range is enlarged from the interval [0,1] to [-1,1].

The objective of this paper is to introduce the concept of bipolar vague Baire space. Also we classify some of its properties with suitable examples. And study the conditions under which a bipolar vague topological space become a bipolar vague Baire space

2. PRELIMINARIES

Definition 2.1[6]: Let X be the universe. Then a bipolar valued fuzzy sets, A on X is defined by positive membership function μ_A^+, i.e., $\mu_A^+ : X \to [0,1]$, and a negative membership function μ_A^-, i.e., $\mu_A^- : X \to [-1,0]$. For the sake of simplicity, we shall use the symbol $A = \{\langle x, \mu_A^+(x), \mu_A^-(x)\rangle : x \in X\}$.

Definition 2.2[4]: A vague set A in the universe of discourse U is a pair (t_A, f_A) where $t_A : U \to [0,1]$, $f_A : U \to [0,1]$ are the mapping such that $t_A + f_A \le 1$ for all $u \in U$ .The function t_A and f_A are called true membership function and false membership function respectively. The interval $[t_A, 1 - f_A]$ is called the vague value of u in A, and denoted by $v_A(u)$, i.e $v_A(u) = [t_A(u), 1 - f(u)]$.

Definition 2.3[2]: Let X be the universe of discourse. A bipolar-valued vague set A in X is an object having the form $A = \{\langle x, [t_A^+(x), 1 - f_A^+(x)], [-1 - f_A^-(x), t_A^-(x)]\rangle : x \in X\}$ where $[t_A^+, 1 - f_A^+] : X \to [0,1]$ and $[-1 - f_A^-, t_A^-] : X \to [-1,0]$ are the mapping such that $t_A^+ + f_A^+ \le 1$ and $-1 \le t_A^- + f_A^-$. The positive membership degree $[t_A^+(x), 1 - f_A^+(x)]$ denotes the satisfaction region of an element x to the property corresponding to a bipolar-valued set A and the negative membership degree $[-1 - f_A^-(x), t_A^-(x)]$ denotes the satisfaction region of x to some implicit counter property of A. For a sake of simplicity, we shall use the notion of bipolar vague set $v_A^+ = [t_A^+, 1 - f_A^+]$ and $v_A^- = [-1 - f_A^-, t_A^-]$.

Definition 2.4[2]: A bipolar vague set $A = [v_A^+, v_A^-]$ of a set U with $v_A^+ = 0$ implies that $t_A^+ = 0, 1 - f_A^+ = 0$ and $v_A^- = 0$ implies that $t_A^- = 0, -1 - f_A^- = 0$ for all $x \in U$ is called zero bipolar vague set and it is denoted by 0.

Definition 2.5[2]: A bipolar vague set $A = [v_A^+, v_A^-]$ of a set U with $v_A^+ = 1$ implies that $t_A^+ = 1, 1 - f_A^+ = 1$ and $v_A^- = -1$ implies that $t_A^- = -1, -1 - f_A^- = -1$ for all $x \in U$ is called unit bipolar vague set and it is denoted by 1.

Definition 2.6[2]: Let $A = < x, [t_A^+, 1 - f_A^+][-1 - f_A^-, t_A^-] >$ and $B = < x, [t_B^+, 1 - f_B^+][-1 - f_B^-, t_B^-] >$ be two bipolar vague sets then their union, intersection and complement are defined as follows:

1. $A \cup B = \{(x, [t_{A\cup B}^+(x), 1 - f_{A\cup B}^+(x)], [-1 - f_{A\cup B}^-(x), t_{A\cup B}^-(x)]) / x \in X\}$ where

$$t_{A\cup B}^+(x) = \max\{t_A^+(x), t_B^+(x)\}, \quad t_{A\cup B}^-(x) = \min\{t_A^-(x), t_B^-(x)\} \text{ and}$$

$$1 - f_{A\cup B}^+(x) = \max\{1 - f_A^+(x), 1 - f_B^+(x)\},$$

$$-1 - f_{A\cup B}^-(x) = \min\{-1 - f_A^-(x), -1 - f_B^-(x)\}.$$

2. $A \cap B = \{(x, [t_{A\cap B}^+(x), 1 - f_{A\cap B}^+(x)], [-1 - f_{A\cap B}^-(x), t_{A\cap B}^-(x)]) / x \in X\}$ where

$$t_{A\cap B}^+(x) = \min\{t_A^+(x), t_B^+(x)\}, \quad t_{A\cap B}^-(x) = \max\{t_A^-(x), t_B^-(x)\} \text{ and}$$

$$1-f^{+}_{A\cap B}(x)=\min\{1-f^{+}_{A}(x),1-f^{+}_{B}(x)\},$$

$$-1-f^{-}_{A\cap B}(x)=\max\{-1-f^{-}_{A}(x),-1-f^{-}_{B}(x)\}.$$

3. $\overline{A}=\{(x,[f^{+}_{A}(x),1-t^{+}_{A}(x)],[-1-t^{-}_{A}(x),f^{-}_{A}(x)])/x\in X\}$ for all $x\in X$.

Definition 2.7[2]: Let (X,BV_{τ}) be a bipolar vague topological space and $A=\langle x,[t^{+}_{A},1-f^{+}_{A}][-1-f^{-}_{A},t^{-}_{A}]\rangle$ be a BVS in X. Then the bipolar vague interior and bipolar vague closure of A are defined by,

$$Bvcl(A)=\cap\{K:K\ is\ a\ BVCS\ in\ X\ and\ A\subseteq K\},$$

$$Bv\operatorname{int}(A)=\cup\{G:G\ is\ a\ BVOS\ in\ X\ and\ G\subseteq A\}$$

Note that $bvcl(A)$ is a BVCS and $bv\operatorname{int}(A)$ is a BVOS in X. Further,

1. A is a BVCSin X iff $Bvcl(A)=A$,
2. A is a BVOS in X iff $Bv\operatorname{int}(A)=A$

Definition 2.8[2]: Let (X,BV_{τ}) and (Y,BV_{σ}) be two bipolar vague topological spaces and $\psi:X\to Y$ be a function. Then ψ is said to be bipolar vague continuous iff the preimage of each bipolar vague open set in Y is a bipolar vague open set in X.

3. Bipolar Vague Baire Space

Definition 3.1: A bipolar vague set A in a bipolar vague topological space (X,τ) is called bipolar vague dense if there exists no bipolar vague closed B in (X,τ) such that $A\subset B\subset 1$.

Definition 3.2: A bipolar vague set A in a bipolar vague topological space (X,τ) is called bipolar vague nowhere dense set if there exists no bipolar vague open set B in (X,τ) such that $B\subset BVcl(A)$. i.e, $BV\operatorname{int}(BVcl(A))=0$.

Example 3.3: Let $X=\{a,b,c\}$. Define the bipolar vague sets A and B as follows:

$$A=\left\langle x,\left(\frac{a}{[0.4,0.8][-0.4,-0.2]},\frac{b}{[0.6,0.8][-0.4,-0.2]},\frac{c}{[0.2,0.4][-0.7,-0.5]}\right)\right\rangle,$$

$$B=\left\langle x,\left(\frac{a}{[0.6,0.8][-0.5,-0.3]},\frac{b}{[0.6,0.9][-0.7,-0.3]},\frac{c}{[0.3,0.5][-0.8,-0.6]}\right)\right\rangle.$$ Then the family $\tau=\{0,1,A\}$ is a bipolar vague topology on X. Now $BV\operatorname{int}(BVcl(\overline{A}))=0$, $BV\operatorname{int}(BVcl(\overline{B}))=0$ but $BV\operatorname{int}(BVcl(B))=1\neq 0$. Hence $\overline{A},\overline{B}$ are bipolar vague nowhere dense sets in (X,τ). Where B is not a bipolar vague nowhere dense in (X,τ).

Theorem 3.4: Let A be a bipolar vague set. If A is a bipolar vague closed set in (X,τ) with $BV\operatorname{int}(A)=0$, then A is a bipolar vague nowhere dense set in (X,τ).

Proof: Let A be a bipolar vague closed set in (X,τ). Then $BVcl(A)=A$. Now $BV\,\text{int}(BVcl(A))=BV\,\text{int}(A)=0$ and hence A is a vague nowhere dense set in (X,τ).

Theorem 3.5: If A is a bipolar vague nowhere dense set in bipolar vague topological space (X,τ), then $BVcl(A)$ is also a bipolar vague nowhere dense set in (X,τ).

Proof: Let A be a bipolar vague nowhere dense set in a (X,τ). Then, $BV\,\text{int}(BVcl(A))=0$.Now $BVcl(BVcl(A))=Bvcl(A)$. Hence $BV\,\text{int}(BVcl(BVcl(A)))=BV\,\text{int}(BVcl(A))=0$. Therefore $BVcl(A)$ is also a bipolar vague nowhere dense set in (X,τ).

Theorem 3.6: If A is a bipolar vague dense and bipolar vague open set in a bipolar vague topological space (X,τ) and if $B\subseteq\overline{A}$, then B is a bipolar vague nowhere dense set in (X,τ).

Proof: Let A be a bipolar vague dense and bipolar vague open set in (X,τ). Then we have $BVcl(A)=1$ and $BV\,\text{int}(A)=A$. Now $B\subseteq\overline{A}$, implies that $BVcl(B)\subseteq BVcl(\overline{A})$. Then $BVcl(B)\subseteq\overline{BV\,\text{int}(A)}=\overline{A}$. Hence $BVcl(B)\subseteq\overline{A}$, which implies that $BV\,\text{int}(BVcl(B))\subseteq BV\,\text{int}(\overline{A})=\overline{BVcl(A)}=\overline{1}=0$. That is, $BV\,\text{int}(BVcl(B))=0$. Hence B is bipolar vague nowhere dense set in (X,τ).

Theorem 3.7: If A is a bipolar vague nowhere dense set in a bipolar vague topological space (X,τ) then $\overline{A}$ is a bipolar vague dense set in (X,τ).

Proof: Let A be a bipolar vague nowhere dense set in (X,τ). Then, $BV\,\text{int}(BVcl(A))=0$. Now $A\subseteq BVcl(A)$ implies that $BV\,\text{int}(A)\subseteq BV\,\text{int}(BVcl(A))=0$. Then $BV\,\text{int}(A)=0$ and $BVcl(\overline{A})=\overline{BV\,\text{int}(A)}=1$ and hence then $\overline{A}$ is a bipolar vague dense set in (X,τ).

Theorem 3.8: If the bipolar vague sets A and B are bipolar vague nowhere dense sets in a bipolar vague topological space (X,τ), then $A\cap B$ is a bipolar vague nowhere dense set in (X,τ).

Proof: Let the bipolar vague sets A and B be a bipolar vague nowhere dense sets in (X,τ). Now $BV\,\text{int}(BVcl(A\cap B))\le BV\,\text{int}[BVcl(A)\cap BVcl(B)]\le BV\,\text{int}(BVcl(A))\cap BV\,\text{int}(BVcl(B))\le 0\cap 0$ [since $BV\,\text{int}(BVcl(A))=0$ and $BV\,\text{int}(BVcl(B))=0$]. That is, $BV\,\text{int}(BVcl(A\cap B))=0$. Hence, $A\cap B$ is a bipolar vague nowhere dense set in (X,τ).

Theorem 3.9: Let A be a bipolar vague dense set in a bipolar vague topological space (X,τ). If B is a bipolar vague set in (X,τ), then B is a bipolar vague nowhere dense set in (X,τ) if and only if $A\cap B$ is a bipolar vague nowhere dense set in (X,τ).

Proof: Let B be a bipolar vague nowhere dense set in (X,τ).Then $BV\,\text{int}(BVcl(B))=0$, Now $BV\,\text{int}(BVcl(A\cap B))\subseteq BV\,\text{int}(BVcl(A)\cap BVcl(B))=BV\,\text{int}(BVcl(A))\cap BV\,\text{int}(BVcl(B))=$ $BV\,\text{int}(BVcl(A))\cap 0=0$. That is $BV\,\text{int}(BVcl(A\cap B))=0$. Therefore $A\cap B$ is a bipolar vague nowhere dense set in (X,τ).

Conversely, let $A \cap B$ be a bipolar vague nowhere dense set in (X,τ). Then $BV\,\mathrm{int}(BVcl(A \cap B) = 0$ implies that $BV\,\mathrm{int}(BVcl(A) \cap BVcl(B)) = 0$. Since A is a bipolar vague dense set in (X,τ) $BVcl(A) = 1$. Then $BV\,\mathrm{int}(1 \cap BVcl(B)) = 0$ and therefore $BV\,\mathrm{int}(BVcl(B)) = 0$ which means that B is a bipolar vague nowhere dense set in (X,τ).

Definition 3.10: Let (X,τ) be a bipolar vague topological space. A vague set A in (X,τ) is called bipolar vague first category if $A = \bigcup_{i=1}^{\infty} A_i$, where A_i's are bipolar vague nowhere dense sets in (X,τ).

The complement of a bipolar vague first category sets in (X,τ) is a bipolar vague residual set in (X,τ).

Definition 3.11: Let (X,τ) be bipolar vague topological space. Then (X,τ) is said to bipolar vague Baire space if $BV\,\mathrm{int}(\bigcup_{i=1}^{\infty} A_i) = 0$, where A_i's are bipolar vague nowhere dense sets in (X,τ).

Example 3.12: Let $X = \{a,b,c\}$. Define the bipolar vague sets A, B and C as follows:

$$A = \left\langle x, \left(\frac{a}{[0.6,0.8][-0.6,-0.3]}, \frac{b}{[0.4,0.6][-0.4,-0.2]}, \frac{c}{[0.3,0.5][-0.6,-0.4]} \right) \right\rangle,$$

$$B = \left\langle x, \left(\frac{a}{[0.7,0.9][-0.7,-0.4]}, \frac{b}{[0.7,0.8][-0.6,-0.4]}, \frac{c}{[0.5,0.6][-0.7,-0.6]} \right) \right\rangle.$$

$$C = \left\langle x, \left(\frac{a}{[0.6,0.8][-0.6,-0.3]}, \frac{b}{[0.4,0.7][-0.4,-0.3]}, \frac{c}{[0.7,0.8][-0.8,-0.6]} \right) \right\rangle$$

Then the family $\tau = \{0,1,A\}$ is a bipolar vague topology on X. Thus (X,τ) is a bipolar vague topological spaces. Now $\overline{A}$, $\overline{B}$ and $\overline{C}$ are bipolar vague nowhere dense sets in (X,τ). Also $BV\,\mathrm{int}(\overline{A},\overline{B},\overline{C}) = 0$. Hence (X,τ) is a bipolar vague Baire space.

Theorem 3.13: Let A be a bipolar vague first category set in (X,τ), then $\overline{A} = \bigcap_{i=1}^{\infty} B_i$ where $BVcl(B_i) = 1$.

Proof: Let A be a bipolar vague first category set in (X,τ). Then $A = \bigcup_{i=1}^{\infty} A_i$, where A_i's are bipolar vague nowhere dense sets in (X,τ) . Now $\overline{A} = \overline{\bigcup_{i=1}^{\infty} A_i} = \bigcap_{i=1}^{\infty}(\overline{A_i})$. Since A_i is a bipolar vague nowhere dense set in (X,τ). then by Theorem 3.7, we have $\overline{A_i}$ is a bipolar vague dense set in (X,τ). Let us put $B_i = \overline{A_i}$. Then $\overline{A} = \bigcap_{i=1}^{\infty} B_i$ where $BVcl(B_i) = 1$.

Theorem 3.14: If $BV\,\mathrm{int}(\bigcup_{i=1}^{\infty} A_i) = 0$, where $BV\,\mathrm{int}(A_i) = 0$ and A_i's are bipolar vague closed sets in a bipolar vague topological space (X,τ), then is (X,τ) is a bipolar vague Baire space.

Proof: Let A_i be a bipolar vague closed sets in (X,τ). Since $BV\operatorname{int}(A_i)=0$ by Theorem 3.4, A_i's are bipolar vague nowhere dense sets in (X,τ). Therefore we have $BV\operatorname{int}(\bigcup_{i=1}^{\infty}A_i)=0$ where A_i's are bipolar vague nowhere dense sets in (X,τ). Hence (X,τ) is a bipolar vague Baire space.

Theorem 3.15: If $BVcl(\bigcap_{i=1}^{\infty}A_i)=1$ where A_i's are bipolar vague dense and bipolar vague open sets in (X,τ), then (X,τ) is a bipolar vague Baire space.

Proof: Now $BVcl(\bigcap_{i=1}^{\infty}A_i)=1$ implies that $\overline{BVcl(\bigcap_{i=1}^{\infty}A_i)}=0$. Then we have $BV\operatorname{int}(\overline{\bigcap_{i=1}^{\infty}A_i})=0$. Which implies that $BV\operatorname{int}(\bigcup_{i=1}^{\infty}\overline{A_i})=0$. Let $B_i=\overline{A_i}$. Then $BV\operatorname{int}(\bigcup_{i=1}^{\infty}B_i)=0$. Now $\overline{A_i}$ is a bipolar vague closed set in (X,τ) and hence B_i is a bipolar vague closed and
$BV\operatorname{int}(B_i)=BV\operatorname{int}(\overline{A_i})=\overline{BVcl(A_i)}=0$. Hence by Theorem 3.14, B_i is a bipolar vague nowhere dense sets, implies that (X,τ) is a bipolar vague Baire space.

Theorem 3.15: Let (X,τ) be a bipolar vague topological space. Then the following are equivalent
(i) (X,τ) is a bipolar vague Baire space.
(ii) $BV\operatorname{int}(A)=0$, for every bipolar vague first category set A in (X,τ).
(iii) $BVcl(B)=1$, for every bipolar vague residual set B in (X,τ).

Proof:(i)$\Rightarrow$(ii) Let A be a bipolar first category set in (X,τ). Then $A=\bigcup_{i=1}^{\infty}A_i$, where A_i's are bipolar vague nowhere dense sets in (X,τ). Now $BV\operatorname{int}(A)=BV\operatorname{int}(\bigcup_{i=1}^{\infty}A_i)=0$. Since (X,τ) is a bipolar vague Baire space. Therefore $BV\operatorname{int}(A)=0$.
(ii)$\Rightarrow$(iii) Let B be a bipolar vague residual set in (X,τ). Then $\overline{B}$ is a bipolar vague first category set in (X,τ). By hypothesis $BV\operatorname{int}(\overline{B})=0$ which implies that $\overline{BVcl(B)}=0$. Hence $BVcl(B)=1$.
(iii)$\Rightarrow$(i) Let A be a bipolar vague first category set in (X,τ). Then $A=\bigcup_{i=1}^{\infty}A_i$, where A_i's are bipolar vague nowhere dense sets in (X,τ). Now A is a bipolar vague first category set implies that $\overline{A}$ is a bipolar vague residual set in (X,τ). By hypothesis, we have $BVcl(\overline{A})=1$ which implies that $\overline{BV\operatorname{int}(A)=1}$. Hence $BV\operatorname{int}(A)=0$. That is, $BV\operatorname{int}(\bigcup_{i=1}^{\infty}A_i)=0$, where A_i's are bipolar vague nowhere dense sets in (X,τ). Hence (X,τ) is a bipolar vague Baire space.

Theorem 3.16: If A is a bipolar vague first category set in a bipolar vague topological space (X,τ) such that $BV\operatorname{int}(BVcl(A))=0$, then (X,τ) is a bipolar vague Baire space.

Proof: Let A be a bipolar vague first category set in a bipolar vague topological space (X,τ). Then $A=\bigcup_{i=1}^{\infty}A_i$, where A_i's are bipolar vague nowhere dense sets in (X,τ). Now $BV\operatorname{int}(BVcl(A))=0$ and $BV\operatorname{int}(A)\subseteq BV\operatorname{int}(BVcl(A))$, implies that $BV\operatorname{int}(A)=0$. Hence $BV\operatorname{int}(\bigcup_{i=1}^{\infty}A_i)=0$, where

A_i's are bipolar vague nowhere dense sets in (X,τ). Therefore (X,τ) is a bipolar vague Baire space.

4. Bipolar vague D-Baire Space

Definition 4.1: A bipolar vague topological space (X,τ) is called a bipolar vague D-Baire space if every bipolar vague first category set in (X,τ) is a bipolar vague nowhere dense set in (X,τ). That is, (X,τ) is a bipolar vague D-Baire space if $BV\operatorname{int}(BVcl(A))=0$ for each bipolar vague first category set A in (X,τ).

Theorem 4.2: If (X,τ) is a bipolar vague D-Baire space, then (X,τ) is a bipolar vague Baire space.

Proof: Let A be a bipolar vague first category set in a bipolar vague D-Baire space (X,τ). Then $A=\bigcup_{i=1}^{\infty}A_i$, where A_i's are bipolar vague nowhere dense sets in (X,τ), and A is a bipolar vague nowhere dense set in (X,τ). Then we have $BV\operatorname{int}(BVcl(A)=0$ and $BV\operatorname{int}(A)\subseteq BV\operatorname{int}(BVcl(A))$, implies that $BV\operatorname{int}(A)=0$. Hence $BV\operatorname{int}(\bigcup_{i=1}^{\infty}A_i)=0$, where A_i's are bipolar vague nowhere dense sets in (X,τ) . Therefore (X,τ) is a bipolar vague Baire space.

Theorem 4.3: If $BVcl(BV\operatorname{int}(A))=1$ for every fuzzy residual set A in a bipolar vague topological space, then (X,τ) is a D-Baire space.

Proof: Let A be a bipolar vague residual set in (X,τ). Then $\overline{A}$ is a bipolar vague first category set in (X,τ). Now, $BVcl(BV\operatorname{int}(A))=1$ implies that $\overline{BVcl(BV\operatorname{int}(A)}=0$. Then we have $BV\operatorname{int}(BVcl(\overline{A}))=0$. Hence for a bipolar vague first category set $\overline{A}$ in (X,τ) , we have $BV\operatorname{int}(BVcl(\overline{A}))=0$. Therefore (X,τ) is a D-Baire space.

Theorem 4.4: Let the function $f:(X,\tau)\to(Y,S)$ from a bipolar vague topological space (X,τ) into another topological space (Y,S) be bipolar vague continuous, bipolar vague open, 1-1 and onto function, then for any bipolar vague set A in (X,τ) , A is a bipolar vague nowhere dense set in (X,τ) if and only if $f(A)$ is a bipolar vague nowhere dense set in (Y,S).

Proof: Let A be a bipolar vague nowhere dense set in (X,τ). Then, $BV\operatorname{int}(BVcl(A))=0$. Since f is a bipolar vague continuous, bipolar vague open, 1-1, and onto function, we have $f(BV\operatorname{int}(BVcl(A)))=BV\operatorname{int}(BVcl(f(A)))$. Then, $BV\operatorname{int}(BVcl(f(A)))=f(0)=0$.Hence $f(A)$ is a bipolar vague nowhere dense set in (Y,S).

Conversely, let $f(A)$ be a bipolar vague nowhere dense set in (Y,S). Then, $BV\operatorname{int}(BVcl(f(A)))=0$. Hence $f(BV\operatorname{int}(BVcl(A)))=B\operatorname{int}(BVcl(f(A)))$ implies that $f(BV\operatorname{int}(BVcl(A))=0$. Therefore $ff^{-1}(BV\operatorname{int}(BVcl(A)))=f^{-1}(0)=0$. Since f is 1-1, $BV\operatorname{int}(BVcl(A))=0$. Hence A is a bipolar vague nowhere dense set in (X,τ).

Theorem4.5: If the function $f:(X,\tau)\to(Y,S)$ from a bipolar vague topological space (X,τ) into another topological space (Y,S) be bipolar vague continuous, bipolar vague open, 1-1 and onto function, then a bipolar vague set A in (Y,S) is a bipolar vague nowhere dense set if and only if $f^{-1}(A)$ is a bipolar vague nowhere dense set in (X,τ).

Proof: The proof follows from the proof of Theorem 4.4.

Theorem4.6: If the function $f:(X,\tau)\to(Y,S)$ from a bipolar vague topological space (X,τ) into another topological space (Y,S) be bipolar vague continuous, bipolar vague open, 1-1 and onto function, then a bipolar vague set A in (X,τ), A is a bipolar vague first category set if and only if $f(A)$ is a bipolar vague first category set in (Y,S).

Proof: Let A be a bipolar vague first category set in a bipolar vague topological space (X,τ). Then $A=\bigcup_{i=1}^{\infty}A_i$, where A_i's are bipolar vague nowhere dense sets in (X,τ). Then $f(A)=f(\bigcup_{i=1}^{\infty}A_i)=\bigcup_{i=1}^{\infty}f(A_i)$. Since A_i is a bipolar vague nowhere dense set in (X,τ), by Theorem 4.4 $f(A_i)$ is a bipolar vague nowhere dense set in (X,τ). Hence $f(A)$ is a bipolar vague first category set in (Y,S).

Conversely, let $f(A)$ be a bipolar vague first category set in (Y,S). Then we have $f(A)=\bigcup_{i=1}^{\infty}\delta_i$ where $\delta_i{}'s$ are bipolar vague nowhere dense sets in (Y,S). Now $f^{-1}f(A)=f^{-1}(\bigcup_{i=1}^{\infty}\delta_i)=\bigcup_{i=1}^{\infty}(f^{-1}(\delta_i))$. Since f is 1-1, we have $A=\bigcup_{i=1}^{\infty}f^{-1}(\delta_i)$. By Theorem 4.5, $f^{-1}(\delta_i)$ is a bipolar vague nowhere dense sets in (X,τ) and $A=\bigcup_{i=1}^{\infty}f^{-1}(\delta_i)$, where $f^{-1}(\delta_i){}'s$ are bipolar vague nowhere dense sets in (X,τ), implies that A is a bipolar vague first category set in a bipolar vague topological space (X,τ).

Theorem 4.7: If a function $f:(X,\tau)\to(Y,S)$ from a bipolar vague topological space (X,τ) into another topological space (Y,S) is bipolar vague continuous, bipolar vague open, 1-1 and onto function and if (X,τ) is a bipolar vague D-Baire space, then (Y,S) is a bipolar vague D-Baire space.

Proof: Let A be a bipolar vague first category set in (X,τ). Since (X,τ) is a bipolar vague D-Baire space $BV\operatorname{int}(BVcl(A))=0$, By Theorem 4.6, $f(A)$ is a bipolar vague first category set in (Y,S). We have $f(BV\operatorname{int}(BVcl(A)))=BV\operatorname{int}(BVcl(f(A)))$ implies that $BV\operatorname{int}(BVcl(f(A)))=f(0)=0$. Hence for the bipolar vague first category set $f(A)$ in (Y,S), we have $BV\operatorname{int}(BVcl(f(A)))=0$. This implies that (Y,S) is a bipolar vague D-Baire space.

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