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RESEARCH ARTICLE

ACHARYA POLYNOMIAL OF SOME GRAPH TRANSFORMATIONS

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ABSTRACT

Let G be a connected graph of order n, the ***Acharya Polynomial AP(G, λ)*** is defined as $\sum_{\substack{1\le d\le n-1 \\ 1\le i\le p}} \mu(d,i)\cdot x^i$ of graph G and x is a parameter. In the present work we determine Acharya polynomial and Acharya index of subdivision graph of wheel graph W_n and the para-line graph the wheel $L(S(W_n))$ for all positive integers $n \ge 3$. Further we find Hosoya and Terminal Hosoya Polynomial for Kragujevac tree.

Key words: Line graph, subdivision graph, para-line graph,wheel graph, Wiener index, Terminal Wiener index, Acharya index.

1. Introduction

In the present paper all graphs considered are simple, finite and connected. Let G be a such graph with vertex set $V(G)$ and edge set $E(G)$. The number of edges incident on vertex v is called *degree*, $d(v)$ of vertex. The *distance* between the vertices u and v is the length of shortest path. It is denoted by $d_G(u,v)$. The diameter *diam*(G)= p is maximum distance between two vertices of a graph G [**1**]. The *line graph* $L(G)$ is the graph whose vertices corresponds to edges of G with two vertices being adjacent if and only if the corresponding edges in G have common vertex. *Subdivision graph* $S(G)$ is the graph obtained from G by replacing each of its edges by a path of length two or equivalently, by inserting an additional vertex of degree two into each edge of G. The line graph of subdivision graph is called para-line graph. A *wheel graph* with n vertices, W_n is the graph formed by connecting a single vertex to vertices of a cycle C_{n-1}. A wheel graph with n vertices has $2(n-1)$ edges. The diameter of wheel graph is 2.

In the theoretical chemistry, the physico-chemical properties of chemical compounds studied by means of graph based structural descriptors, which are known as topological indices. The oldest structural descriptor is defined in 1947 by H. Wiener [**2**]. This index is known as *Wiener index*. It is defined as sum of distances between all pair of vertices of graph

$$W = W(G) = \sum_{(u,v)\subseteq V(G)} d_G(u,v)$$

The *Hosoya polynomial* of *G* is defined as

$$(H,x) = \sum_{i>1} d(G,i)\cdot x^i$$

where *d*(*G*,*i*) is the number of vertices of the graph *G* whose distance is *i*, and *x* is some real number. The *Terminal Wiener index* of a graph [**3**] is defined as the sum of the distances between all the pair of pendant vertices .

$$TW = TW(G) = \sum_{1<i<j<k} d(v_i, v_j / G)$$

The *Terminal Hosoya Polynomial*

$$TH(G,x) = \sum_{k\geq 1} d_T(G,i)\cdot x^i$$

Details on chemical application and mathematical properties of Wiener index and terminal Wiener index see the reviews and references quoted therein [**4**,**5**,**6**,**7**].

Present authors have defined novel topological index viz *Acharya index* $AI_\lambda(G)$ of a graph *G* [**8**] as the sum of distances between all pair of degree *d* vertices i.e.

$$AI_\lambda(G) = \sum_{\substack{1\leq d\leq n-1\\ 1\leq i\leq p}} \mu(d,i)\cdot i$$

Where *μ*(*d*,*i*) denotes pair of degree *d* vertices at distance *i, p=diam(G)*

Acharya Polynomial AP(*G*,*i*) is defined as follows

$$AP(G,i) = \sum_{\substack{1\leq d\leq n-1\\ 1\leq i\leq p}} \mu(d,i)\cdot x^i$$

The following relationship between Acharya index and Acharya Polynomial is noted in **[8,9]**

$$AI_\lambda(G) = AP^1(G,i)$$

2. On the chemical applicability of Acharya Index.

The productivity of Acharya index was tested using a dataset of 74 alkanes from ethane to nonanes found at [**10,11**]. The alkane data set consists of the following data: boiling points (bp), molar volumes (mv) at 20°C, molar refractions (mr) at 20°C, heats of vaporization (hv) at 25°C, critical temperature (ct), critical pressure (cp) surface tensions (st) 20°C and melting points (mp). The Acharya index was correlated with each of these physical properties and surprisingly, we can see that the Acharya index shows good correlation with boiling points (bp), molar volume (mv) and molar refraction (mr) with correlation coefficient value 0.839, 0.841 and 0.849 respectively.

Further, we have verified these physical properties with the so called Terminal Wiener index, which shows weak correlation, compared to Acharya index with correlation coefficient values lies between 0.192 to 0.652. Hence, the predicting power of Acharya index is better than the terminal Wiener index.

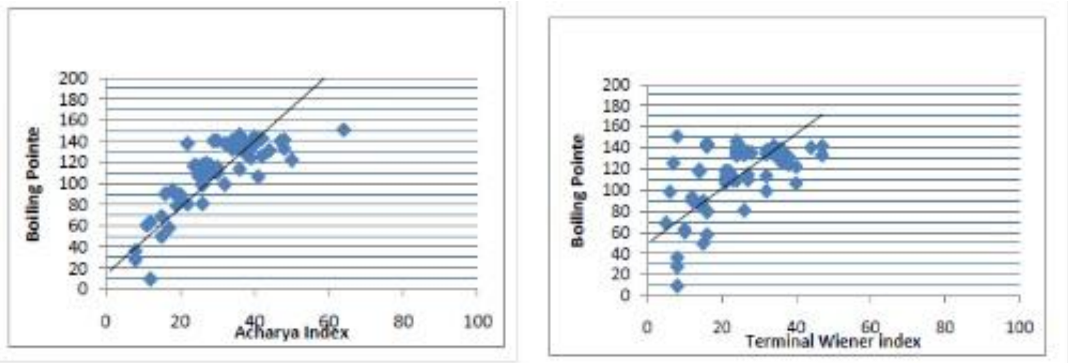

Figure1: Correlation between Acharya index, terminal Wiener index with physical properties (Bp) of 74 alkanes

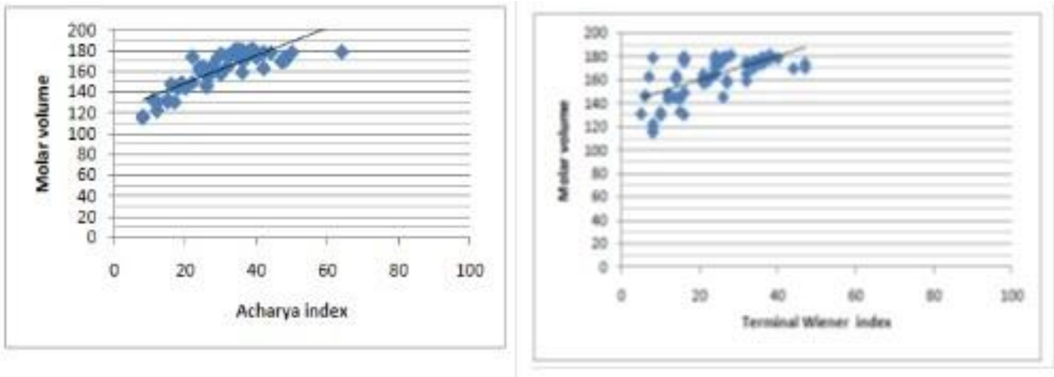

Figure2: Correlation between Acharya index, terminal Wiener index with physical properties molar volume (mv) of 74 alkanes.

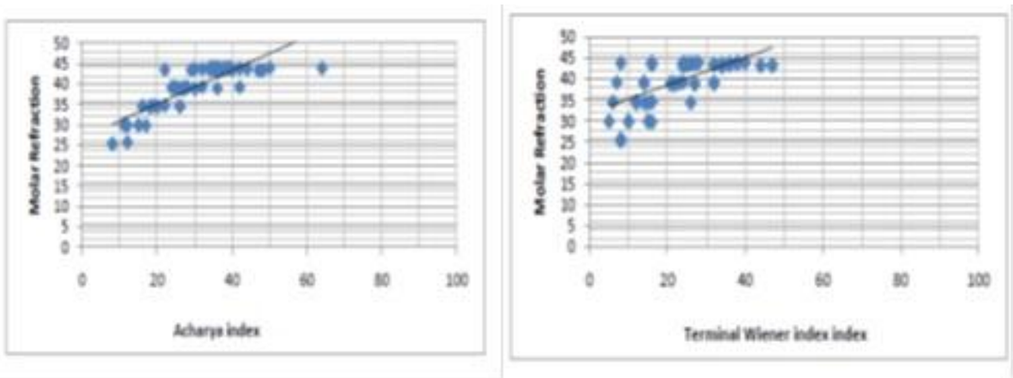

Figure3: Correlation between Acharya index, terminal Wiener index with physical properties molar refraction (mr) of 74 alkanes.

3. Acharya index and Acharya Polynomial of subdivision graph of wheel and para-line graph of wheel

Theorem 3.1: Let W_n is wheel graph with order n, $AI_{\lambda}(W_n) = n^2 - n$.

Proof: Since $W_n = C_{n-1}+K_1$, let v be a central vertex of wheel such that $deg(v) = n-1$ and $deg(u_i)=3$, $1 \le i \le n-1$. Therefore to compute Acharya index we need to count only (n-1) vertices of degree 3. For any vertex u_i , $1 \le i \le n-1$ except adjacent vertices all are at distance 2. Therefore $n(n-3)/2$ vertices are at distance 2 and n edges between adjacent vertices. Hence,

$$AI_{\lambda}(W_n) = n \times 1 + \frac{n(n-3)}{2} \cdot 2 = n^2 - n.$$

Corollary 3.2: Let W_n is wheel graph with order n, $AP(W_n, x) = n \cdot x + \frac{n(n-3)}{2} \cdot x^2$

We need the following auxiliary result to prove our next result.

Theorem A [8]: If G is regular graph then Acharya index is equal to Wiener index.

Theorem 3.3: Let $L(W_n)$ be the line graph of wheel graph of order n then

$AI_\lambda(L(W_n)) = W(C_{n-1}) + W(K_{n-1})$.

Proof: The line graph of wheel graph can be divided into two disjoint induced subgraphs C_{n-1} and K_{n-1}, Since C_{n-1} and K_{n-1} are regular graphs. Therefore by Theorem A, Acharya index is equal sum of Wiener index of C_{n-1} and K_{n-1}.

Corollary 3.4: Let $L(W_n)$ be the line graph of wheel graph of order n then

$AP(L(W_n), x) = H(C_{n-1}, x) + H(K_{n-1}, x)$

Theorem 3.5: If $S(W_n)$ is subdivision graph of wheel graph with order n. Then the Acharya Polynomial and the Acharya index of $S(W_n)$) given by

$$AP(S(W_n).x) = \frac{n^2+7n}{2}x^2 + \frac{3n^2-5n}{2}x^4 + \frac{n^2-5n}{2}x^6$$

$$AI_\lambda(S(W_n)) \ = 10n^2 - 18n$$

Proof: Let $S(W_n)$ be the subdivision graph of wheel graph W_n .The partition of $V(S(W_n))$ is given in Table1.

Table 1: Degree of vertices and number of vertices

$u \in V(S(W_n))$	$deg(u)=n$	$deg(u)=2$	$deg(u)=3$
Number of vertices	1	$2n$	n

The vertex set and edge set of subdivision graph of Wheel graph $S(W_n)$ are given by

$$|V(S(W_n))| = 3n+1 \quad \text{and} \quad E(S(W_n)) = \frac{n\times 1 + 2\times 2n + 3\times n}{2} = 4n$$

Table 2: Degree of vertices and number of vertices at a distance

Degree of vertices	**Number of vertices at a distances**					
	1	**2**	**3**	**4**	**5**	**6=*diam***
2	0	(½)(n^2+5n)	0	n^2-n	0	(½)(n^2-5n)
3	0	n	0	(½)(n^2-3n)	0	0

Hence by employing the results in Table 1 and Table 2 we get Acharya Polynomial as

$$AP(S(W_n), x) = \sum_1^6 \mu(d,i)\cdot x^i$$

$$= \mu(d,1)\cdot x^1 + \mu(d,2)\cdot x^2 + \mu(d,3)\cdot x^3 + \mu(d,4)\cdot x^4 + \mu(d,5)\cdot x^5 + \mu(d,6)\cdot x^6$$

$$= 0\cdot x + \frac{n^2+7n}{2}\cdot x^2 + 0\cdot x^3 + \frac{3n^2-5n}{2}\cdot x^4 + 0\cdot x^5 + \frac{n^2-5n}{2}\cdot x^6$$

$$= \frac{n^2+7n}{2}x^2 + \frac{3n^2-5n}{2}x^4 + \frac{n^2-5n}{2}x^6.$$

Also

$$AI_\lambda(S(W_n)) = \frac{\partial}{\partial x}\left(AP(S(W_n), x\right) \ at \ x=1$$

$$= \left[\frac{1}{2}(n^2+5n)+n\right]\cdot 2 + \left[(n^2-n)+\frac{1}{2}(n^2-3n)\right]\cdot 4 + \frac{1}{2}(n^2+5n)\cdot 6$$

$$= 10n^2 - 18n$$

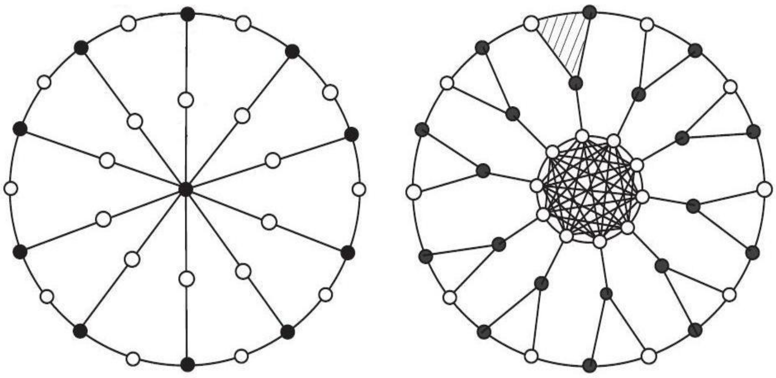

Figure4. The subdivision graph of the wheel graph of order W_n and the para-line graph of the wheel $L(S(W_n))$ for all positive integers $n \geq 3$

Theorem 3.6 : If $L(S(W_n))$ be a para-line graph of wheel graph. Then the Acharya polynomial and the Acharya index of $L(S(W_n))$ given by

$$AP(S(W_n).x) = \frac{n^2+7n}{2} \cdot x + 4n \cdot x^2 + \frac{n^2+7n}{2} \cdot x^3 + (2n^2-4n) \cdot x^4 + (n^2-9n) \cdot x^5$$

$$AI_\lambda(S(W_n)) \;\; = 20n^2 - 39n$$

Proof: Let $L(S(W_n))$ be the para-line graph of wheel graph $S(W_n$ **)**. The Vertex partition of $V(L(S(W_n)))$ is given in Table3.

Table 3: Degree of vertices and number of vertices

$u \in V(L(S(W_n)))$	*deg*(*u*)=3	*deg*(*u*)=*n*
Number of vertices	3*n*	*n*

The vertex set and edge set para-line graph of Wheel graph $L(S(W_n))$ are given by

$$|V(L(S(W_n)))| = 4n \quad \text{and} \quad E(S(W_n)) = \frac{n \times n + 3 \times 3n}{2} = \frac{1}{2}n(n+9)$$

Table 4:Degree of vertices and number of vertices at a distance

Degree of vertices	**Number of vertices at a distances**				
	1	**2**	**3**	**4**	**5=*diam***
3	4*n*	4*n*	(½)(n^2+7*n*)	2n^2-4*n*	n^2-9*n*
n	(½)(n^2-*n*)	0	0	0	0

Hence by using results in Table 3 and Table 4 we get Acharya Polynomial and Acharya Index as

$$AP(L(S(W_n)),x) = \sum_1^5 \mu(d,i) \cdot x^i$$

$$= \mu(d,1) \cdot x^1 + \mu(d,2) \cdot x^2 + \mu(d,3) \cdot x^3 + \mu(d,4) \cdot x^4 + \mu(d,5) \cdot x^5$$

$$= \frac{n^2+7n}{2} \cdot x + 4n \cdot x^2 + \frac{n^2+7n}{2} \cdot x^3 + (2n^2-4n) \cdot x^4 + (n^2-9n) \cdot x^5$$

$$AI_\lambda(S(W_n)) = \frac{\partial}{\partial x}\left(AP(L(S(W_n)),x\right) \;\; at \;\; x=1$$

$$= \frac{n^2+7n}{2} \cdot 1 + 4n \cdot 2 + \frac{n^2+7n}{2} \cdot 3 + (2n^2-4n) \cdot 4 + (n^2-9n) \cdot 5$$

$$= 20n^2 - 39n$$

4. Hosoya Polynomial, Terminal Hosoya Polynomial of Kragujevac trees.

Definition 4.1: [**12**] Let P_3 be the 3-vertex tree, rooted at one of its terminal vertices, For k = 2,3,... ., construct the rooted tree B_k by identifying the roots of k copies of P_3. The vertex obtained by identifying the roots of P_3-trees is the root of B_k .

Examples illustrating the structure of the rooted tree B_k are depicted in Fig. 2.

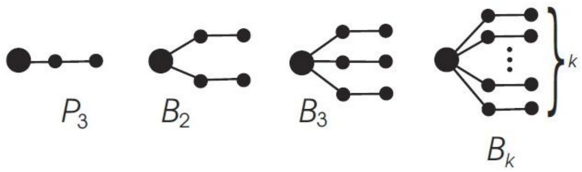

Figure5. The rooted trees B_2 , B_3 , and B_k , obtained respectively, by identifying the roots of 2, 3, and k copies of P_3 . Their roots are indicated by large dots.

Definition 4.2: [**12**] Let $d \geq 2$ be an integer . Let $\beta_1, \beta_2, \cdots \beta_d$, be the rooted trees specified in the previous definition. *i.e* $\beta_1, \beta_2, \cdots \beta_d \in \{B_1, B_2, \cdots\}$. A Kragujevac tree T is a tree possessing a vertex of degree d , adjacent to the roots of $\beta_1, \beta_2, \cdots \beta_d$ .This vertex is said to be central vertex of T , whereas d is the degree of T. The sub graphs $\beta_1, \beta_2, \cdots \beta_d$ are the braches of T .

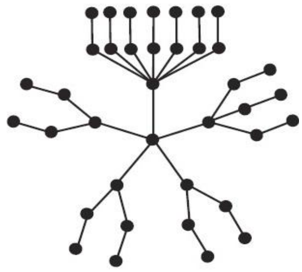

Figure 6. A typical Kragujevac tree.

Theorem 4.3: If G is a Kragujevac tree, If $d(G,i)$ is pair of vertices a distance i and $d \geq 2$ is degree of central vertex at then the Hosoya Polynomial is

$$W(G) = \sum_{i=1}^{p} d(G,i) x^i$$

$$= d(G,1)x^1 + d(G,2)x^2 + d(G,3)x^3 + d(G,4)x^4 + d(G,5)x^5 + d(G,6)x^6$$

$$= \sum_{i=1}^{d} (2k_i + 1) \cdot x^1 + \left(2\sum_{i=1}^{d} k_i + \sum_{i=1}^{d} {}^{k_i}c_2 + {}^{d}c_2 \right) \cdot x^2 + \left\{ d\sum_{i=1}^{d} k_i + \sum_{i=1}^{d} k_i (k_i - 1) \right\} \cdot x^3$$

$$+ \left\{ (d-1)\sum_{i=1}^{d} k_i + \sum_{i=1}^{d} {}^{k_i}c_2 + \sum_{1 \leq i \leq j \leq n} k_i k_j \right\} \cdot x^4 + \left(2 \sum_{1 \leq i \leq j \leq d} k_i k_j \right) \cdot x^5 + \left(\sum_{1 \leq i \leq j \leq d} k_i k_j \right) \cdot x^6$$

Proof: Consider of Kragajevac tree of degree $d \geq 2$. The size of Kragajevac tree is $\sum_{i=1}^{d} (2k_i + 1)$

Therefore the pair of vertices at distance one is equals to the number of edges in Kragajevac tree, so is the first term in expression**.** Now for each branch B_{k_i} , i=1,2,.... . of Kragajevac tree contains k_i pendant vertices, k_i degree two vertices and rooted vertex. The k_i degree two vertices and k_i pendant vertices are at distance two from central vertex and rooted vertex respectively. Thus $2\sum_{i=1}^{d} k_i$ pair of vertices at distances two. And also vertices of degree two in a branch B_{k_i} and all rooted vertices are at distance two. Therefore the total number of pair of vertices at a distance two is

$2\sum_{i=1}^{d} k_i + \sum_{i=1}^{d} {}^{k_i}c_2 + {}^{d}c_2$. To find pair of vertices at distance three, consider the distance between central vertex and pendant vertices, we note $\sum_{i=1}^{d} k_i$ are at distance 3, the distance of rooted vertex B_{k_i} to the vertices of degree two in B_{k_j} also three, which is given by $\sum_{i=1}^{d} k_i(k_i-1)$ and pendant vertices to the vertices of degree two in the same branch are at distance 3 given by $\sum_{i=1}^{d}\left(\sum k_i\right)-k_i$. Hence the total pair of vertices at distance 3 are $d\sum_{i=1}^{d} k_i + \sum_{i=1}^{d} k_i(k_i-1)$.

The distance between pendant in a branch and the distance of degree two vertices of B_{k_i} to the degree two vertices in B_{k_j} is four. These are given by $\sum_{i=1}^{d} {}^{k_i}c_2 + \sum_{1\le i\le j\le n} k_i k_j$. the distance of rooted vertex B_{k_i} to the vertices of pendant vertices in B_{k_j} is four given by $\sum_{i=1}^{d}\left(\sum k_i\right)-k_i$ Hence we have $(d-1)\sum_{i=1}^{d} k_i + \sum_{i=1}^{d} {}^{k_i}c_2 + \sum_{1\le i\le j\le n} k_i k_j$ pair of vertices at distance four. Now distance of pendant vertex in B_{k_i} to the vertices of degree two in B_{k_j} and distance of degree two vertex in B_{k_i} to the pendant vertices in B_{k_j} is five, given by $2\sum_{1\le i\le j\le d} k_i k_j$ .Now the distance between pendant vertices is six, i.e $\sum_{1\le i\le j\le n} k_i k_j$ pair of vertices at a distance six. which are given by the last two terms in expression.

Theorem 4.4: If *G* is a Kragujevac tree, If *d* ≥2is degree of central vertex and *p* = *diam*(*G*)=6 ,then Terminal Hosoya polynomial is given by

$$TW(G) = \sum_{i=1}^{d} \frac{k_i(k_i-1)}{2} \cdot x^4 + \sum_{1\le i\le j\le n} k_i k_j \cdot x^6$$

$$TW(G) = \sum_{k=1}^{p} a_k x^k = a_1 x^1 + a_2 x^2 + \dots\dots\dots + a_p x^p$$

$$a_1 = a_2 = a_3 = 0 \ , a_4 = {}^{k_i}C_2 = \frac{k_i(k_i-1)}{2} \ for\, d \ge 2 \quad \text{and} \quad a_p = a_6 = k_i k_j$$

Proof: By the definition of Kragajevac tree of degree *d*≥2 contains branches B_{k_i} and each branch B_{k_i} *i*=1,2,.... and each contains k_i pendant vertices. Each of these pendant vertices at distance four. Therefore, there are ${}^{k_i}c_2$ pair of pendant vertices at four . The branches B_{k_i} and $B_{k_j}, i \neq j$ with k_i and k_j pendant vertices. Each of these are at a distance 6. Since there are $k_i \times k_j$ pair of vertices of this kind. This gives the second term in expression.

Acharya index is degree distance based topological index. Hence depending on the B_{k_i} and degree *d*≥2 of the central vertex. We have the following cases,

Theorem 4.5: If *G* is a Kragujevac tree, If *d*≥2 is the degree of central vertex, then Acharya polynomial is given by

Case1: For all the branches of *G* , then if $B_{k_i} = B_{k_j} \ \forall \ i \neq j$

$$AP(G,x)=\left(\sum_{i=1}^{d}\frac{k_i(k_i-1)}{2}+{}^{d}c_2\right)\cdot x^2+\left(\sum_{i=1}^{d}\frac{k_i(k_i-1)}{2}+\sum_{1\le i\le j\le d}k_ik_j\right)\cdot x^4+\sum_{1\le i\le j\le n}k_ik_j\cdot x^6$$

Case2: For all the branches of G , then if $B_{k_i}\neq B_{k_j}\ \forall\ i\neq j$

$$AP(G,x)=\sum_{i=1}^{d}\frac{k_i(k_i-1)}{2}\cdot x^2+\left(\sum_{i=1}^{d}\frac{k_i(k_i-1)}{2}+\sum_{1\le i\le j\le d}k_ik_j\right)\cdot x^4+\sum_{1\le i\le j\le n}k_ik_j\cdot x^6$$

Case3: For some the branches of *G.* If $k_i = d$ (say some i=1,2,3...*m*)

$$AP(G,x)=\sum_{1}^{m}k_i\cdot x+\sum_{i=1}^{d}\frac{k_i(k_i-1)}{2}\cdot x^2+\left(\sum_{i=1}^{d}\frac{k_i(k_i-1)}{2}+\sum_{1\le i\le j\le d}k_ik_j\right)\cdot x^4+\sum_{1\le i\le j\le n}k_ik_j\cdot x^6$$

Case4: For some the branches of *G.* If $k_i = k_j \neq d$(say some i=1,2,3...*m*)

$$AP(G,x)=\left(\sum_{i=1}^{d}\frac{k_i(k_i-1)}{2}+\sum_{i=1}^{m}{}^{m}c_2\right)\cdot x^2+\left(\sum_{i=1}^{d}\frac{k_i(k_i-1)}{2}+\sum_{1\le i\le j\le d}k_ik_j\right)\cdot x^4+\sum_{1\le i\le j\le n}k_ik_j\cdot x^6$$

5. Conclusion

In this paper ,we have shown the chemical applicability of Acharya index and then formulate, Acharya Polynomial and Acharya index of subdivision graph of wheel graph W_n and para-line graph of wheel graph $L(S(W_n))$ for all positive integers n≥3 and also obtained Hosoya Polynomial, Terminal Hosoya Polynomial and Acharya Polynomial for Kragujevac trees.

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