Vol.5.Issue.3.2017 (July-Sept)
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BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com
Email:editorbomsr@gmail.com

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

RESEARCH ARTICLE

INTEGRAL SOLUTIONS OF NON-HOMOGENEOUS QUINTIC DIOPHANTINE EQUATION

$$x^3 + y^3 = 2z^5$$

A.VIJAYASANKAR[1], M.A.GOPALAN[2], V.KRITHIKA[3]

[1]Assistant Professor, Department of Mathematics,National College, Trichy-620001,Tamilnadu,India.
e-mail:avsankar70@yahoo.com
Professor, Department of Mathematics, SIGC, Trichy-620002, Tamilnadu,India.
e-mail:mayilgopalan@gmail.com
[3]Research Scholar, Dept. of Mathematics, National College, Trichy-620001, Tamilnadu, India.
e-mail:krithijay.24@gmail.com

V.KRITHIKA

ABSTRACT

The non-homogeneous Quintic Diophantine equation is represented by $x^3 + y^3 = 2z^5$ is analyzed for its non-zero distinct integral solutions. A few interesting relations among the solutions are exhibited.

Keywords: Quintic equation with three unknowns, integral solutions.

1. INTRODUCTION

Diophantine equations, homogeneous and non-homogeneous, have aroused the interest of numerous mathematicians since antiquity as can be seen from [1-4]. The problem of finding all integer solutions of a Diophantine equation with three or more variables and degree atleast three, in general, presents a good deal of difficulties. Cubic equations in two variables fall into the theory of elliptic curves which is a very developed theory but still an important topic of current research. A lot is known about equations in two variables in higher degrees. For equations with more than three variables and degree atleast three, very little is known. It is worth to note that undesirability appears in equations, even perhaps at degree four with fairly small coefficients. In [5-7], ternary quintic diophantine equations are studied and in [8-9], quintic diophantine equations with four unknowns are analyzed. In [10-12], a few quintic diophantine equation with five unknowns are observed.

In this communication, a typical ternary fifth degree Diophantine equation $x^3 + y^3 = 2z^5$ is considered and different choices of integral solutions are obtained. A few interesting relations among the solutions are presented.

2. METHOD OF ANALYSIS

The Diophantine equation to be solved for its non-zero distinct integral solutions is given by

$$x^3 + y^3 = 2z^5 \tag{1}$$

Introducing the transformations

$$x = u + v, y = u - v, z = u, \quad u \neq v \neq 0 \tag{2}$$

in (1), can be written as

$$u^4 - u^2 - 3v^2 = 0 \tag{3}$$

As the above equation is quadratic in 'u' , we have

$$u^2 = \frac{1}{2}\left[1 \pm \sqrt{1+12v^2}\right]$$

$$u^2 = \frac{1}{2}\left[1 + \sqrt{1+12v^2}\right] \text{ \{as the other value is inadmissible\}} \tag{4}$$

Choose $\alpha^2 = 1+12v^2$ (5)

Using (5) in (4), we obtain

$$u^2 = \frac{1}{2}[1+\alpha] \tag{6}$$

From (5), we get the initial solution as

$$\alpha_0 = 7, v_0 = 2$$

The other solutions of (5), can be derived from the relations

$$\left.\begin{aligned} \alpha_n &= \frac{f_n}{2} \\ v_n &= \frac{g_n}{4\sqrt{3}} \end{aligned}\right\} \tag{7}$$

where

$$\left.\begin{aligned} f_n &= (7+4\sqrt{3})^{n+1} + (7-4\sqrt{3})^{n+1} \\ g_n &= (7+4\sqrt{3})^{n+1} - (7-4\sqrt{3})^{n+1} \end{aligned}\right\} \tag{8}$$

Substituting α_n in (6), we obtain

$$\left.\begin{aligned} u_n &= \frac{\left(2+\sqrt{3}\right)^{n+1} + \left(2-\sqrt{3}\right)^{n+1}}{2} \\ v_n &= \frac{\left(2+\sqrt{3}\right)^{2n+2} - \left(2-\sqrt{3}\right)^{2n+2}}{4\sqrt{3}} \end{aligned}\right\} \tag{9}$$

By using (9) in (2), we get the integral solutions of (1) are found to be

$$\left.\begin{aligned} 4\sqrt{3}\,x_n &= 2\sqrt{3}F_n + g_n \\ 4\sqrt{3}\,y_n &= 2\sqrt{3}F_n - g_n \\ 2z_n &= F_n \end{aligned}\right\} \qquad (10)$$

where $F_n = \left(2+\sqrt{3}\right)^{n+1} + \left(2-\sqrt{3}\right)^{n+1}$

2.1 PROPERTIES

- $3\left(x_n^2 - y_n^2\right) = 16 z_n^4 \left(z_n^2 - 1\right)$
- $2\left(1 + z_{2n+1}\right)$ is a perfect square.
- $27\left(x_n^3 - y_n^3\right)^2 = 4z_n^6\left(z_n^2 - 1\right)\left(z_n^2 + 8\right)^2$
- $10z_{n+1} = 14x_{n+1} - x_{n+2} - x_n$.
- $z_{n+1} - z_n - z_{n+2} = 0.$
- $\{2\left(z_{4n+3} - 1\right) + 4 * a\ perfect\ square\}$ is a biquadratic integer.
- $4z_{n+1} - z_{n+2} - z_n = 0.$
- $2z_{3n+2} + 3\left\{\left(2+\sqrt{3}\right)^{n+1} + \left(2-\sqrt{3}\right)^{n+1}\right\}$ is a cubical integer.
- $10z_{n+2} = 56x_{n+1} - 10z_n - 4\left(x_{n+2} + x_n\right)$
- Each of the following expressions represents a hyperbola:

Hyperbola	$\left(\alpha_{n,}\beta_n\right)$
$3\alpha_n^2 - \beta_n^2 = 12$	$\left(2z_n, 2z_{n+1} - 4z_n\right)$
$3\alpha_n^2 - \beta_n^2 = 3$	$\left(4z_{n+1} - z_{n+2}, 2z_{n+2} - 7z_{n+1}\right)$
$12\alpha_n^2 - \beta_n^2 = 48$	$\left(2z_n, z_{n+2} - 7z_n\right)$

c

- Each of the following expressions represents a parabola:

Parabola	$\left(\alpha_{n,}\beta_n\right)$
$\beta_n^2 = 6\alpha_n - 12$	$\left(1 + z_{2n+1}, 2z_{n+1} - 4z_n\right)$
$2\beta_n^2 = 3\alpha_n - 6$	$\left(4z_{2n+2} - z_{2n+3} + 1, 2z_{n+2} - 7z_{n+1}\right)$
$\beta_n^2 = 24\alpha_n - 48$	$\left(1 + z_{2n+1}, z_{n+2} - 7z_n\right)$

3. CONCLUSION

In this paper, we have presented sets of infinitely many non-zero distinct integer solutions to the quintic equation with three unknowns given by $x^3 + y^3 = 2z^5$. As Diophantine equations are rich in variety due to their definition. One may attempt to find integer solutions to higher degree Diophantine equation with multiple variables.

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