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RESEARCH ARTICLE

COUPLED LUCAS SEQUENCE

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ABSTRACT

In this paper we have introduced interlinked coupled recurrence relation of Lucas second order sequence and deduced some of its properties

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Key words: Fibonacci numbers, Lucas numbers, Fibonacci sequence, Lucas sequence, 2F Sequence

1. INTRODUCTION

Atanassov [1] and Suman, Amitava, k sisodiya introduce respectively the interlinked second order recurrence relation and interlinked Jacobsthal Sequence by constructing two sequences $\{\alpha\}_{i=0}^{\infty}$ and $\{\beta\}_{i=0}^{\infty}$ naming them as 2F Sequences.

According to the scheme, $\alpha_{n+2} = \beta_{n+1} + \beta_n, n \geq 0$, $\beta_{n+2} = \alpha_{n+1} + \alpha_n, n \geq 0$

Taking $\alpha_0 = a$, $\beta_0 = b$, $\alpha_1 = c$, $\beta_1 = d$,where a,b,c,d are integers, he extended his research in the same direction which can be seen in [1],[3] and [5].Hirschhorn in [6] and [2] present explicit solution to the longlasting problems on the second and third order recurrence relations posed by Atanassov[5]. Recently Singh, Sikhwal and Jain deduced coupled recurrence relations of order five[7].Carlitz, et.el,[8] had also given a representation for a special sequence.

2. COUPLED LUCAS SEQUENCE

Taking Lucas Sequence

$L_{n+2} = L_{n+1} + 2L_n$ where , $L_0 = 0, L_1 = 1$

$l_{n+2} = l_{n+1} + 2l_n$ where , $l_0 = 2, l_1 = 1, n \geq 0$.

The Koken and Bozkurth in [1] and [2] have given some matrix properties of Jacobsthal-Lucas numbers.

We have introduced coupled order recurrence relations for Lucas number and Lucas sequence and called them as 2-L Sequences.

$L_{n+2} = l_{n+1} + 2l_n, n \geq 0$

$l_{n+2} = L_{n+1} + 2L_n, n \geq 0$

$L_0 = a, L_1 = b, l_0 = c, l_1 = d$ (2.1)

If we set a = b and c = d then the sequence $\{L_i\}_{i=0}^{\infty}$ and $\{L_i\}_{i=0}^{\infty}$ shall coincide with each other and the sequence $\{L_i\}_{i=0}^{\infty}$ shall becomes a generalized Lucas sequence where,

$L_0(a,c) = a, L_1(a,c) = c$

$L_{n+2}(a,c) = l_{n+1}(a,c) + l_n(a,c)$

$L_n = a, b, d + 2c, b + 2a + 2d$

$l_n = c, d, b + 2a, d + 2c + 2b$

By examining the above terms we obtain the following properties:

Theorem 1:For every integers $n \geq 0$

(a) $L_{4n}/l_0 = l_{4n}/L_0$

(b) $L_{4n+1} + l_1 = l_{4n+1} + L_1$

(c) $L_{4n+3} + l_0 + l_1 = l_{4n+3} + L_0 + L_1$

Proof:

For (c) the statement is obviously true for n=0.

Assuming that the statement is true for some integer, $n \geq 0$, by the given scheme (1)

$L_{4n+3} + l_0 + l_1 = l_{4n+2} + 2l_{4n+1} + l_0 + l_1$

$= L_{4n+1} + 2L_{4n} + 2l_{4n+1} + l_0 + l_1$

(by inductive hypothesis)

$= L_{4n+1} + L_{4n+2} + l_{4m+1} + l_1 + l_0$

$= l_{4n+3} + l_1 + l_0$

Hence the statement is true for all integers $n \geq 0$

Similar proofs can be given for parts (a) and (b).Adding the first n terms of $\{L_i\}_{i=0}^{\infty}$ and $\{L_i\}_{i=0}^{\infty}$ yields the following results.

Theorem 2: For all integers $k \geq 0$

(a) $l_{3k+5} = \sum_{i=1}^{3k} L_{3k+i} + \sum_{i=-1}^{k+1} l_{3k+i} + \sum_{i=1}^{2k} l_{3k+i} + l_{3k-i}$

(b) $L_{3k+5} = \sum_{i=1}^{3k} l_{3k+i} + \sum_{i=-1}^{k+1} L_{3k+i} + \sum_{i=1}^{2k} l_{3k+i} + l_{3k-i}$

Proof(a):

$l_{3k+5} = L_{3k+4} + 2L_{3k+3}$

$= l_{3k+3} + 2l_{3k+2} + 2L_{3k+3}$ (1)

$= L_{3k+2} + 2L_{3k+1} + 2l_{3k+2} + 2l_{3k+2} + 4l_{3k+1}$

$= L_{3k+2} + 2L_{3k+1} + 2l_{3k+2} + 2L_{3k+3}$

$= \sum_{i=1}^{3k} L_{3k+i} + L_{3k+1} + L_{3k+3} + l_{3k+2}$ by(1)

$$= \sum_{i=1}^{3k} L_{3k+i} + L_{3k+1} + 2L_{3k+2} + 2l_{3k+1} + l_{3k+2}$$

$$= \sum_{i=1}^{3k} L_{3k+i} + l_{3k} + 2l_{3k-1} + 2l_{3k+2} + 2l_{3k+1} + l_{3k+2}$$

$$= \sum_{i=1}^{3k} L_{3k+i} + l_{3k} + 2l_{3k-1} + 2l_{3k+2} + 2l_{3k+1} + l_{3k+2}$$

$$= \sum_{i=1}^{3k} L_{3k+i} + \sum_{i=-1}^{k+1} l_{3k+i} + l_{3k-i} + l_{3k+1} + l_{3k+2}$$

$$= \sum_{i=1}^{3k} L_{3k+i} + \sum_{i=-1}^{k+1} l_{3k+i} + \sum_{i=1}^{2k} L_{3k+i} + l_{3k-i}$$

The proof of (b) is similar to the proof of (a), hence omitted for the sake of brevity. Adding the first n terms with even or odd subscripts for each sequence $\{L_i\}_{i=0}^{\infty}$ and $\{l_i\}_{i=0}^{\infty}$.

3. TWO INFINITE SEQUENCES

Let us assume two infinite sequences of second order $\{a_i\}_{i=0}^{\infty}$ and $\{b_i\}_{i=0}^{\infty}$ with the initial values a, c and b, d $\in R$

Out of the many schemes that emerge we study two of them

Scheme 3.1

$$a_{n+2} = b_{n+1} + 2a_n : b_{n+2} = a_{n+1} + 2b_n, n \geq 0$$

$$a_0 = a, b_0 = b, a_1 = c, b_1 = d$$

Setting a – b , c – d, the sequence $\{a_i\}$ and $\{b_i\}$ coincides and from a generalized Lucas sequence L_i

Consider,

n	a_n	b_n
0	a	b
1	c	d
2	d+2a	c+2b
3	3c+2b	3d+2a

Theorem3.1: $a_n - b_n = (-1)^{n-1}(a_1 - b_1)L_n + (-1)^n.2.(a_0 - b_0)L_{n-1}$

Proof: Using the principle of mathematical induction we get, for n=2

$$a_2 - b_2 = (d + 2a) - (c + 2b)$$

$$= -(c - d) + 2(a - b)$$

$$= (-1)^{2-1}.(c - d).1 + (-1)^2, 2.(a - b).1$$

$$= (-1)^{2-1}.(c - d).L_2 + (-1)^2, 2.(a_0 - b_0).L_{2-1}$$

If the statement is true for n=k

That is , $a_k - b_k = (-1)^{k-1}(a_1 - b_1)L_k + (-1)^k.2.(a_0 - b_0)L_{k-1}$

Hence for n=k+1 , we get

$$(1)^{k+1-1}(a_1 b_1)L_{k+1} + (1)^{k+1} 2.(a_0 b_0)L_{k+1-1}$$

$$= (-1)^k (a_1 - b_1)L_k + (-1)^{k+1}.2.(a_0 - b_0)L_k$$

$$= (-1)^k (a_1 - b_1)(L_k + 2L_{k-1}) + (-1)^{k+1}(a_0 - b_0)(2L_{k-1} + 2L_{k-2})$$

$$= (-1)^k (a_1 - b_1)(L_k) + (-1)^k (a_1 - b_1)(2L_{k-1}) + (-1)^{k+1}(a_0 - b_0)(2L_{k-1}) + (-1)^{k-1}(a_0 - b_0)(4L_{k-2})]$$

$$= -[(-1)^{k-1}(a_1 - b_1)(L_k) + (-1)^k (a_0 - b_0)(2L_{k-1})] + (-1)^2[(-1)^{k-2}(a_1 - b_1)(L_{k-1}) + (-1)^{k-1}(a_0 - b_0)(2L_{k-2})]$$
$$= -(a_k - b_k) + 2[a_{k-1} - b_{k-1}]$$
$$= a_{k+1} - b_{k+1}$$

Scheme 3.2

$$a_{n+2} = a_{n+1} + 2a_n : b_{n+2} = b_{n+1} + 2b_n, n \geq 0$$

Consider ,

n	a_n	b_n
0	a	b
1	c	d
2	c+2a	d+2b
3	3c+2a	3d+2b

Theorem 3.3: $a_n - b_n = L_n(a_1 - b_1) + 2L_{n-1}(a_0 - b_0)$

Proof:-By the principal of mathematical induction we get for n=2

For n=2

$$a_2 - b_2 = (c - d) + 2(a - b)$$
$$a_2 - b_2 = L_2(a_1 - b_1) + 2L_1(a_0 - b_0)$$

Now, Supposing that the statement is true for n=k

$$a_k - b_k = L_k(a_1 - b_1) + 2L_{k-1}(a_0 - b_0)$$

Thus, for ,n=k+1,we get

$$= L_{k+1}(a_1 - b_1) + 2L_{k+1-1}(a_0 - b_0)$$
$$= [L_k + 2L_{k-1}](a_1 - b_1) + 2.[L_{k-1} + 2L_{k-2}](a_0 - b_0)$$
$$= L_k(a_1 - b_1) + 2L_{k-1}.(a_1 - b_1) + 2L_{k-1}(a_0 - b_0) + 4.L_{k-2}(a_0 - b_0)$$
$$= L_k(a_1 - b_1) + 2L_{k-1}.(a_0 - b_0) + 2[L_{k-1}(a_1 - b_1) + 2.L_{k-2}(a_0 - b_0)]$$
$$= (a_k - b_k) + 2[a_{k-1} - b_{k-1}]$$
$$= [a_k + 2a_{k-1}] - [b_k + 2b_{k-1}]$$
$$= a_{k+1} - b_{k+1}$$

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