Vol.5.Issue.4.2017 (Oct-Dec)

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com

Email:editorbomsr@gmail.com

RESEARCH ARTICLE

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

COMMON FIXED POINTS IN FUZZY METRIC SPACES

SERVET KUTUKCU[1*], ABDULKADIR ERGUN[2]

[1,2]Department of Mathematics, Ondokuz Mayis University, 55139 Kurupelit, Samsun, Turkey

*E-mail: skutukcu@omu.edu.tr

ABSTRACT

In this paper, we prove common fixed point theorems for six maps using some conditions in fuzzy metric spaces in the sense of George and Veremani [6], which turns out to be a material generalization of the results of Kutukcu et al. [20]. We also give an example to illustrate our main theorem.

Keywords: Fuzzy metric space, common fixed point, compatible maps of type β.

AMS(2010) Subject Classification: 47H10, 54H25.

1. INTRODUCTION

The notion of fuzzy sets was introduced by Zadeh [1]. Deng [2], Erceg [3]Kaleva and Seikkala[4] and Kramosil and Michalek[5] have introduced the concept of fuzzy metric spaces in different ways. George and Veeramani [6] modified the concept of fuzzy metric spaces introduced by Kramosil and Michalek [5] in order to get the Hausdorff topology.

Grabiec [7] extended the fixed point theorems of Banach [8] and Edelstein [9] to fuzzy metric spaces in the sense of Kramosil and Michalek[5] whose study is useful in the field of fixed point theorems of contractive type maps. Since then Fang [10] proved some fixed point theorems in fuzzy metric spaces, which improve, generalize and extend some main resultsof [8,9,11-13,14]. Sessa [15] defined a generalization of commutativity, which called weak commutativity. Further Jungck[16] introduced more generalized commutativty, so called compatibility. Following Grabiec [7], Kramosil and Michalek[5] and Mishra et al. [17] obtained common fixed point theorems for compatible maps and asymptotically commuting maps on fuzyy metric spaces which generalize, extend and fuzzify several fixed point theorems for contractive-type maps on metric spaces and other spaces. Pathak et al. [18] introduced the concept of compatible maps of type (P) in metric spaces, which is equivalent to the concept of compatible maps under some conditions and proved common fixed point theorems in metric spaces. Cho et al. [19] introduced the notion of compatible,maps of type (β) on fuzzy metric spaces. Many authors have studied the fixed point theory in fuzzymetric spaces. The most interesting references are [7,10,11,12,17,20,21,22]. Recently, Sharma et al. [22] proved a common fixed point theorem for six maps under the condition of compatible maps of type (β) on fuzzy metric spaces.

In this paper, we prove common fixed point theorems for six maps using some conditions in fuzzy metric spaces in the sense of George and Veremani [6], which turns out to be a material generalization of the results of Kutukcu et al. [20]. We also give an example to illustrate our main theorem.

2. PRELIMINARIES

Definition 2.1 ([14]). A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if $*$ satisfies the following conditions; for all $a, b, c, d \in [0,1]$,

(1) $a * 1 = a$,
(2) $a * b = b * a$,
(3) $(a * b) * c = a * (b * c)$,
(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$.

Basic $t$riangular norms are as follows;

$a * b = min\{a, b\}$ (min$t$-norm)
$a * b = a.b$ (product$t$-norm)
$a * b = maks\{a + b - 1,0\}$ (Lukasiewicz t-norm)

There exists the following inequality between these triangular norms, from weak to strong;

$$maks\{a + b - 1,0\} \leq a.b \leq min\{a, b\}.$$

Example 2.2 $a * b = min\{a, b\}$ is a t-norm for all $a, b \in [0,1]$.Really,

(1) $a * 1 = min\{a, 1\} = a$.
(2) $a * b = \min\{a, b\} = \min\{b, a\} = b * a$.
(3) $a * (b * c) = a * min\{b, c\} = min\{a, min\{b, c\}\} = min\{a, b, c\}$
$$= min\{min\{a, b\}, c\}$$
$$= min\{a * b, c\}$$
$$= (a * b) * c.$$
(4) Let $a \leq b$ and $c \leq d$ for all $a, b, c, d \in [0,1]$. Then,
1. $a \leq b \leq c \leq d \Longrightarrow a * c = min\{a, c\} = a \leq b = min\{b, d\} = b * d$.
2. $a \leq d \leq c \leq b \Longrightarrow a * c = min\{a, c\} = a \leq d = min\{b, d\} = b * d$.
3. $a \leq c \leq b \leq d \Longrightarrow a * c = min\{a, c\} = a \leq b = min\{b, d\} = b * d$.
4. $c \leq a \leq b \leq d \Longrightarrow a * c = min\{a, c\} = c \leq b = min\{b, d\} = b * d$.
5. $c \leq a \leq d \leq b \Longrightarrow a * c = min\{a, c\} = c \leq b = min\{b, d\} = b * d$.
6. $c \leq d \leq a \leq b \Longrightarrow a * c = min\{a, c\} = c \leq d = min\{b, d\} = b * d$.

Definition 2.3 ([6]). A 3-tuple $(X, M, *)$ is said to be a fuzzy metric space if X isan arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on X^2x$[0, \infty)$ satisfying the following conditions; for all $x, y, z \in X$and $t, s > 0$,

(1) $M(x, y, t) > 0$,
(2) $M(x, y, t) = 1 \iff x = y$,
(3) $M(x, y, t) = M(y, x, t)$,
(4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(5) $M(x, y, .): [0, \infty) \to [0, 1]$is continuous.

Example 2.4 Let (X, d) be a metric space and $a * b = a.b$ for all $a, b \in [0, 1]$. Define$M: X^2$x$[0, \infty) \to [0, 1]$by

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}$$

for all $x, y \in X$ and $t > 0$. Then $(X, M_d, *)$ is a fuzzy metric space. This fuzzy metric space is called the standart fuzzy metric space .

Example 2.5 Let $X = N^+$ and $a * b = a.b$ for all $a, b \in [0, 1]$. Define $M: X^2\text{x}[0, \infty) \to [0, 1]$ by

$$M(x, y, t) = \begin{cases} \frac{x}{y}, & x \le y \\ \frac{y}{x}, & y \le x \end{cases}$$

for all $x, y \in X$ and $t > 0$. In this case,$(X, M, *)$ is a fuzzy metric space. There exists no metric on Xsatisfying $M(x, y, t)$.

Lemma 2.6 ([7]). The mapping $M(x, y, .)$ is nondecreasingfor all $x, y \in X$,

Proof. We must show when$t < s$,$M(x, y, t) \le M(x, y, s)$ for all $t, s > 0$. Assume that $M(x, y, s) > M(x, y, t)$ for $t > s$. Then,

$$\begin{aligned} M(x, y, s) &> M(x, y, t) \\ &\ge M(x, y, s) * M(y, y, t - s) \\ &> M(x, y, s) \end{aligned}$$

Thus, we get a contradiction. Then $M(x, y, .)$ is nondecreasing.

Lemma 2.7 ([17], [19]). Let $\{y_n\}$ be a sequence in an FM-space $(X, M, *)$ with $t * t \ge t$ for all $t \in [0, 1]$. If there is a number $k \in (0,1)$ such that

$$M(y_{n+2}, y_{n+1,} kt) \ge M(y_{n+1}, y_n, t)$$

for all $t > 0$ and n=1,2,3,…..then $\{y_n\}$ is a Cauchy sequence in X.

Definition 2.8 ([6]). Let$(X, M, *)$ be a fuzzy metric space. We define open ball $B(x, r, t)$with center $x \in X$ and radius r, $0 < r < 1$, as

$$B(x, r, t) = \{y \in X: M(x, y, t > 1 - r\}$$

for $t > 0$.

Definition 2.9 ([7]). Let$(X, M, *)$ be a fuzzy metric space.

(1) A sequence $\{x_n\}$ in X converges to a point x in Xif$\lim_{n\to\infty} M(x_n, x , t)$=1 for all t$> 0$.

(2) A sequence $\{x_n\}$ in X is called a Cauchy sequence if $\lim_{n\to\infty} M(x_{n+p}, x_n, t)$=1 for all $t > 0$ and $p > 0$.

(3) A fuzzy metric space in which every Cauchy sequence is convergent is called complete.

3. COMPATİBLE MAPS OF TYPE (β)

In this section, we give the concept of compatible maps of type (β) and some properties of these maps in fuzzy metric spaces.

Definition 3.1 ([17]). Self maps A and B offuzzy metric space $(X, M, *)$ are said to be compatible if $\lim_{n\to\infty} M(ABx_n, BAx_n, t) = 1$ for all $t > 0$whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} A\, x_n$=$\lim_{n\to\infty} B\, x_n$= z, for some $z \in X$.

Definition 3.2 ([19]). Self maps A and B of fuzzy metric space $(X, M, *)$ are said to be compatible of type (β) if $\lim_{n\to\infty} M(AAx_n, BBx_n, t) = 1$ for all t>0 whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} A\, x_n$=$\lim_{n\to\infty} B\, x_n$= z, for some $z \in X$.

Proposition 3.3 ([19]). Let$(X, M, *)$ be a fuzzy metric space with t $* t \ge t$ for all $t \in [0,1]$ and A, B be continuous maps from X into itself. Then A and $B$are compatible if and only if they are compatible of type (β).

Proof. Suppose that A and B are compatible and let $\{x_n\}$ be sequence in X such that $\lim_{n\to\infty} A\, x_n = \lim_{n\to\infty} B\, x_n = z$ for some $z \in X$. Since A and B are continuous, we have

$\lim_{n\to\infty} AAx_n = \lim_{n\to\infty} ABx_n = Az$,

$\lim_{n\to\infty} BAx_n = \lim_{n\to\infty} BBx_n = Bz$.

Further, since A and B are compatible,

$$\lim_{n\to\infty} M(AB\,x_n, BAx_n, t) = 1$$

for all $t > 0$. Now, since

$M(AAx_n, BBx_n, t) \geq M(AAx_n, ABx_n, \frac{t}{2}) * M(ABx_n, BBx_n, \frac{t}{2})$,

$M(AAx_n, BBx_n, t) \geq M\left(AAx_n, ABx_n, \frac{t}{2}\right) * M\left(ABx_n, BAx_n, \frac{t}{4}\right) * M\left(BAx_n, BBx_n, \frac{t}{4}\right)$

for all $t > 0$, it follows that

$$\lim_{n\to\infty} M(AAx_n\,, BBx_n, t) \geq 1 * 1 * 1 \geq 1 * 1 \geq 1$$

which implies that

$\lim_{n\to\infty} M(AA\,x_n, BBx_n, t) = 1.$

Therefore, A and B are compatible of type (β).

Conversely, suppose that A and B are compatible type (β) and let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} A\,x_n = \lim_{n\to\infty} B\,x_n = z$ for some $z \in X$. Since A and B are continuous, we have

$\lim_{n\to\infty} AAx_n = \lim_{n\to\infty} ABx_n = Az$,

$\lim_{n\to\infty} BAx_n = \lim_{n\to\infty} BBx_n = Bz$.

Further, since A and B are compatible of type (β), we have, for all $t > 0$,

$$\lim_{n\to\infty} M(AAx_n\,, BBx_n, t) = 1.$$

Thus, from the inequality

$M(ABx_n, BAx_n, t) \geq M(ABx_n, AAx_n, \frac{t}{2}) * M(AAx_n, BAx_n, \frac{t}{2})$

$\quad \geq M\left(ABx_n, AAx_n, \frac{t}{2}\right) * M\left(AAx_n, BBx_n, \frac{t}{4}\right) * M\left(BBx_n, BAx_n, \frac{t}{4}\right)$,

it follows that

$$\lim_{n\to\infty} M(ABx_n\,, BAx_n, t) \geq 1 * 1 * 1 \geq 1 * 1 \geq 1$$

for all $t > 0$, which implies that

$$\lim_{n\to\infty} M(AB\,x_n, BAx_n, t) = 1.$$

Therefore, A and B are compatible. This completes the proof.

Proposition 3.4 ([19]). Let$(X, M,*)$ be afuzzy metric space with $t * t \geq t$ for all $t \in [0,1]$ and A, B be maps from X into itself. If A and B is compatible of type (β) and $Az = Bz$ for some $z \in X$, then $ABz = BBz = BAz = AAz$.

Proof. Suppose that $\{x_n\}$ is a sequence in X defined by $x_n = z$, for some $z \in X$ and $n = 1,2,3, \ldots$ and $Az = Bz$. Then we have

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = Az.$$

Since A and B be compatible of type (β), we have for all $t > 0$,

$$M(AAz, BBz, t) = \lim_{n\to\infty} M(AA\,x_n, BBx_n, t) = 1$$

So $ABz = BBz = BAz = AAz$. This completes the proof.

Proposition 3.5 ([19]). Let$(X, M,*)$ be a fuzzy metric space with $t * t \geq t$ for all $t \in [0,1]$ and A, B be maps from X into itself. Let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} Ax_n$=$\lim_{n\to\infty} Bx_n = z$ for some $z \in X$. Then we have the followings:

(i) $\lim_{n\to\infty} BBx_n$=Az if A is continuous at z,

(ii) $\lim_{n\to\infty} AAx_n$=Bz if B is continuous at z,

(iii) ABz=BAz and Az= Bz if A and B are continuous at z.

Proof.(1)Suppose that A is continuous at z. From $\lim_{n\to\infty} Ax_n = z$ for some $z \in X$, it follows that $\lim_{n\to\infty} AAx_n = Az$. Further since A and B are compatible of type (β), for all $t > 0$,

$$\lim_{n\to\infty} M(AAx_n, BBx_n, t) = 1.$$

Thus, from the inequality

$$M(BBx_n, Az, t) \geq M(BBx_n, AAx_n, \frac{t}{2}) * M(AAx_n, Az, \frac{t}{2})$$

for all $t > 0$, it follows that

$\lim_{n\to\infty} M(BBx_n, Az, t) \geq 1 * 1 \geq 1$.

Therefore, we have $\lim_{n\to\infty} BBx_n = Az$.

(2) The proof of $\lim_{n\to\infty} AAx_n = Bz$ follows from the same proof lines of (1).

(3) Suppose that A and B are continuous at z. Since A is continuous at z, by (1), we have $\lim_{n\to\infty} BBx_n = Az$. On the other hand, since $\lim_{n\to\infty} Bx_n = z$ for some $z \in X$ and B is continuous at z, $\lim_{n\to\infty} BBx_n = Bz$. Therefore, by the uniqueness of the limit, we have $Az = Bz$ and so, by Proposition 2, it follows that $ABz = BAz$. This completes the proof.

Example 3.6. Let$X = [0, \infty)$ with the metric d defined by $d(x, y) = |x - y|$ and $M(x, y, t)$=$\frac{t}{t+d(x,y)}$for all $x, y \in X$and $t > 0$. Clearly $(X, M,*)$ is a fuzzy metric space where $*$ is defined by $a * b = a.b$. Define $A, B: X \to X$ by $Ax = 1$for $x \in [0,1]$, $Ax = 1 + x$ for $x \in (1, \infty)$and $Bx = 1 + x$ for$x \in [1, \infty)$, $Bx = x$ for $x \in [1, \infty)$. Then both A and B are discontinuous at $x = 1$. Consider the sequence $\{x_n\}$in X defined by $x_n = \frac{1}{n}$, $n = 1,2, \ldots$ Then, we have $\lim_{n\to\infty} Ax_n$=$\lim_{n\to\infty} Bx_n$=1. Therefore A and B are compatible of type (β) but they are not compatible.

Example 3.7. Let$X = \mathbb{R}$ with the metric d defined by $d(x, y) = |x - y|$ and define $M(x, y, t)$=$\frac{t}{t+d(x,y)}$for all $x, y \in X$ and t>0.Clearly $(X, M,*)$ is a fuzzy metric space where $*$ is defined by $a * b = a.b$.Define $A, B: X \to X$by $Ax = \frac{1}{x^3}$, for $x \neq 0$, $Ax = 1$ for $x = 0$ and $Bx = \frac{1}{x^2}$ for $x \neq 0$, $Bx = 2$ for $x = 0$. Then both A and B are discontinuous at $x = 0$.Consider the sequence $\{x_n\}$ in X defined by $x_n = n$, $n = 1,2, \ldots$ Then, we have $\lim_{n\to\infty} Ax_n$=$\lim_{n\to\infty} Bx_n$=0. Therefore A and B are compatible but they are not compatible of type (β).

4. MAIN RESULTS

Theorem 4.1. Let $(X, M,*)$ be a complete fuzzy metric space with $t * t \geq t$ for all $t \in [0,1]$ and A, B, P, S, T and Q be maps from X into itself such that

(1) $P(ST)(X) \cup Q(AB)(X) \subset AB(ST)(X)$,

(2) There exists a constant k ∈ (0,1) such that

$M^2(Px,Qy, kt) * [M(ABx, Px, kt)M(STy, Qy, kt)]$+$\alpha M(STy, Qy, kt)M(ABx, Qy, 2kt) \geq$
$[pM(ABx, Px, t) + qM(ABx, STy, t)]M(ABx, Qy, 2kt)$

for all x, y in X and $t > 0$where $0 < p, q < 1, 0 \leq \alpha < 1$, such that $p + q - \alpha = 1$,

(3) $AB = BA, PB = BP, TQ = QT, ST = TS, AB(ST) = ST(AB)$,

(4) A, B, S and T are continuous,

(5) The pairs (P, AB) and (Q, ST) are compatible of type (β).

Then A, B, P, S, T and Q have unique common fixed point in X.

Proof. Let x_0 be an arbitrary point of X. By (1), we can construct a sequence $\{x_n\}$ in X as follows;

$P(ST)x_{2n} = AB(ST)x_{2n+1,}Q(AB)x_{2n+1} = AB(ST)x_{2n+2}$.

Now, let $z_n = AB(ST)x_n$.Then, by (2), we have

$M^2(P(ST)x_{2n},Q(AB)x_{2n+1}, kt) *$
$[M(AB(ST)x_{2n}, P, (ST)x_{2n}kt)M(ST(AB)x_{2n+1}, Q(AB)x_{2n+1}, kt)]$+
$\alpha M(ST(AB)x_{2n+1}, Q(AB)x_{2n+1}, kt)M(AB(ST)x_{2n}, Q(ST)x_{2n}, 2kt) \geq$
$[pM(AB(ST)x_{2n}, P(ST)x_{2n}, t) +$
qMABSTx2n,STABx2n+1,tM(ABSTx2n,QABx2n+1,2kt)

and

$M^2(AB(ST)x_{2n+1}, AB(ST)x_{2n+2}, kt) *$
$[M(z_{2n}, AB(ST)x_{2n+1}, kt)M(z_{2n+1}, AB(ST)x_{2n+2}, kt)]+$
$\alpha M(z_{2n+1}, AB(ST)x_{2n+2}, kt)M(z_{2n}, AB(ST)x_{2n+2}, 2kt) \geq [pM(z_{2n}, AB(ST)x_{2n+1}, t) +$
qMz2n,z2n+1,tMz2n,z2n+2,2kt.
Then
$M^2(z_{2n+1}, z_{2n+2}, kt) * [M(z_{2n}, z_{2n+1}, kt)M(z_{2n+1}, z_{2n+2}, kt)] +$
$\alpha M(z_{2n+1}, z_{2n+2}, kt)M(z_{2n}, z_{2n+2}, 2kt) \geq$
$[pM(z_{2n}, z_{2n+1}, t) + qM(z_{2n}, z_{2n+1}, t)]M(z_{2n}, z_{2n+2}, 2kt)$
and

$$M(z_{2n+1}, z_{2n+2}, kt) * M(z_{2n+1}, z_{2n+2}, kt) * [M(z_{2n}, z_{2n+1}, kt) * M(z_{2n+1}, z_{2n+2}, kt)] + \alpha M(z_{2n+1}, z_{2n+2}, kt)M(z_{2n}, z_{2n+2}, 2kt) \geq (p+q)M(z_{2n}, z_{2n+1}, t)M(z_{2n}, z_{2n+2}, 2kt).$$

So

$$M(z_{2n+1}, z_{2n+2}, kt)M(z_{2n}, z_{2n+2}, 2kt) + \alpha M(z_{2n+1}, z_{2n+2}, kt)M(z_{2n}, z_{2n+2}, 2kt) \geq (p+q)M(z_{2n}, z_{2n+1}, t)M(z_{2n}, z_{2n+2}, 2kt).$$

Thus, it follows that

$$M(z_{2n+1}, z_{2n+2}, kt) \geq \left(\frac{p+q}{1+\alpha}\right)M(z_{2n}, z_{2n+1}, t)$$

for$0 < k < 1$ and $t > 0$.Similary $P(ST)x_{2n+1} = AB(ST)x_{2n+2}$, $Q(AB)x_{2n+2} = AB(ST)x_{2n+3}$and $z_n = AB(ST)x_n$. We also have

$$M(z_{2n+2}, z_{2n+3}, kt) \geq M(z_{2n+1}, z_{2n+2}, t)$$

for$0 < k < 1$ and $t > 0$.In general, for $m = 1,2,3, \ldots$, we have

$$M(z_{m+1}, z_{m+2}, kt) \geq M(z_m, z_{m+1}, t)$$

for$0 < k < 1$ and $t > 0$. Hence, by Lemma 2.7, $\{z_n\}$ is a Cauchy sequence in X. Since $(X, M, *)$ is complete, it converges to a point z in X. Since $\{P(ST)x_{2n}\}$ and $\{Q(AB)x_{2n+1}\}$ are subsequences of $\{z_n\}$, $P(ST)x_{2n} \to z$ and $Q(AB)x_{2n+1} \to z$ as $n \to \infty$.
Let $y_n = STx_n$ and $w_n = ABx_n$ for n = 1,2,... Then, we have $y_{2n} \to z, ABy_{2n} \to z, STw_{2n+1} \to z$and $Qw_{2n+1} \to z$. Since the pairs $(P, AB)$and (Q, ST) are compatible of type (β), we have $M(P\,Py_{2n}, AB(AB)y_{2n}, t) \to 1$ and $M(Q\,Qw_{2n+1}, ST(ST)w_{2n+1}, t) \to 1$as $n \to \infty$. Moreover, by the continuity of A, B and Proposition 3.5, we have
$AB(P)y_{2n} \to ABz$ and$P\,Py_{2n} \to ABz$
as $n \to \infty$. Similary,
$ST(Q)\,w_{2n+1} \to STz$ve$Q\,Qw_{2n+1} \to STz$
as $n \to \infty$.Now, taking $x = y_{2n}$ and $y = Qw_{2n+1}$ in (2), we have
$M^2(Py_{2n}, Qw_{2n+1}, kt)[M(ABy_{2n}, Py_{2n}, kt)M(ST(Q)w_{2n+1}, QQw_{2n+1}, kt)]+$
$\alpha M(ST(Q)w_{2n+1}, QQw_{2n+1}, kt)M(ABy_{2n}, QQw_{2n+1}, 2kt) \geq$
$[pM(ABy_{2n}Py_{2n}, t) + qM(ABy_{2n}, ST(Q)w_{2n+1}, t)]M(ABy_{2n}, QQw_{2n+1}, 2kt)$
and
$M^2(z, STz, kt) * [M(z, z, kt)M(STz, STz, kt)]+\alpha M(STz, STz, kt)M(z, STz, 2kt) \geq [pM(z, z, t) +$
qMz,STz,tMz,STz,2kt.
Then, it follows that
$M^2(z, STz, kt\,)+ \alpha M(z, STz, 2kt) \geq [p + qM(z, STz, t]M(z, STz, 2kt)$.
Since $M(x, y, .)$ is non-decreasing for all x, y in X, we have

$$M(z, STz, 2kt)M(z, STz, t) + \alpha M(z, STz, 2kt) \geq [p + qM(z, STz, t)]M(z, STz, 2kt)$$

and

$$M(z, STz, t) + \alpha \geq p + qM(z, STz, t).$$

Thus

$$M(z,STz,t) - qM(z,STz,t) \geq p - \alpha$$
$$M(z,STz,t)[1-q] \geq p - \alpha$$
$$M(z,STz,t) \geq \frac{p-\alpha}{1-q} = 1$$

for all $t > 0$. So $z = STz$. Similary for$x = y_{2n}$ an $y = AB(Q)w_{2n+1}$ we have $z = ABz$.
Now, taking $x = y_{2n}$ and $y = z$ in (2), we have
$M^2(Py_{2n}, Qz, kt) * [M(ABy_{2n}, Py_{2n}, kt)M(STz, Qz, kt)]$
$\alpha M(STz, Qz, kt)M(ABy_{2n}, Qz, 2kt)$
$\geq [pM(ABy_{2n}Py_{2n}, t) + qM(ABy_{2n}, STz, t)]M(ABy_{2n}, Qz, 2kt)$
and
$M^2(z, Qz, kt) * [M(z, z, kt)M(z, Qz, kt)]$+$\alpha M(z, Qz, kt)M(z, Qz, 2kt) \geq [pM(z, z, t) +$
qMz,z,tMz,Qz,2kt.
Then
$M^2(z, Qz, kt) * [M(z, Qz, kt)]$+$\alpha M(z, Qz, kt)M(z, Qz, 2kt) \geq [p + q]M(z, Qz, 2kt)$.
So
$M(z, Qz, kt)[M(z, Qz, kt) * 1] + \alpha M(z, Qz, kt)M(z, Qz, 2kt) \geq (p + q)M(z, Qz, 2kt)$
and since $M(x, y, .)$ is non-decreasing for all x, y in X, we have
$M(z, Qz, 2kt)M(z, Qz, kt) + \alpha M(z, Qz, kt)M(z, Qz, 2kt) \geq (p + q)M(z, Qz, 2kt)$.
Thus it follows that
$M(z, Qz, kt)$+$\alpha M(z, Qz, kt) \geq p + q$
and

$$M(z, Qz, kt) \geq \frac{p+q}{1+\alpha} = 1$$

for$0 < k < 1$ and $t > 0$. So $z = Qz$. Similaryfor$x = (P)y_{2n}$, $y = z$ we have $z = Pz$.
Now, taking $x = Bz$ and $y = z$ in (2), we have
$M^2(P(B)z, Qz, kt)[M(AB(B)z, P(B)z, kt)M(STz, Qz, kt)] +$
$\alpha M(STz, Qz, kt)M(AB(B)z, Qz, 2kt) \geq$
$[pM(AB(B)z, P(B)z, t) + qM(AB(B)z, STz, t)]M(AB(B)z, Qz, 2kt)$
and
$M^2(BPz, Qz, kt) * [M(Bz, BPz, kt)M(z, z, kt)]$+$\alpha M(z, z, kt)M(Bz, z, 2kt) \geq [pM(Bz, BPz, t) +$
qMBz,z,tMBz,z,2kt.
Then
$M^2(Bz, z, kt) * M(Bz, Bz, kt)$+$\alpha M(Bz, z, 2kt) \geq [pM(Bz, BPz, t) + qM(Bz, z, t)]M(Bz, z, 2kt)$.
So
$M^2(Bz, z, kt)$+$\alpha M(Bz, z, 2kt) \geq [p + qM(Bz, z, t)]M(Bz, z, 2kt)$
and since $M(x, y, .)$ is non-decreasing for all x, y in X, we have
$M(Bz, z, 2kt)M(Bz, z, t)\alpha M(Bz, z, 2kt) \geq [p + qM(Bz, z, t)]M(Bz, z, 2kt)$.
Thus it follows that

$$M(Bz, z, t) + \alpha \geq p + qM(Bz, z, t)$$

and

$$M(Bz, z, t) - qM(Bz, z, t) \geq p - \alpha$$
$$M(Bz, z, t)[1-q] \geq p - \alpha$$
$$M(Bz, z, kt) \geq \frac{p-\alpha}{1-q} = 1$$

for$0 < k < 1$ and all $t > 0$. So $z = Bz$.Similary for$x = z, y = Tz$ we have $z = Tz$. Since $ABz = z$ and $STz = z$, we also have $Az = z$ and $Sz = z$. Therefore, z is a common fixed point of A, B, P, Q, S and T.

Let $v(v \neq z)$ be another common fixed point of A, B, P, Q, S and T. Then using inequality (2), we have

$M^2(Pz, Qv, kt) * [M(ABz, Pz, kt)M(STv, Qv, kt)] +$
$\alpha M(STv, Qv, kt)M(ABz, Qv, 2kt) \geq$
$[pM(ABz, Pz, t) + qM(ABz, STv, t)]M(ABz, Qv, 2kt)$.

So

$M^2(z, v, kt) * [M(z, z, kt)M(v, v, kt)]+\alpha M(v, v, kt)M(z, v, 2kt) \geq$
$[pM(z, z, t) + qM(z, v, t)]M(z, v, 2kt)$
$M^2(z, v, kt\ ) * M(z, v, 2kt) \geq [p + qM(z, v, t)]M(z, v, 2kt)$

and

$$M(z, v, t)M(z, v, 2kt) + \alpha M(z, v, 2kt) \geq [p + qM(z, v, t)]M(z, v, 2kt).$$

Thus, it follows that

$$M(z, v, 2kt)[M(z, v, t) + \alpha] \geq [p + qM(z, v, t)]M(z, v, 2kt)$$
$$M(z, v, t) - qM(z, v, t) \geq p - \alpha$$
$$M(z, v, t)[1 - q] \geq p - \alpha$$
$$M(z, v, t) \geq \frac{p - \alpha}{1 - q} = 1$$

for all $t > 0$. So $z = v$. Hence A, B, P, S, T and Q have unique common fixed point.

If we put $\alpha = 0$ in Theorem 4.1, we have the following result:

Corollary 4.2. Let $(X, M,*)$ be a complete fuzzy metric space with $t * t \geq t$ for all $t \in [0,1]$ and A, B, P, S, T and Q be maps from X into itself such that the conditions (1), (3), (4) and (5) of the Theorem 4.1 hold and there exists a constant $k \in (0,1)$ such that

$M^2(Px,Qy, kt) * [M(ABx, Px, kt)M(STy, Qy, kt)] \geq$
$[pM(ABx, Px, t) + qM(ABx, STy, t)]M(ABx, Qy, 2kt)$

for all x, y in X and $t > 0$ where $0 < p, q < 1$ such that $p + q = 1$. Then A, B, P, S, T and Q have unique common fixed point in X.

If we put $B = T = I$ (the identity map on X) in Theorem 4.1, we have the following result in Kütükcü et al. ([20]);

Corollary 4.2. Let $(X, M,*)$ be a complete fuzzy metric space with $t * t \geq t$ for all $t \in [0,1]$ and A, B, P, S, T and Q be maps from X into itself such that

(1) $PS(X) \cup QA(X) \subset AS(X)$

(2) there exists a constant $k \in (0,1)$ such that

$M^2(Px,Qy, kt$
$) * [M(Ax, Px, kt)M(Sy, Qy, kt)]+\alpha M(Sy, Qy, kt)M(Ax, Qy, 2kt) \geq$
$[pM(Ax, Px, t) + qM(Ax, Sy, t)]M(Ax, Qy, 2kt)$

for all x, y in X and $t > 0$ where $0 < p, q < 1$, $0 \leq \alpha < 1$ such that $p + q - \alpha = 1$,

(3) A, S are continuous and $AS = SA$,

(4) the pairs (P, A) and (Q, S) are compatible of type (β).

Then A, P, S and Q have unique common fixed point in X.

If we put $A = S, B = T$ and $P = Q$ in Theorem 4.1, we have the following result:

Corollary 4.3. Let $(X, M,*)$ be a complete fuzzy metric space with $t * t \geq t$ for all $t \in [0,1]$ and A, B and P be maps from X into itself such that

(1) $P(X)\ \subset\ AB(X)$,

(2) there exists a constant $k \in (0,1)$ such that

$M^2(Px, Py, kt) * [M(ABx, Px, kt)M(ABy, Py, kt)] + \alpha M(ABy, Py, kt)M(ABx, Py, 2kt) \geq [pM(ABx, Px, t) + qM(ABx, ABy, t)]M(ABx, Py, 2kt)$

for all x, y in X and $t > 0$ where $0 < p, q < 1$, $0 \leq \alpha < 1$ such that $p + q - \alpha = 1$,

(3) $AB = BA$,

(4) A and B is continuous

(5) the pair (P, AB) is compatible of type (β).

Then A, B and P have unique common fixed point in X.

The following example illustrates our main Threorem.

Example 4.4. Let $X = \left\{\frac{1}{n} : n \in N\right\} \cup \{0\}$with the metric d defined by $d(x, y) = |x - y|$and

$$M(x, y, t) = \frac{t}{t + d(x, y)}$$

for all $x, y \in X$ and$t > 0$. Clearly,$(X, M, *)$ is a complete fuzzy metric space where $*$ is defined by $a * b = a.b$. Let A, B, P, S, T and Q be maps from X into itself defined as $Ax = x, Bx = \frac{x}{2}, Px = \frac{x}{4}, Sx = \frac{x}{5}, Tx = \frac{x}{6}, Qx = 0$ for all $x \in X$. Then

$P(ST)(X) \cup Q(AB)(X) = \left\{\frac{1}{120n} : n \in N\right\} \cup \{0\} \subset \left\{\frac{1}{60n} : n \in N\right\} \cup \{0\} = AB(ST)(X)$.

Clearly,$AB = BA, PB = BP, TQ = QT, ST = TS, AB(ST) = ST(AB)$ and A, B, S, T are continuous.If we take $k = \frac{1}{2}$and $t = 1$, we see that the condition (2) of the main theorem is also satisfied. Moreover, the maps P and AB are compatible of type (β) if $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} ABx_n = 0$for $0 \in X$. Similary, the maps Q and ST are also compatible of type (β). Thus all the conditions of main of main theorem are satisfied and 0 is the unique common fixed point of $A, B, P, S, T$and Q.

5. REFERENCES

[1]. Zadeh, L.A., 1965. Fuzzy sets, *Inform ve Control*. 8, 338-353.

[2]. Deng, Z.K., 1982. Fuzzy pseduo-metric spaces, *J.Math. Anal. Appl.* 86, 74-95.

[3]. Erceg, M. A., 1979.Metric spaces in fuzzy set theory, *J.Math.Anal.Appl.* 69, 205-230.

[4]. Kaleva, O., Seikala, S., 1984. On fuzzy metric spaces,*Fuzzy Sets and Systems*. 12, 215-229.

[5]. Kromosil, O., Michalek, J., 1975. Fuzzy metric and statiscal metric spaces, *Kybernetika (Praha).* 11, 326-334.

[6]. George, A., Veeramani, P., 1994. On some results in fuzzy metric spaces, *Fuzzy Sets and Systems*. 64, 395-399.

[7]. Grabiec, M., 1988. Fixed points in fuzzy metric space,*Fuzzy Sets ve Systems*. 27, 385-389.

[8]. Banach S., 1932.Theorie les operations linearies,*ManograieMathematyezne*, Warsaw,Poland.

[9]. Edelstein, M., 1962. On fixed and periodic points under contraction mappings, *J.London Math.Soc.*37, 74-79.

[10]. Fang, X. J., 1992. On fixed point theorems in fuzzy metric spaces, *Fuzzy Sets and Systems*. 46, 107-113.

[11]. Hadzic, O., Pap, E., 2001. Fixed point theory in probabilistic metric spaces, *Kluwer Acad.Publ.*

[12]. Hadzic, O., Pap. E., 2002.A fixed point theorem for multivalued mappins in probabilistic metric spaces and an application in fuzzy metric spaces, Fuzzy sets and sytems. 127, 333-334.

[13]. Istratescu, I., 1981.A fixed point theorem for mappings with a probabilistic contractive iterate,*Rev.Roumaire Math.* Pure appl. 26, 431-435.

[14]. Schweizer, B., Sklar, A. 1960.Statistical Metric Spaces, *Pacific Journal of Mathematics.*(10); 1, 313-334.

[15]. Sessa, S., 1982. On weak commutativity condition of mappings fixed point considerations, *Publ Inst.Math.Beagrad*. 32: (46), 149-153.

[16]. Jungck, G., 1976. Commuting mappings and fixed points, *Amer.Math.Monthly.* 83, 261-263.

[17]. Mishra, S. N., Sharma, N., Singh, S.L., 1994. Common fixed points of maps fuzzy metric spaces, *İnternat.J.Math.Sci.* 17, 253-258.

[18]. Pathak, H. K., Cho, Y. J., Chang, S. S., Kang, S. M.,Compatible mappings of type (P),*Rev.Res. Univ. Novi Sad., to appear.*

[19]. Cho, Y. J., 1997.Fixed points in fuzzy metric spaces, *J.Fuzzy Math.* 4, 949-962.

[20]. Kütükcü, S., Turkoglu,D., Yıldız, C., 2006.Common fıxed points of compatible (β) on fuzzy metric spaces, *Commun.Korean Math.Soc.*21, 89-100.

[21]. Rhoades, B. E., 1977.A comparison of various definitions of contractive mappings.*Trans Amer. Math. Soch.* 226, 257-290.

[22]. Sharma,S., Deshpande,B., 2002.Common fixed points of compatible maps of type (β) on fuzzy metric spaces, *Demonstratio MathXXXV.* 1, 165-174.