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RESEARCH ARTICLE

A MANPOWER MODEL WHEN COMPLETE LENGTH OF SERVICE FOLLOWS MIXED RIGHT TRUNCATED EXPONENTIAL DISTRIBUTION

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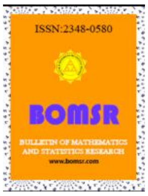

ABSTRACT

In this paper a manpower model when Complete Length of Service (CLS) follows mixed right truncated exponential distribution was considered, as it best suited to the practical situations prevailing in many of the private sector organizations. The behaviour of the model characteristics are obtained and analyzed through numerical illustrations and graphs. The results indicate that mixture right exponential distribution fits the data better compare to than the mixed exponential distribution, particularly at the low values of CLS.

Keywords: Manpower Models, Manpower planning, Truncated distributions, Exponential distributions, Truncated Distributions, mixed failure models.

1. Introduction

A manpower model is a representation of the physical phenomena of the manpower existing in the organization regarding. The manpower models generate the mechanism of flows such as recruitment, promotion, demotion, retirement, wastage etc., which are having stochastic in nature. Therefore the statistical treatment of manpower system through stochastic modelling is inevitable in order to have efficient design and accurate prediction of the manpower system. The manpower model reveals that most of the manpower models have been analyzed through its Complete Length of Service (CLS) distribution of the organization. The CLS distribution is determined by push and pulls flows which are stochastic.

Most of the research on manpower models deals with positively skewed CLS distributions having various infinite range distributions. However, if the length of service of an employee is relatively small, then estimating manpower models with infinite range distributions may not yields optimal results. In such situations, truncated distributions are best suited for estimating the CLS of the organization. In any organization, it is observed/ experienced that the CLS is influenced by the environmental factors existing in the organization as well as psychological behaviour/factor of the individuals. These are other independent factors that influence to continue in the organization or leave the organization. Hence the mixed effect of these factors will decide CLS. Therefore, CLS is best estimated by mixed right truncated distribution. Also since exponential distributions are best suited for estimating CLS in general, in these paper mixtures truncated exponential distribution was considered for estimating the CLS of the organization. The behaviour of the model characteristics are obtained and analyzed through numerical illustrations

2. Review of Literature

The lifetime data such as employees' entry and leaving in an organization plays an important role in manpower planning in order to take precautionary measures in any organization. The lifetime distributions have been extensively used in various applications e.g. agriculture, engineering Human resource management, health etc. to study data arising from time to unit failure. The exponential distribution is widely used model for survival times research (Epstein, 1958). The truncated exponential distribution may occur in various ways (Bain and Gao 1996), (Joshi 1979). Saralees, et al, (2006) developed various models using different truncated distribution by using 'R-language'. Faris (2008) discussed on the properties of the mean estimation of the truncated exponential distribution. Rao and Murthy (2016) discussed a manpower model using right truncated exponential distribution and estimated their properties.

There is a need to study the problems related to mixed failure distributions to investigate the lifetime data. Mixture models are used in statistics for estimation of problems (Robins, 1980, Blum and Susarla 1977). Seacock (1954) first proposed the mixed exponential distributions. To describe the distribution of the Complete Length of Service on leaving Bartholomew (1959, 1971) suggested mixed exponential distribution. Mixture exponential models are effective for a relatively small number of values (Congdon, 2003; Fruhwirth-Schnatter; 2006).In lifetime data analysis, mixed exponential distributions play an important role (Zhang, 2015). Zhang (2015) discussed the estimate of the parameters for the three mixed exponential distribution.

3. Manpower planning model with Mixed Right Truncated Exponential Distribution

In this paper, Manpower Planning model when CLS follows Mixed Right Truncated distributions was proposed and its characteristics were studied such as probability density function, the cumulative distribution function, survival function, Renewal equation and analyzed through numerical illustrations and graphs.

3.1 Assumptions

1. Let 'T', denote the Complete Length of Service (CLS) and it is continuous non-negative random variable
2. Individuals leaving the organization are considered as wastage and it is independent and identically distributed random variables
3. The Complete Length of Service (CLS) or life lengths are assumed to be independent and identically distributed
4. The Complete Length of Service (CLS) of an employee in the organization follows a Mixed Right Truncated Exponential Distribution (MRTED).

3.2 Notations

f(t): Probability Density Function (PDF) of the CLS

F(t): Cumulative Distribution Function CDF of the CLS

A: Truncated value of't', t will be truncated on the right at 'A', a known constant

S (t): Reliability or Survival function is 1 - F (t) = Pr (T > ***t)*** for t > 0. The probability that the employee serves in the organization in the time interval (0, t] and is still serving in the organization at time t.

R (t): $f(t)+\frac{F^2(t)}{\int_0^t S(t).dt}$, Renewal equation or Renewal Density

E (t): Mean Time to Failure (MTTF) i.e. means (expected) value of t λ_1 and λ_2 are failure rates

3.3 Manpower planning models having Mixed Right Truncated Exponential Distribution

Using the notations mentioned in section 3.1 and under the aassumptions specified in section 3.2, various characteristics of Mixed Right Truncated Exponential Distribution **(**MRTED) are obtained and their behavioral characteristics were through numerical illustrations and graphs. These were given in Table 1 and diagrammatically represented in figure 1 to figure 4.

The following are the different characteristics of the manpower model when Complete Length of service (CLS) distribution of an organization follows the mixture of the right truncated exponential distribution (MRTED) with parameters λ_1 and λ_2.

The Density function is

$$f(t) = p\ \frac{\lambda_1\, e^{-\lambda_1 T}}{1-e^{-\lambda_1 A}} + (1-p)\frac{\lambda_2\, e^{-\lambda_2 T}}{1-e^{-\lambda_2 A}} \quad ,\text{where } \lambda_1 , \lambda_2>0 , 0< T\leq A, 0< p<1 \qquad (1)$$

The Distribution function is

$$F(t) = p\ \frac{1-e^{-\lambda_1 T}}{1-e^{-\lambda_1 A}} + (1-p)\frac{1-e^{-\lambda_2 T}}{1-e^{-\lambda_2 A}} \qquad (2)$$

The Mean Time to Failure is

$$E(t) = p\ \left[\frac{1}{\lambda_1} - \frac{A\, e^{-\lambda_1 T}}{1-e^{-\lambda_1 A}}\right] + (1-p)\left[\frac{1}{\lambda_2} - \frac{A\, e^{-\lambda_2 T}}{1-e^{-\lambda_2 A}}\right] \qquad (3)$$

The Survival or Reliability function is

$$S(t)= \frac{[(1-e^{-\lambda_1 A})(1-e^{-\lambda_2 A}) - p(1-e^{-\lambda_2 A})(1-e^{-\lambda_1 T}) - (1-p)(1-e^{-\lambda_1 A})(1-e^{-\lambda_2 T})]}{(1-e^{-\lambda_1 A})(1-e^{-\lambda_2 A})} \qquad (4)$$

The Renewal density is

$$R(t) = \frac{1}{r_1}\left[r_2 + \frac{{r_3}^2}{r_4}\right] \qquad (5)$$

Where,

$r_1 = \left(1-e^{-\lambda_1 A}\right)\left(1-e^{-\lambda_2 A}\right)$

$r_2 = p\, \lambda_1\ e^{-\lambda_1 T}\left(1-e^{-\lambda_2 A}\right) + \ (1-p)\, \lambda_2\, e^{-\lambda_2 T}\left(1-e^{-\lambda_1 A}\right)$

$r_3 = p\, \left(1-e^{-\lambda_1 T}\right)\left(1-e^{-\lambda_2 A}\right) + \ (1-p)\left(1-e^{-\lambda_2 T}\right)\left(1-e^{-\lambda_1 A}\right)$

$r_4 = T\, \left(1-e^{-\lambda_1 A}\right)\left(1-e^{-\lambda_2 A}\right) - p\left(1-e^{-\lambda_2 A}\right)\left(T - \frac{1-e^{-\lambda_1 T}}{\lambda_1}\right) - (1-p)\left(1-e^{-\lambda_1 A}\right)\left(T - \frac{1-e^{-\lambda_2 T}}{\lambda_2}\right)$

4. Results and Discussions

The numerical calculation of the characteristics of the manpower planning models having Mixed Right Truncated Exponential Distribution (MRTED) as CLS for various values of λ_1 , λ_2 , p and 'A' are given in Table 1 to Table 4. The behaviour of this distribution is also analyzed with the help of graphs given from figure 1 to figure 4.

Table 1 gives values of time, when λ_1 =1 & 3 and other parapets are fixed at λ_2 =0.5, p=0.5 and A=5 and 10. Table 2 gives values of time, when λ_2 =1 & 3 and other parapets are fixed at λ_1 =0.5, p=0.5 and A=5 and 10. Table 2 gives the CLS studied when p value is fixed and other parameters are changed. The behaviour of Mixed Right Truncated Exponential Distribution (MRTED) for the values given in Table 1 & 2 and Figures 1 and 2. From Table 1 and Figure 1 , it was observed that ,the renewal density R(t) increasing for small values of 'A' and it is constant when 'λ_1' increases, as 'A' increases the renewal density is showing decreasing and then after some it is increasing very slowly. The result is same in case of fixed and changing values for 'λ'. The same result found for Table 2 and figure 2 also.

Table 3 gives values of time, when p=0.05 and 0.8 and other parapets are fixed at λ_1 =0.5 λ_2 =0.5 and A=5 and 10. Table 4 gives values of time, when λ_2 =0.5 and other parapets are fixed at λ_1 =0.5, p=0.5 and A=5 and 10. In Tables 1 & 2 , the CLS studied when p value is changed and other parameters are fixed. The behaviour of CLS of MRTED studied through the values given in Table 3 & 4 and Figures 3 and 4, From Table 3 and Figure 3, it was observed that, the renewal density is showing increasing for small values of 'A' and it is constant when 'λ' value increases. The renewal density is showing decreasing and then after some it is increasing very slowly. The result is same in case of fixed and changing values for 'λ' and p. The value of p increases the graph shows that renewal density is also increasing. The Table 4 and Figure 4 show that renewal density is increasing for smaller and larger values of 'A'. It is also increasing when P values changes and other parameters are fixed.

The results show that there is a significant change in Model characteristics of Manpower model having CLS as Mixed Right Truncated Exponential Distribution (MRTED), for an example for larger values of 'A', the renewal equation gives λ. Therefore, this paper concluded that there is a significant difference in truncated distributions as compared to the non-truncated distributions; particularly for smaller values of 'A' .This type of situation is very common in private organizations, where the leaving of employees is common as compared to the government or public sector organizations.

5. Conclusions

In this paper, Man power Planning Model when the CLS follows Mixed Right Truncated Exponential Distribution (MRTED) with the assumption that the length of service of an employee in an organization is relatively small was developed and various life time characteristics are obtained. The model is further analysed through numerical results and figures. The results indicates that mixture right exponential distribution fits the data better compare to than the mixed exponential distribution, particularly at the low values of CLS.

Table 1: The values of Renewal density for fixed p and for different values of λ_1 and, λ2 and for different values of the truncated parameter A.

	λ_1= 1,λ_2= 0.5, A=5,p=0.5	**λ_1= 1,λ_2= 0.5, A=10,p=0.5**	**λ_1= 3,λ_2= 0.5, A=5,p=0.5**	**λ_1= 3,λ_2= 0.5, A=10,p=0.5**
0.5	0.762	0.727	1.291	1.243
1	0.756	0.71	1.116	1.05
1.5	0.756	0.698	1.054	0.972
2	0.76	0.691	1.032	0.935
2.5	0.766	0.686	1.027	0.915
3	0.774	0.683	1.031	0.902
3.5	0.784	0.681	1.032	0.894
4	0.795	0.68	1.031	0.889
4.5	0.807	0.679	1.031	0.885
5	0.82	0.679	1.031	0.883
5.5		0.679		0.881
6		0.679		0.88
6.5		0.68		0.88
7		0.68		0.88
7.5		0.681		0.88
8		0.681		0.881
8.5		0.682		0.882

9		0.682		0.883
9.5		0.683		0.883
10		0.684		0.885

Table 2: The values of Renewal density for fixed p and for different values of λ_1 and, λ2 and for different values of the truncated parameter A.

T	λ_1= 0.5,λ_2= 1, A=5,p=0.5	λ_1= 0.5,λ_2= 1, A=10,p=0.5	λ_1= 0.5,λ_2= 3, A=5,p=0.5	λ_1= 0.5,λ_2= 3, A=10,p=0.5
0.5	0.762	0.727	1.291	1.243
1	0.756	0.71	1.116	1.05
1.5	0.756	0.698	1.054	0.972
2	0.76	0.691	1.032	0.935
2.5	0.766	0.686	1.027	0.915
3	0.774	0.683	1.031	0.902
3.5	0.784	0.681	1.039	0.894
4	0.795	0.68	1.051	0.889
4.5	0.807	0.679	1.066	0.885
5	0.82	0.679	1.083	0.883
5.5		0.679		0.881
6		0.679		0.88
6.5		0.68		0.88
7		0.68		0.88
7.5		0.681		0.88
8		0.681		0.881
8.5		0.682		0.882
9		0.682		0.883
9.5		0.683		0.883
10		0.684		0.885

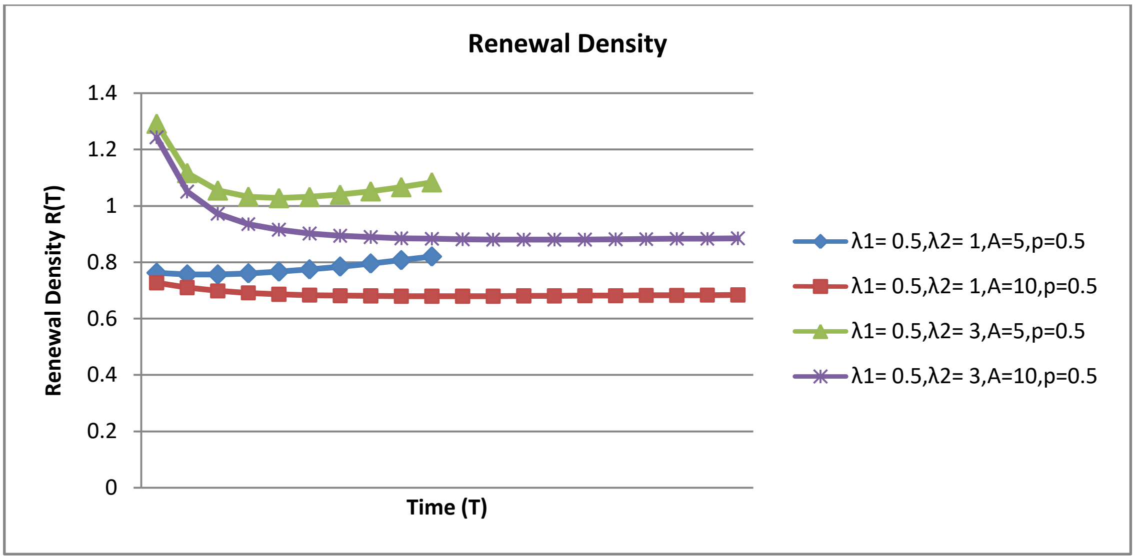

Figure 2: A graph of Renewal Density when p=0.5, A=5 and 10 , λ_1=0.5 ,λ_2=1 and 3

Table 3. The values of Renewal density for different values of p and for fixed values of of λ_1 and, λ2 and for different values of the truncated parameter A.

T	λ_1= 0.05,λ_2=0.5 , A=5,p=0.05	λ_1= 0.05,λ_2=0.5 , A=10,p=0.05	λ_1= 0.05, λ_2=0.5, A=5,p=0.8	λ_1= 0.05, λ_2=0.5, A=10,p=0.8
0.5	0.539	0.482	0.302	0.196
1	0.551	0.481	0.318	0.193
1.5	0.563	0.479	0.337	0.191
2	0.576	0.478	0.36	0.191
2.5	0.59	0.477	0.386	0.193
3	0.605	0.476	0.417	0.195
3.5	0.621	0.476	0.453	0.198
4	0.639	0.476	0.494	0.203
4.5	0.658	0.476	0.542	0.208
5	0.678	0.476	0.599	0.214
5.5		0.477		0.22
6		0.478		0.227
6.5		0.479		0.235
7		0.48		0.243
7.5		0.481		0.252
8		0.483		0.262
8.5		0.485		0.272
9		0.487		0.284
9.5		0.489		0.296
10		0.491		0.309

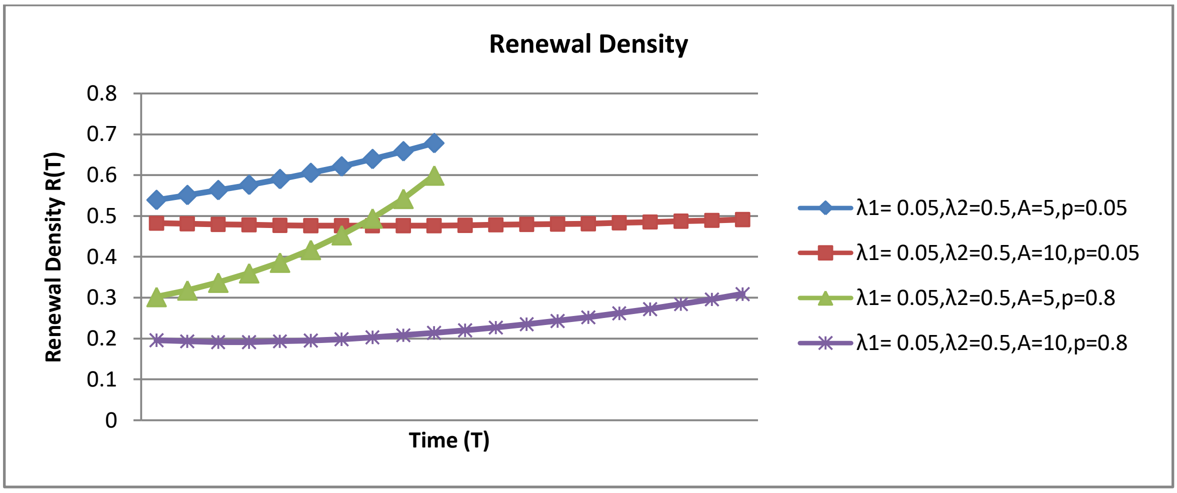

Figure 3: A graph of Renewal Density when p=0.05 and 0.8, A=5 and 10 , λ_1=0.05 ,λ_2=0.5

Table 4. The values of Renewal density for different values of p and for fixed values of λ1 and, λ2 and for different values of the truncated parameter A.

T	λ_1= 0.5,λ_2=0.05, A=5,p=0.05	λ1= 0.5,λ2=0.05, A=10,p=0.05	λ_1= 0.5, λ_2=0.05, A=5,p=0.8	λ1= 0.5,λ2=0.05, A=10,p=0.8
0.5	0.26	0.148	0.488	0.419
1	0.281	0.15	0.496	0.412
1.5	0.305	0.154	0.507	0.405
2	0.331	0.158	0.52	0.4
2.5	0.362	0.163	0.536	0.396
3	0.396	0.168	0.553	0.393
3.5	0.435	0.174	0.573	0.391
4	0.48	0.18	0.596	0.389
4.5	0.533	0.187	0.622	0.389
5	0.594	0.194	0.652	0.389
5.5		0.201		0.39
6		0.209		0.391
6.5		0.218		0.393
7		0.227		0.396
7.5		0.237		0.4
8		0.248		0.403
8.5		0.259		0.408
9		0.271		0.413
9.5		0.284		0.418
10		0.298		0.424

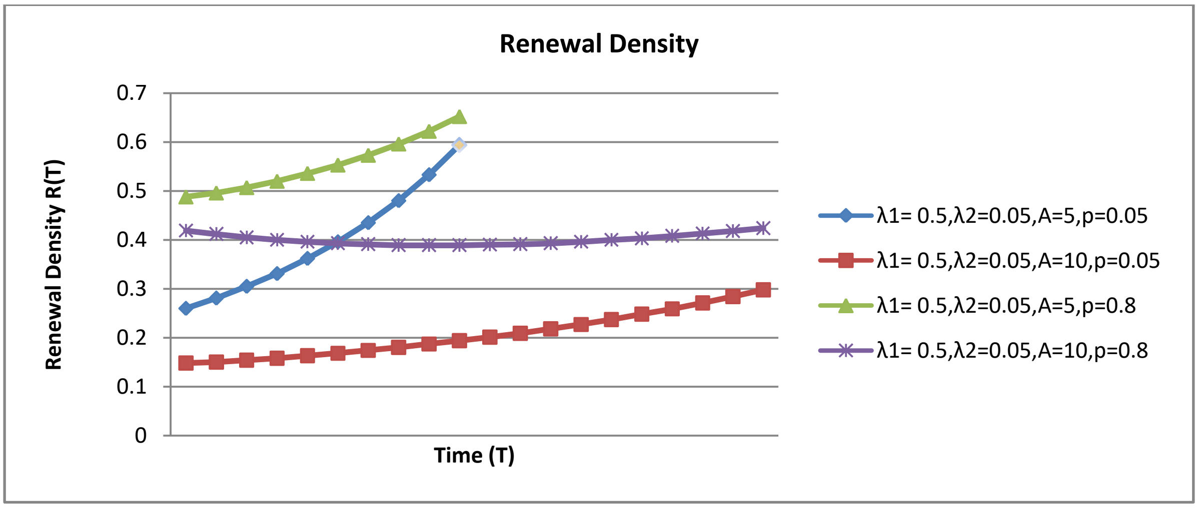

Figure 4: A graph of Renewal Density when p=0.05 and 0.8, A=5 and 10 , λ1=0.5 ,λ2=0.05

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