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GENERALIZED SQUARE ROOT TRANSFORMATION AND EXPONENTIAL RATIO TYPE ESTIMATORS FOR FINITE POPULATION MEAN IN SAMPLE SURVEY

SITANSHU SEKHAR MISHRA
Lecturer in Statistics, Department of Statistics, Salipur (Autonomous) College, Salipur, Cuttack
Odisha, India
E-mail: sitanshumishra0@gmail.com

ABSTRACT

In this paper, a general class of exponential ratio type estimators have been proposed for estimating the population mean of the study variable under simple random sampling scheme. Expressions for the bias and MSE of the proposed classes of estimators have been derived upto first order approximation. The optimum MSE and bias of the proposed estimators have been obtained. The efficiencies of the proposed estimators have been compared theoretically and with numerical illustrations.

Keywords : Ratio estimator, Auxiliary variable, Bias, Exponential estimator, Mean square error, Efficiency.

AMS Classification : 62D05

1. Introduction

The use of auxiliary information is commonly adopted in sample surveys to achieve higher precision for the estimates of some population parameters such as population total, population mean, population ratio, etc. In survey sampling when study variable '*y*' is highly positively correlated with the auxiliary variable (*x*), we use ratio estimator and when it is negatively correlated, we use product estimator for estimating population parameters.

In survey sampling literature, a great variety of techniques using auxiliary information by means of ratio, product and regression methods have been used. Recent development in ratio and product methods of estimation along with their variety of modified forms are lucidly described by several authors etc. The development is continued in the form of exponential estimators for ratio and product type estimators such as Bahl and Tuteja (1991), Singh and Vishwakarma (2007), Noor Ul Amin and Hanif (2012), Sanaullahet al. (2014) among others. The aim of this paper is to develop a generalized square root transformation ratio type estimator proposed by Swain (2014) with the alternative exponential estimator proposed by Swain (2013) as regards bias and efficiency.

Let $U = \{U_1, U_2, ..., U_N\}$ be a finite population of size *N*. To each unit $U_i, (i = 1, 2, ..., N)$ in the population paired values (y_i, x_i) corresponding to the study variable *y* and an auxiliary variable *x*, correlated with the study variable *y* are attached. Let $(\bar{Y}, \bar{X})$ be the population means of the study variable *y* and auxiliary variable *x* respectively and is defined as:

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i \ , \ \bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i \ .$$

Thus the population ratio is defined as $R = \frac{\bar{Y}}{\bar{X}}$. Further, define the finite population variances of *y* and *x* and their covariance as

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2 \ , \ S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{X})^2 ,$$

$S_{yx} = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})(x_i - \bar{X})$ respectively.

Also, the coefficient of variations of *y*, *x* and their coefficient of covariations are defined by

$C_y = \frac{S_y}{\bar{Y}}$, $C_x = \frac{S_x}{\bar{X}}$, and $C_{yx} = \frac{S_{yx}}{\bar{Y}\,\bar{X}} = \rho_{yx} C_y C_x$, respectively.

where 'ρ' is the simple correlation between *y* and *x*.

A simple random sample '*s*' of size *n* is selected from *U* without replacement and the values $(y_i, x_i), i = 1, 2, ..., n$ are observed on the sampled units. Assume that *y* and *x* are positively correlated.

The classical ratio estimator of the population mean $\bar{Y}$, using auxiliary information on *x* is given by

$$\bar{y}_R = \frac{\bar{y}}{\bar{x}}\bar{X} . \tag{1.1}$$

where $\bar{y}$ and $\bar{x}$ are sample means of *y* and *x* respectively, defined by

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \ , \ \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \ .$$

The ratio estimator $\bar{y}_R$ envisages advance knowledge of $\bar{X}$.

To $O\left(\frac{1}{n}\right)$,

$B(\bar{y}_R) = \theta\bar{Y}\left(C_x^2 - C_{yx}\right)$ (seeSukhatme, et al., 1970) (1.2)

and $MSE(\bar{y}_R) = \theta\bar{Y}^2\left(C_y^2 + C_x^2 - 2C_{yx}\right)$ (see Sukhatme, et al, 1970) (1.3)

where $\theta = \left(\frac{1}{n} - \frac{1}{N}\right)$.

The ratio estimator $\bar{y}_R$ is a biased estimator and the bias decreases with increase in sample size.

In large samples $\bar{y}_R$ is more efficient than the simple mean per unit estimator $\bar{y}$ if $K > \frac{1}{2}$, where

$$K = \rho \frac{C_y}{C_x}.$$

Srivastava (1967) proposed a modified ratio estimator using power transformation as

$$\bar{y}_{SR} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha} \tag{1.4}$$

where α is a real constant to be suitably chosen.

Bahl and Tuteja (1991) proposed a ratio type exponential estimator as

$$\bar{y}_{BTR} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right). \tag{1.5}$$

Swain (2014) proposed a square root transformation ratio type estimator as

$$\bar{y}_{SQR} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)^{\frac{1}{2}}. \tag{1.6}$$

Swain (2014) has shown that the Srivastava's (1967) estimator with $\alpha = \frac{1}{2}$ has the same bias and efficiency as that of the Bahl-Tuteja's (1991) estimator to $O(\frac{1}{n})$.

Swain (2013) proposed a generalized exponential ratio type estimator as

$$\bar{y}_{sw} = \bar{y}\, e^{\alpha\left(\frac{\bar{X}-\bar{x}}{\bar{X}}\right)}. \tag{1.7}$$

where α is a real number to be suitably chosen.

To $O(\frac{1}{n})$, the bias and mean square error of $\bar{y}_{sw}$ are given by

$$B(\bar{y}_{sw}) = \theta\bar{Y}\left(\frac{\alpha^2}{2}C_x^2 - \alpha C_{yx}\right). \tag{1.8}$$

$$MSE(\bar{y}_{sw}) = \theta\bar{Y}^2\left(C_y^2 + \alpha^2 C_x^2 - 2\alpha C_{yx}\right). \tag{1.9}$$

To $O(\frac{1}{n})$, $\bar{y}_{SR}$ and $\bar{y}_{sw}$ are equally efficient and $\bar{y}_{SR}$ is less biased than $\bar{y}_{sw}$ if

$$K > \frac{(2\alpha+1)}{4}. \tag{1.10}$$

For optimum α, the above inequality reduces to $K > \frac{1}{2}$.

In the following we suggest a more generalized square root transformation ratio type estimator and exponential ratio type estimator and compare it as regards bias and mean square error.

2. Proposed Estimators:

Define the generalized square root transformation estimator

(i) $$t_{g1} = \bar{y}\left[\alpha\left(\frac{\bar{X}}{\bar{x}}\right)^{\frac{1}{2}} + (1-\alpha)\left(\frac{\bar{x}}{\bar{X}}\right)^{\frac{1}{2}}\right] \tag{2.1}$$

where, α is a real constant to be suitably chosen.

Further, the generalized exponential estimator is defined as

(ii) $$t_{g_2} = \bar{y}\left[\alpha\, e^{\left(\frac{\bar{X}-\bar{x}}{\bar{X}}\right)} + (1-\alpha)\, e^{\left(\frac{\bar{x}-\bar{X}}{\bar{X}}\right)}\right]. \qquad (2.2)$$

Let $\bar{y} = \bar{Y}(1+e_0)$.

$\bar{x} = \bar{X}(1+e_1)$,

where, $E(e_0) = E(e_1) = 0$, $V(e_0) = \theta C_y^2, V(e_i) = \theta C_x^2$

$COV(e_0\, e_1) = \theta\, C_{yx}$.

Bias and Mean square Error of t_{g1}:

Expanding t_{g1} in power series and keeping upto second-degree terms we have,

$$t_{g1} = \bar{Y}\left(1 + \frac{1}{2}e_1 - \frac{1}{8}e_1^2 - \alpha e_1 + \frac{1}{2}\alpha e_1^2 + e_0 + \frac{1}{2}e_0 e_1 - \frac{1}{8}e_0 e_1^2 - \alpha e_0 e_1 + \frac{1}{2}e_1^2 e_0\right)$$

$$= \bar{Y}\left(1 + \left(\frac{1}{2} - \alpha\right)e_1 + \left(\frac{\alpha}{2} - \frac{1}{8}\right)e_1^2 + e_0 + \left(\frac{1}{2} - \alpha\right)e_0 e_1 - \frac{1}{8}e_0 e_1^2 + \frac{1}{2}\alpha e_1^2 e_0\right)$$

Thus the bias of t_{g1} to $O(\frac{1}{n})$ is given by

$$B(t_{g1}) = \bar{Y}\left[\left(\frac{\alpha}{2} - \frac{1}{8}\right)E\left(e_1^2\right) + \left(\frac{1}{2} - \alpha\right)E\left(e_0 e_1\right)\right]$$

$$= \bar{Y}\left[\left(\frac{\alpha}{2} - \frac{1}{8}\right)\theta C_x^2 + \left(\frac{1}{2} - \alpha\right)\theta C_{yx}\right]$$

$$= \theta\bar{Y}\left[\left(\frac{\alpha}{2} - \frac{1}{8}\right)C_x^2 + \left(\frac{1}{2} - \alpha\right)C_{yx}\right] \qquad (2.3)$$

$$\therefore\ MSE(t_{g1}) = E(t_{g1} - \bar{Y})^2$$

$$\cong \bar{Y}^2\, E\left(\left(\frac{1}{2} - \alpha\right)e_1 + e_0\right)^2$$

$$= \bar{Y}^2\,\theta\left[C_y^2 + \left(\frac{1-2\alpha}{2}\right)^2 C_x^2 + (1-2\alpha)C_{yx}\right] \qquad (2.4)$$

The optimum value of α is obtained by minimizing $MSE(t_{g1})$ with respect to α , which gives

$$\frac{\partial\, MSE(t_{g1})}{\partial \alpha} = 0\ ,$$

from which $\alpha_{opt} = K + \frac{1}{2}$.

Substituting the optimum value of α in $MSE(t_{g1})$, we have to $O(\frac{1}{n})$

$$MSE(t_{g1})_{opt} = \theta\bar{Y}^2\, C_y^2\,(1-\rho^2). \qquad (2.5)$$

$$B(t_{g1})_{opt} = \theta\bar{Y}\left[\left(\frac{\alpha_{opt}}{2}-\frac{1}{8}\right)C_x^2+\left(\frac{1}{2}-\alpha_{opt}\right)C_{yx}\right]$$

$$= \theta\bar{Y}\left[-K^2+\left(\frac{1}{2}K+\frac{1}{8}\right)\right]C_x^2 . \quad (2.6)$$

Bias and mean square Error of t_{g2}:

We propose a generalized exponential ratio type estimator as

$$t_{g2} = \bar{y}\left[\alpha\, e^{\left(\frac{\bar{X}-\bar{x}}{\bar{X}}\right)} + (1-\alpha)e^{\left(\frac{\bar{x}-\bar{X}}{\bar{X}}\right)}\right]$$

$$= \bar{Y}(1+e_0)\left[\alpha e^{-e_1}+(1-\alpha)e^{e_1}\right]$$

$$= \bar{Y}(1+e_0)\left[\alpha\left(1-e_1+\frac{e_1^2}{2}+\dots\right)+(1-\alpha)\left(1+e_1+\frac{e_1^2}{2}+\dots\right)\right]$$

$$= \bar{Y}\left[1+e_0+(1-2\alpha)e_1+\frac{e_1^2}{2}+(1-2\alpha)e_0e_1+e_0\frac{e_1^2}{2}\right].$$

$$B(t_{g2}) = E\left(t_{g2}\right)-\bar{Y}.$$

$$= \bar{Y}\left[\frac{E(e_1^2)}{2}+(1-2\alpha)\ E(e_0e_1)\right]$$

$$= \theta\bar{Y}\left[\frac{1}{2}C_x^2+(1-2\alpha)C_{yx}\right]. \quad (2.7)$$

$$MSE(t_{g2}) = E\left(t_{g2}-\bar{Y}\right)^2$$

$$= \bar{Y}^2\, E\left[e_0+(1-2\alpha)e_1\right]^2$$

$$= \bar{Y}^2\left[E(e_0^2)+(1-2\alpha)^2 E\left(e_1^2\right)+2(1-2\alpha)E\left(e_0e_1\right)\right]$$

$$= \theta\bar{Y}^2\left[C_y^2+(1-2\alpha)^2C_x^2+2(1-2\alpha)C_{yx}\right]. \quad (2.8)$$

Minimizing $MSE(t_{g2})$ with respect to α which gives

$$\alpha_{opt} = \frac{K+1}{2}.$$

Substituting the optimum value of α in $MSE(t_{g2})$, we have

$$MSE(t_{g2})_{opt} = \theta\bar{Y}^2 C_y^2(1-\rho^2) \quad (2.9)$$

which equals the large sample mean square error of the linear regression estimator.

$$\bar{y}_{lr} = \bar{y}+b(\bar{X}-\bar{x})$$

where*b* is the sample regression coefficient of *y* on *x*.

Substituting of the optimum value of α in $B(t_{g2})$, we have

$B(t_{g2})_{opt}$

$$= \theta\overline{Y}\left[\frac{1}{2}C_x^2 + \left(1 - 2.\frac{K+1}{2}\right)C_{yx}\right]$$

$$= \theta\overline{Y}\left[\frac{1}{2}C_x^2 - K\,C_{yx}\right]$$

$$= \theta\overline{Y}\left[\frac{1}{2}C_x^2 - K^2C_x^2\right]$$

$$= \theta\overline{Y}\left[-K^2\,C_x^2 + \frac{1}{2}C_x^2\right]$$

$$= \theta\overline{Y}\left[-K^2 + \frac{1}{2}\right]C_x^2 .$$

3. Comparison of Biases and Mean square Errors of $\mathbf{t_{g1}}$ and $\mathbf{t_{g2}}$:

$$B(t_{g1})_{opt} = \theta\overline{Y}\left[-K^2 + \left(\frac{1}{2}K + \frac{1}{8}\right)\right]C_x^2 .$$

$$B(t_{g2})_{opt} = \theta\overline{Y}\left[-K^2 + \frac{1}{2}\right]C_x^2 .$$

$$MSE\,(t_{g1})_{opt} = MSE\,(t_{g2})_{opt} = \theta\overline{Y}^2\,C_y^2\,(1-\rho^2).$$

Thus, to $O\left(\frac{1}{n}\right), t_{g1}$ and t_{g2} are equally efficient.

4. Numerical Illustrations :

Considering 5 natural populations described in Table-1 we compare the approximate absolute biases of square root transformation ratio type estimator with exponential ratio type estimator in Table-2.

Table 1: Description of Populations

Pop^n. No.	Description	N	y	x	ρ_{yx}	C_x	C_y	K
1	Sampford (1962)	17	Acreage under Oats in 1957	Acreage crops and grass in 1947	0.4	0.22	0.45	0.818
2	Singh and Chaudhary (1986)	16	Area under Wheat 1979-80	Total cultivated area during 1978-79	0.96	0.74	0.69	0.895
3	Konijn (1973)	16	Food expenditure	Total expenditure	0.95	0.08	0.11	1.3062
4	Murthy (1967)	16	Output for Factories (000 Rs)	Fixed capital (000 Rs.)	0.84	0.15	0.09	0.504
5	Swain (2003)	19	No. of Milk Cows in 1957	No. of Milk Cows Census 1956	0.72	1.14	1.12	0.7073

Table : 2 Absolute Biases of Estimators without constant multiplier $\theta\overline{Y}$

Bias / Population	$B(\bar{y}_R)$	$B(t_{g1})$	$B(t_{g2})$
1	0.008809	+0.0065	+0.0081

2	0.057498	+0.1251	+0.1648
3	0.00196	+0.0059	+0.0077
4	0.01116	0.0027	0.0055
5	0.38039	+0.0280	+0.0002

Comments :

(i) t_{g1} is less biased than t_{g2} for population 1,2,3,4.

(ii) t_{g2} is less biased than t_{g1} for population 5.

Conclusion :

(i) To $O\left(\frac{1}{n}\right), t_{g1}$ and t_{g2} are equally efficient.

(ii) For most of the natural population t_{g1} is approximately less biased than t_{g2} to first order approximation.

(iii) Further work with computations involving a number of natural and artificial populations is necessary to chose the preferable one between the estimators in question.

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