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EFFECTS OF THERMAL MODULATION AND ROTATION IN A POROUS MEDIUM SATURATED BY A DIELECTRIC FLUID ON THE ONSET OF MAGNETOELECTROCONVECTION

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ABSTRACT

Combined effects of thermal modulation and rotation in the presence of uniform electric field and magnetic field on the Onset of Magnetoelectroconvection in a dielectric fluid Saturated horizontal porous layer is investigated for fluctuates time periodic temperature modulations. The dielectric constant is assumed to be a linear function of temperature. Modified Darcy equation that incorporates the Coriolis expression is utilized to depict the flow in a porous medium and the subsequent eigenvalue problem is tackled utilizing a regular perturbation technique with small amplitude approximation. The effects of various physical quantities on the onset of Magnetoelectroconvection have been examined. It is observed that correction Rayleigh number R_{a2} decreases as the Taylor number T_a, Hartmann number M_a, Prandtl number P_r increases on symmetric, asymmetric and lower wall temperature modulation. It is also found that the R_{a2} increases as the electric Rayleigh number R_e and Darcy number D_a increases irrespective of the type of modulation.

Keywords: Magnetoelectroconvection, Thermal modulation, Rotation, Porous medium, Perturbation Procedure.

Nomenclature

P_r	Prandtl number
D_a	Darcy number
T_a	Taylor number
R_a	Thermal Rayleigh number
M_a	Hartmann number
R_e	Electric Rayleigh number
A	Specific heat ratio
K	Permeability of the porous medium(m^2)
T	Temperature (kelvin)
t	Time (sec)
$\vec{q} = (u, v, w)$	velocity vector (m/s)
$\vec{J}$	Electric current density (amp / m^2)
K	Wave number (m^{-1})
$\vec{B}$	Magnetic field (tesla)
$\vec{g}$	Acceleration due to gravity
$\hat{k}$	Unit vector in z -direction

Greek symbols

α	Coefficient of thermal expansion (per Kelvin)
ε	Dielectric constant
φ	Electric potential (volts)
η	Expansion coefficient of electric permittivity(farad/m)
ν	Kinematic viscosity (m^2/s)
δ	Porosity
χ	Thermal diffusivity(m^2/s)
σ	Electric conductivity(mho)
$\bar{\phi}$	Phase angle
Ω	Frequency of modulation
$\bar{\Omega}$	Angular velocity

Subscripts/superscripts

b	Base state
c	critical value
*	Non-dimensional quantity

1 Introduction

Magnetoelectroconvection in a fluid saturated porous medium has gained considerable attention in recent years because of its relevance in a wide range of application such as water movements in geothermal energy utilization, underground spreading of chemicals waste, nuclear waste repository, enhanced recovery of petroleum from reservoirs and so on. Because of these applications several studies have been undertaken to investigate the effects of different phenomena connected with such media. An excellent review of most of these studies has been reported by [1, 2, 3, 4, 7] and so on.

In the above mentioned applications rotation plays an important role. It is realized that earth's crust is essentially a porous medium consisting of different types of fluid like water, oil, gases, etc., the temperature of which increments as one goes deep inside. Also, constant angular velocity of the earth about its geographical axis gives rise to Coriolis force. Subsequently any attempt to examine convective currents in geothermal frameworks will prompt the issue of finding the effect of rotation on the stability of liquid dynamical system in a porous layer.

The effect of rotation on the onset of convection in a horizontal fluid layer is well known for ordinary viscous fluids [6] have considered large amplitude convection in a rotating fluid saturated permeable layer using Darcy model. Palm et al. [10] have shown that the results of thermal convection in a rotating porous layer are equivalent to those of non–rotating anisotropic porous medium. Jou et al. [8] have studied about thermal convection in a porous medium subject to transient heating and rotation. Recently Qin et al. [9] have examined the nonlinear stability of a rotating porous medium by employing the Brinkman model and they have determined the critical energy bounds numerically.

In [11] made a preliminary investigation for a rotating convective fluid bounded by free surfaces. He found that finite amplitude instability occurs because of non-linear effects can adjust some part of the constraint of rotation, thereby reducing the inhibiting effect of rotation on convective motions. Also, it is necessary that Taylor number be sufficiently large for the constraint to be effective in order that the system be able to reduce the effect. These qualitative conclusions of [11] were checked analytically by Rossby, who carried out an extensive investigation of Benard convection of several fluids in both rotating and non-rotating frames. Later Veronis [12] conducted a numerical investigation of two-dimensional Benard problem in a rotating fluid confined between free boundaries. The numerical examinations demonstrate that at low rotation rates the thermal field exhibits a structure similar to that of a non-rotating liquid. Linear stability hypothesis of Benard convection in a rotating fluid has been considered by Chandrasekhar [13] and shown that fluids with $P_r >> 1$ exhibits behaviour markedly different from that of fluids with $P_r << 1$. Rudraiah et al. [14, 15] have studied the effect of the Coriolis force and a non-uniform temperature gradient on the onset of Marangoni and Rayleigh–Benard convection and demonstrated that combined effects of rotation and non-uniform temperature gradient are to control convection. Wong et al. [17] and Roberts [16] have examined convective system in a dielectric fluid in the presence of a transverse electric field without porous medium. The corresponding problems in porous medium have not been given much consideration inspite of its numerous applications cited earlier and its investigation is the main objective this paper.

The stability on the onset of convection in a fluid saturated porous medium subject to both temperature modulation and rotation was investigated by Malashetty et al. [18, 19], Gaikwad et al [20] have investigated the combined effects of thermal regulation and rotation of small amplitude on the convection in a liquid saturated porous layer with saturated fluid layer of Walter's B fluid for an extensive variety of estimations of frequency using linear stability theory and Pranesh et al. [21] have analyzed the effect of thermal modulation on the onset of Rayleigh-Bernard convection in a micorpolar fluid saturated porous layer

Electro thermo convective instability of a dielectric fluid layer saturated with densely packed porous medium in the presence of modulated temperature has been discussed by Rudraiah and Gayathri [22]. Shiva kumara et al. [23] discussed in detail the effect on the onset of Darcy-Brinkman electroconvection in a dielectric fluid saturated porous layer. Gayathri et al. [24] have investigated the effect of electric modulation on the convective instability of electro thermo convection in a dielectric fluid saturated porous medium using modified Darcy. These authors have also extended their work in combined effects of Coriolis force and a vertical AC electric field on the onset of electro thermo convection in a dielectric fluid saturated Brinkman porous medium for a various types of velocity boundary conditions is carried out by Shiva kumara et al. [25] and Shiva kumara et al. [26]

studied the effects of rotation in couple stress dielectric fluid layer on the onset of electro hydrodynamics instability. Recently, Rana et al. [27] considered the effect of rotation on the electro hydrodynamics instability of a fluid layer with an electrical conductivity gradient.

This paper presents the stability analysis of a dielectric fluid saturated porous layer subject to boundary temperature modulation, uniform electric field and rotation. We try to provide a fundamental understanding of how rotation and electric field would influence the onset of thermal convection arising from sinusoidally heated boundaries. Although the effect of rotation on the thermally driven flows is well understood, but some questions such as how the external constraints of rotation and electric fields affect the modulated thermally driven convective instability and whether or not the understanding gained from such studies is pertinent, remain basically unresolved. We intend to answer these questions by solving a thermally modulated system with rotation in the presence of electric field and magnetic field using perturbation technique with small amplitude approximation.

The literature pertaining to temperature modulation, rotation and corresponding study for a dielectric fluid with the combined effects of electric and magnetic fields is missing despite its importance in understanding the control of convection encountered in many scientific and technological problems.

Therefore, the main object of this paper is to study the effect of magnetic field on the convective instability of dielectric fluid saturated with the porous medium in the presence of modulated temperature, rotation and uniform electric field by choosing free-free boundaries.

2 Mathematical Formulations

We study an infinite horizontal layer of an incompressible dielectric fluid saturated with the densely packed porous medium of characteristic thickness h in the presence of a uniform applied magnetic field B_0 in the horizontal direction as shown in Fig.1. The wall temperatures are externally imposed and are taken as

$$T = \frac{\Delta T}{2}\left[1+\beta\cos\Omega t\right] \qquad \text{at } z=0, \tag{1}$$

$$T = -\frac{\Delta T}{2}\left[1-\beta\cos\left(\Omega t+\bar{\phi}\right)\right] \qquad \text{at } z=h, \tag{2}$$

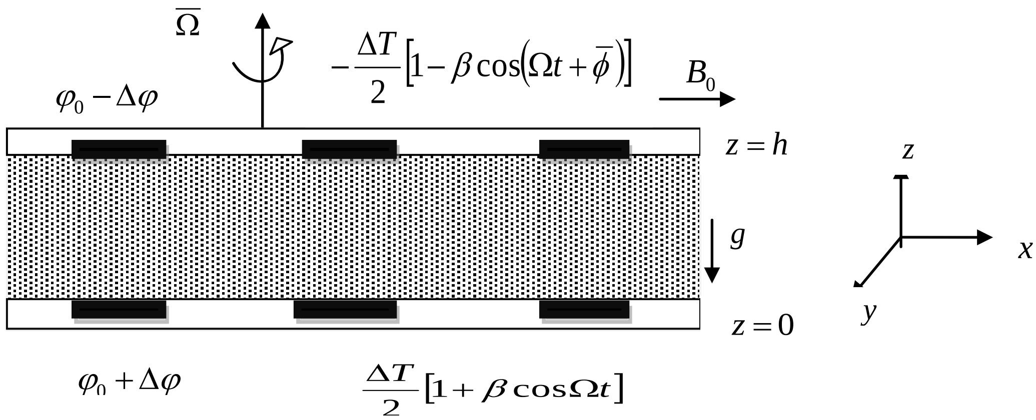

Fig. 1. Physical configuration

The time-dependent parts indicate the modulation imposed on the adverse thermal gradient caused by the temperatures $\frac{\Delta T}{2}$ and $-\frac{\Delta T}{2}$ at the lower and upper walls, respectively. The fluid saturated porous layer is subjected to rotation with an angular velocity $\overline{\Omega}$. The axis of rotation is taken along the z-axis. The walls are embedded with parallel electrodes having applied uniform voltages $\varphi_0 + \Delta\varphi$ at $z = 0$ and $\varphi_0 - \Delta\varphi$ at $z = h$.

The basic equations describing the flow of a Boussinesq incompressible dielectric fluid are

$$\nabla \circ \vec{q}, \tag{3}$$

$$\delta^{-1} \frac{\partial \vec{q}}{\partial t} + \delta^{-2} (\vec{q} \circ \nabla) \vec{q} + 2\delta^{-1} \overline{\Omega} \hat{k} \times \vec{q} = -\nabla P + \frac{\rho}{\rho_0} \vec{g} - \frac{\nu}{K} \vec{q} - \frac{\vec{E}^2}{2\rho_0} \nabla \varepsilon + \frac{(\vec{J} \times \vec{B})}{\rho_0}, \tag{4}$$

$$A \frac{\partial T}{\partial t} + (\vec{q} \circ \nabla) T = \chi \nabla^2 T. \tag{5}$$

$$\rho = \rho_0 (1 - \alpha T), \tag{6}$$

Eq. (4) is the modified Lapwood [5], equation where modification means the addition of the last two terms on the right hand side of Eq. (4). Following Roberts [16], since the fluid is dielectric, we assume that there are no free charges as well as no induced magnetic field. The relevant Maxwell equations are

$$\nabla \times \vec{E} = 0 \quad \text{OR} \quad \vec{E} = -\nabla \varphi, \tag{7}$$

$$\nabla \circ (\varepsilon \vec{E}) = 0, \tag{8}$$

$$\varepsilon = \varepsilon_m (1 + \eta T). \tag{9}$$

The quantities at basic state of the system are described of the form

$$\vec{q} = \vec{q}_b = 0, \quad \varphi = \varphi_b(z), \quad \vec{E} = \vec{E}_b(z), \quad T = T_b(z, t), \quad \varepsilon = \varepsilon_b(z. t). \tag{10}$$

Substituting eq. (10) into the above Eqs, we obtain the quiescent state solutions from (3) – (9), are as follows

$$-\nabla P_b + \frac{\rho_b}{\rho_0} \vec{g} - \frac{E_b{}^2}{2\rho_0} \nabla \varepsilon_b = 0, \tag{11}$$

$$A \frac{\partial T_b}{\partial t} = \chi \nabla^2 T_b, \tag{12}$$

$$\nabla \cdot (\varepsilon_b \vec{E}_b) = 0, \tag{13}$$

$$\varepsilon_b = \varepsilon_m (1 + \eta T_b). \tag{14}$$

Eq. (12) is solved by using the boundary conditions (1) and (2), we obtain the solution which consists of both steady and oscillating parts is given by

$$T_b = \frac{\Delta T}{2h}(h - 2z) + \beta \, \mathrm{Re} \left\{ \left[a(\lambda) e^{\frac{\lambda z}{h}} + a(-\lambda) e^{\frac{-\lambda z}{h}} \right] e^{-i \Omega t} \right\}, \tag{15}$$

where

$$a(\lambda) = \frac{\Delta T}{2} \left(\frac{e^{-i\bar{\varphi}} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}} \right), \quad \lambda = (1 - i) \left(\frac{A \Omega h^2}{2\chi} \right)^{1/2}, \tag{16}$$

and Re stands for the real part.

From Eq. (14), we get

$$\varepsilon_b \approx \varepsilon_m \left(1 - \frac{\Delta T}{h} \eta z \right). \tag{17}$$

Using $E_b = -\nabla \varphi_b$ in Eq. (13) and solving the resulting equation, we get the solution for φ_b in the form

$$\varphi_b = -\frac{2\Delta\varphi}{(1+\eta\Delta T/2)}\frac{z}{h} - \frac{\eta\Delta T\Delta\varphi}{(1+\eta\Delta T/2)}\frac{z^2}{h^2} + \varphi_0 + \Delta\varphi, \tag{18}$$

$$E_b = \frac{2\Delta\varphi}{h}\left(1 + \frac{\Delta T}{h}\eta z\right). \tag{19}$$

We now introducing perturbations on the flow in the form

$$\vec{q} = \vec{q}' = (u', v', w'),\ \varepsilon = \varepsilon_b + \varepsilon',\ P = P_b + P',\ \varphi = \varphi_b + \varphi',\ \vec{E} = \vec{E} + \vec{E}',\ T = T_b + T'. \tag{20}$$

Eq. (20) using in Eqs. (3)- (9) and substituting the solutions given by Eqs. (11)- (14), afterward linearizing, we obtain

$$\delta^{-1}\frac{\partial \vec{q}'}{\partial t} + 2\,\delta^{-1}\,\overline{\Omega}\,\hat{k} \times \vec{q}' = -\nabla\Pi + \alpha\, g T'\,\hat{k} - \frac{\nu}{K}\vec{q}' + Q'\hat{k} - \frac{\sigma}{\rho_0}B_0{}^2 w'\,\hat{k}, \tag{21}$$

$$\left(A\frac{\partial}{\partial t} - \chi\nabla^2\right)T' = -\frac{\partial T_b}{\partial z}w', \tag{22}$$

$$\nabla^2\varphi' = \frac{2\Delta\varphi}{\varepsilon_m h}\frac{\partial \varepsilon'}{\partial z} + \frac{\eta\Delta T}{h}\frac{\partial \varphi'}{\partial z}, \tag{23}$$

$$\varepsilon' = \varepsilon_m\,\eta T', \tag{24}$$

where,

$$Q' = \frac{\Delta T\eta E_b\varepsilon_m}{\rho_0 h}\left(\frac{2\Delta\varphi\varepsilon'}{h\varepsilon_m} - \frac{\partial\varphi'}{\partial z}\right). \tag{25}$$

Following Roberts [16], we assume $\eta\,\Delta T \ll 1$. Accordingly, we omit the second part in Eq. (23) in association with the term $\nabla^2\varphi'$ on the left side because their ratio is of order $\eta\,\Delta T \ll 1$, under this approximation Eqs. (23) and (25), using Eq. (24), become

$$\nabla^2\varphi' = \frac{2\eta\Delta\varphi}{h}\frac{\partial T'}{\partial z}, \tag{26}$$

$$\nabla^2 Q' = \frac{4\eta^2\varepsilon_m\,\Delta T(\Delta\varphi)^2}{\rho_0 h^3}\nabla_H{}^2\,T', \tag{27}$$

where,

$$\nabla_H{}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

Eliminating the pressure from Eq. (21), we take curl twice on it and by retaining the vertical component, we obtain

$$\left(\frac{1}{\delta}\frac{\partial}{\partial t} + \frac{\nu}{K}\right)\nabla^2 w' = \alpha\, g\,\nabla_H{}^2\,T' + \nabla_H{}^2\,Q' - \frac{2\overline{\Omega}}{\delta}\frac{\partial\varsigma}{\partial z} - \frac{\sigma}{\rho_0}B_0{}^2\,\nabla_H{}^2 w' \tag{28}$$

Substituting Eqs. (22) and (27) into Eq. (28), we get the following expression for the vertical velocity component

$$\begin{bmatrix} \left(\frac{1}{\delta}\frac{\partial}{\partial t}+\frac{\nu}{K}\right)^2\left(A\frac{\partial}{\partial t}-\chi\nabla^2\right)\nabla^4+\alpha g\left(\frac{1}{\delta}\frac{\partial}{\partial t}+\frac{\nu}{K}\right)\nabla^2\left(\frac{\partial T_b}{\partial z}\nabla_H{}^2\right) \\ +\frac{4\overline{\Omega}}{\delta^2}\frac{\partial^2}{\partial z^2}\left(A\frac{\partial}{\partial t}-\chi\nabla^2\right)\nabla^2+\frac{\sigma}{\rho_0}B_0{}^2\nabla^2\left(A\frac{\partial}{\partial t}-\chi\nabla^2\right)\left(\frac{1}{\delta}\frac{\partial}{\partial t}+\frac{\nu}{K}\right)\nabla_H{}^2 \end{bmatrix} \tag{29}$$
$$=-\frac{4\eta^2\varepsilon_m\,\Delta T(\Delta\phi)^2}{\rho_0 h^3}\left(\frac{1}{\delta}\frac{\partial}{\partial t}+\frac{\nu}{K}\right)\frac{\partial T_b}{\partial z}\nabla_H{}^4 w'$$

We impose the following conditions appropriate for stress-free, isothermal boundaries.

$$w'=\frac{\partial^2 w'}{\partial z^2}=T'=0 \quad \text{at } z=0,h\ . \tag{30}$$

Non- dimensionalizing the above Eq. (29) by using the transformations.

$$\left(x^*,y^*,z^*\right)=\frac{1}{h}(x,y,z),\ w^*=\frac{w'}{\chi/h},\ t^*=\frac{t}{Ah^2/\chi},\omega=\frac{\Omega}{\chi/Ah^2}, \tag{31}$$

Therefore, the resulting equation (after neglecting asterisks $*$)

$$\begin{bmatrix} \left(\frac{1}{\delta AP_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)^2\left(\frac{\partial}{\partial t}-\nabla^2\right)\nabla^4+R_a\left(\frac{1}{\delta AP_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)\nabla^2\left((-1+\beta f)\ \nabla_H{}^2\right) \\ +T_a\frac{\partial^2}{\partial z^2}\left(\frac{\partial}{\partial t}-\nabla^2\right)\nabla^2+M_a\nabla^2\left(\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{1}{\delta AP_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)\nabla_H{}^2 \end{bmatrix}w=-R_e\left(\frac{1}{\delta AP_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)(-1+\beta f)\nabla_H{}^4 w \tag{32}$$

The boundary conditions given by Eq. (30) should also be made dimensionless using the scales given by Eq. (31) and using $\frac{\partial^4 w'}{\partial z^4}=0$. If w' and T' are zero, we obtain

$$w=\frac{\partial^2 w}{\partial z^2}=\frac{\partial^4 w}{\partial z^4}=0 \quad \text{at } z=0,1. \tag{33}$$

If the horizontal dependence of w in Eq. (32) is assumed to be of the form, where k_x and k_y are the horizontal wave numbers, then w takes the following form:

$$w=W(z,t)\,e^{i(\mathrm{k}_x x+\mathrm{k}_y y)}, \tag{34}$$

It follows that

$$\nabla_H{}^2 w=-\mathrm{k}^2 W,$$

Where $\mathrm{k}^2=\mathrm{k}_x{}^2+\mathrm{k}_y{}^2$. Also Eqs. (32) and (33), respectively become

$$\begin{bmatrix} \left(\frac{1}{\delta AP_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)^2\left(\frac{\partial}{\partial t}-(D^2-\mathrm{k}^2)\right)(D^2-\mathrm{k}^2)^2-R_a k^2\left(\frac{1}{\delta AP_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)(D^2-\mathrm{k}^2)(-1+\beta f) \\ +T_a\left(\frac{\partial}{\partial t}-(D^2-\mathrm{k}^2)\right)(D^2-\mathrm{k}^2)D^2-M_a\mathrm{k}^2\left(\frac{1}{\delta AP_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)\left(\frac{\partial}{\partial t}-(D^2-\mathrm{k}^2)\right)(D^2-\mathrm{k}^2) \end{bmatrix}W \tag{35}$$
$$=-R_e\mathrm{k}^4\left(\frac{1}{\delta AP_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)(-1+\beta f)W$$

$$W=\frac{\partial^2 W}{\partial z^2}=\frac{\partial^4 W}{\partial z^4}=0 \quad \text{at } z=0,1. \tag{36}$$

To obtain an expression for R_a from Eq. (34), we use the following regular perturbation procedure.

3 Regular Perturbation Procedure with Small Amplitude Approximation

We seek the eigenvalue R_a and eigenfunction W of Eq. (35) for a temperature gradient $\frac{\partial T_b}{\partial z} = (-1+\beta f)$ that departs from the linear temperature gradient $\frac{\partial T_b}{\partial z} = -1$ in modulated system by the quantities of the order β. It follows that the eigenvalue R_a and eigenfunction W of the present problem differ from those related with the standard Benard problem by the quantities of order β.

We therefore assume the solution of Eq. (35) as a series expansion of the following form.

$$W = W_0 + \beta W_1 + \beta^2 W_2 + \cdots$$
$$R_a = R_{a0} + \beta R_{a1} + \beta^2 R_{a2} + \cdots \tag{37}$$

Substituting Eq. (37) into Eq. (35) and equating the coefficients of like powers of β, we obtain the following system of equations

$$LW_0 = 0 \tag{38}$$

$$LW_1 = k^2 R_{a0}\left(\frac{1}{\delta A P_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)(D^2-\mathrm{k}^2)(fW_0) - \mathrm{k}^2 R_{a1}\left(\frac{1}{\delta A P_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)(D^2-\mathrm{k}^2)W_0 - R_e\mathrm{k}^4\left(\frac{1}{\delta A P_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)fW_0 , \tag{39}$$

$$\begin{aligned} LW_2 = {} & \mathrm{k}^2 R_{a0}\left(\frac{1}{\delta A P_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)(D^2-\mathrm{k}^2)(fW_1) - \mathrm{k}^2 R_{a2}\left(\frac{1}{\delta A P_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)(D^2-\mathrm{k}^2)W_0 - R_e\mathrm{k}^4\left(\frac{1}{\delta A P_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)(fW_1) \\ & -\mathrm{k}^2 R_{a1}\left(\frac{1}{\delta A P_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)(D^2-\mathrm{k}^2)W_1 + \mathrm{k}^2 R_{a1}\left(\frac{1}{\delta A P_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)(D^2-\mathrm{k}^2)(fW_0) \end{aligned} \tag{40}$$

The marginally stable solutions of Eq. (37) are of the form

$W_0^{(n)} = \sin n\pi z$.

and corresponding to the eigenvalues are as follows

$$R_{a0}^{(n)} = \frac{(n^2\pi^2+\mathrm{k}^2)^2}{D_a\mathrm{k}^2} + \frac{D_a T_a n^2\pi^2(n^2\pi^2+\mathrm{k}^2)}{\mathrm{k}^2} - \frac{R_e\mathrm{k}^2}{(n^2\pi^2+\mathrm{k}^2)} + M_a(n^2\pi^2+\mathrm{k}^2).$$

The least eigenvalue for a fixed wave number k, when $n=1$, is obtained:

$$R_{a0} = \frac{(\pi^2+\mathrm{k}^2)^2}{D_a\mathrm{k}^2} + \frac{D_a T_a \pi^2(\pi^2+\mathrm{k}^2)}{\mathrm{k}^2} - \frac{R_e\mathrm{k}^2}{(\pi^2+\mathrm{k}^2)} + M_a(\pi^2+\mathrm{k}^2), \tag{41}$$

Corresponding to $W_0 = \sin \pi z$.

Suppose the minimum value R_{a0} is R_{a0c}, when $\mathrm{k}=\mathrm{k}_c$ satisfies the equation

$$\begin{aligned} & (1+M_aD_a)(\mathrm{k}_c^{\,2})^4 + (2\pi^2 - D_a^{\,2}T_a\pi^2 + 2M_aD_a\pi^2)(\mathrm{k}_c^{\,2})^3 - (D_a^{\,2}T_a\pi^4 + R_e D_a\pi^2 - M_aD_a\pi^4)(\mathrm{k}_c^{\,2})^2 \\ & \quad - 2\pi^6(1+D_a^{\,2}T_a)\mathrm{k}_c^{\,2} - (1+D_a^{\,2}T_a)\pi^8 = 0 \end{aligned} \tag{42}$$

We observe that, when $R_e = 0,\ T_a = 0$ and $M_a = 0.$ Eqs. (41) and (42), yield the values $R_{a0c} = 4\pi^2$ and $\mathrm{k}_C = \pi$ which are identical with results of Lapwood [5] for convection in the porous layer. Also we observe that in the absence of magnetic field and Coriolis force, the results are similar with the classical results of Rudraiah et al [21] for convection in the presence electric field.

Now from Eq. (39), we have

$$LW_1 = R_{a0}\mathrm{k}^2\left(\frac{1}{\delta AP_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)\left(D^2-\mathrm{k}^2\right)\left(f\sin\pi z\right)+R_{a1}\mathrm{k}^2\left(\frac{1}{\delta AP_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)\left(\pi^2+\mathrm{k}^2\right)\sin\pi z$$
$$-R_e\mathrm{k}^4\left(\frac{1}{\delta AP_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)f\sin\pi z \quad (43)$$

The solubility condition requires that the time-independent part of the right hand side of Eq. (43) should be orthogonal to $\sin\pi z$. Since f varies sinusoidally in time, the only steady term is $R_{a1}\mathrm{k}^2\left(\frac{1}{\delta AP_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)\left(\pi^2+\mathrm{k}^2\right)\sin\pi$ so that R_{a1}=0. It follows that all odd coefficients, i.e., R_{a3}, R_{a5}… in Eq. (37) are zero. Expanding the right hand side of Eq. (43) in a Fourier series of the form

$$e^{\lambda z}\sin m\pi z = \sum_{n=1}^{\infty} g_{nm}(\lambda)\sin n\pi z\,, \quad (44)$$

and obtain $LW_1 = -\,\mathrm{k}^2\,\mathrm{Re}\left(\left(\frac{-i\omega}{\delta AP_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)\sum_{n=1}^{\infty}\left[R_{a0}\left(n^2\pi^2+\mathrm{k}^2\right)+R_e\mathrm{k}^2\right]B_n(\lambda)\sin\,n\,\pi\,z\,e^{-i\omega t}\right)$

So that

$$W_1 = -\,k^2\,\mathrm{Re}\left(\left(\frac{-i\omega}{\delta AP_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)\sum_{n=1}^{\infty}\left[R_{a0}\left(n^2\pi^2+\mathrm{k}^2\right)+R_e\mathrm{k}^2\right]\frac{B_n(\lambda)}{L(\omega,n)}\sin\,n\,\pi\,z\,e^{-i\omega t}\right), \quad (45)$$

We now define $L(\omega,n)=D1-i\omega D2$ (46)

where $g_{nm}(\lambda)$, $B_n(\lambda)$, $L(\omega,n)$ $D1$ and $D2$ are defined in the appendix

The equation for W_2 from Eq. (40) takes the form

$$LW_2 = \mathrm{k}^2R_{a0}\left(\frac{1}{\delta AP_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)\left(D^2-\mathrm{k}^2\right)\left(fW_1\right)+\mathrm{k}^2R_{a2}\left(\frac{1}{\delta AP_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)\left(\pi^2+\mathrm{k}^2\right)\sin\pi z-R_e\mathrm{k}^4\left(\frac{1}{\delta AP_r}\frac{\partial}{\partial t}+\frac{1}{D_a}\right)\left(fW_1\right). \quad (47)$$

Equation (47) is used to determine the first non-zero correction to R_a. The solvability condition requires that the steady part of the right hand side is orthogonal to W_0 and hence we obtain the following expression.

$$R_{a2} = \frac{-k^2D_a}{4\left(\pi^2+\mathrm{k}^2\right)}\left\{R_{a0}\left(\pi^2+\mathrm{k}^2\right)+R_e\mathrm{k}^2\right\}\left(\frac{\omega^2}{\delta^2A^2P_r^2}+\frac{1}{D_a^2}\right)\times\sum_{n=1}^{\infty}\frac{\left\{R_{a0}\left(n^2\pi^2+\mathrm{k}^2\right)+R_e\mathrm{k}^2\right\}\left|B_n(\lambda)\right|^2\left[L(\omega,n)+L^*(\omega,n)\right]}{\left|L(\omega,n)\right|^2}, \quad (48)$$

Where $L^*(\omega,n)$ is the complex conjugate of $L(\omega,n)$.

The critical Rayleigh number R_a is evaluated up to O$\left(\beta^2\right)$ by the expression $R_a = R_{a0}+\beta^2R_{a2}$, here we note that R_{a0} and R_{ae2} are taken from the expressions (41) and (48) respectively and calculated at $\mathrm{k}=\mathrm{k}_c$. for the following three different cases:

Case (i): Oscillating wall temperature filed is symmetric ($\bar{\varphi}=0$).

Case (ii): Oscillating wall temperature field is asymmetric ($\bar{\varphi}=\pi$).

Case (iii): Only the lower wall temperature is modulated while the upper wall being held at fixed constant temperature ($\bar{\varphi}=-i\infty$).

The correction Rayleigh number R_{a2} with ω is computed using MATHEMATICA for different physical parameters and is shown in the Figs. 2 - 10. The results obtained for the above cases are discussed in the next section.

Results and Discussion

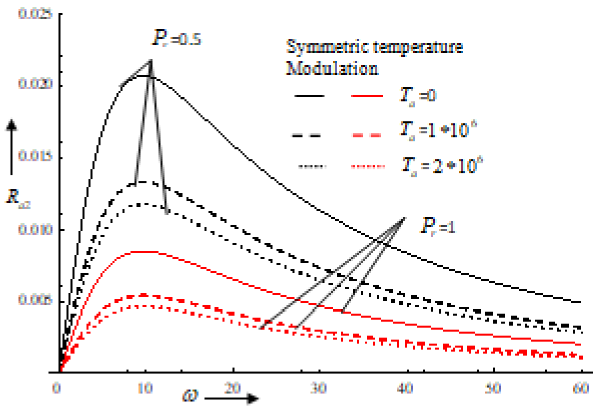

Fig. 2. Plot of R_{a2} with ω for various T_a and P_r while $M_a = 50$, $D_a = 10^{-3}$ and $R_e = 6*10^4$ are fixed

Fig. 2 is a plot of the correction R_{a2} with ω for various estimations of T_a and P_r while $D_a = 10^{-3}$, $M_a = 50$ and $R_e = 6*10^4$ are fixed, when the oscillating temperature field is symmetric. From this figure, it is found that for the value of ω approximately equal to ten R_{a2} increases to its maximum value and then it decreases with further increase in ω. Also, it is observed that R_{a2} is constantly positive showing that the convection sets in at larger estimations of Rayleigh number than those predicted for the unmodulated system.

This figure also demonstrates that R_{a2} decreases as Taylor number T_a increases, indicating that the impact of rotation on the symmetric modulated porous layer is to advance the beginning of Magnetoelectroconvection for the given frequency ω. However, the effect of T_a disappears for fairly large estimations of ω, in which $R_{a2} \to 0$.

We also observe form this Fig. 2 that the peak value of R_{a2} decreases as P_r increases. Hence P_r reduces the stabilizing effect on the system of symmetric modulation.

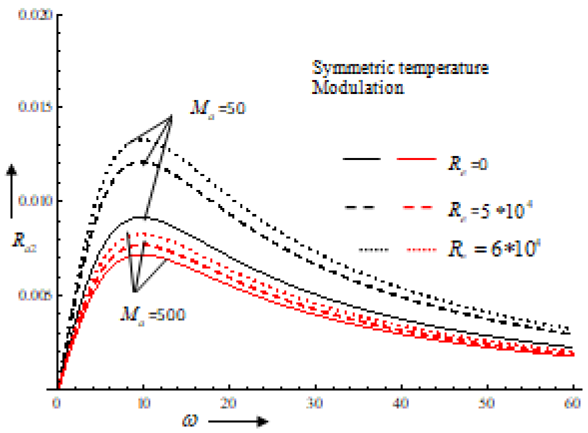

Fig. 3. Variation of R_{a2} versus ω for several values of R_e and M_a with fixed $D_a = 10^{-3}$, $P_r = 0.5$ and $T_a = 1*10^6$.

Fig. 3 is the plot of Rayleigh number R_{a2} with modulation frequency ω for several estimations of R_e and M_a with fixed $D_a = 10^{-3}$, $P_r = 0.5$ and $T_a = 1*10^6$, when the time-periodic boundary temperature is symmetric. It is found that, R_{a2} increases as electric Rayleighnumber R_e increases. Thus the effect of R_e is to delay the onset of Magnetoelectroconvection.

Further, it is also observed that for a given ω, R_{a2} decreases as increases in the value of Hartmann number M_a. Hence the magnetic field effect on the symmetric modulated system is to augment Magnetoelectroconvection.

Fig. 4 presents the effect of D_a on R_{a2}, for fixed $P_r = 0.5$, $T_a = 1*10^6$, $R_e = 6*10^4$ and $M_a = 50$, when the time- periodic boundary temperature is symmetric. From this figure for a given ω, R_{a2} increases as Darcy number D_a increases. Hence the effect of D_a is to delay the onset of Magnetoelectroconvection.

In Fig. 5, the fluctuation of R_{a2} is plotted against ω for various values of T_a and P_r for fixed values of $D_a = 10^{-3}$, $M_a = 50$ and $R_e = 6*10^4$, in the case of time- periodic boundary temperature is asymmetric. From this figure, it is clear that for the value of ω, R_{a2} decreases as T_a increases and hence the effect on the system of rotation on the asymmetric modulated porous layer is to advance the Magnetoelectroconvection.

This figure also shows that as R_{a2} decreases with increasing P_r. Hence P_r reduces the stabilizing effect on the system of asymmetric modulation.

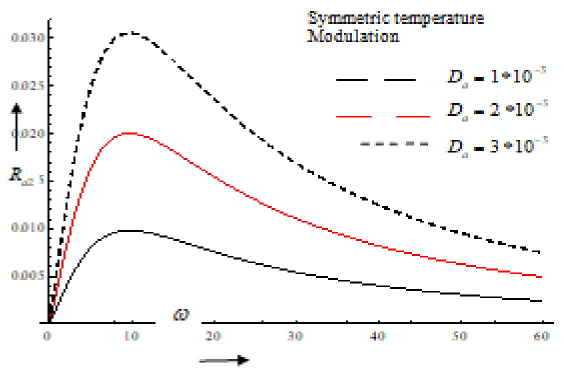

Fig. 4. Fluctuation of R_{a2} with ω while varying D_a with fixed $P_r = 0.5$, $T_a = 1*10^6$, $R_e = 6*10^4$ and $M_a = 50$

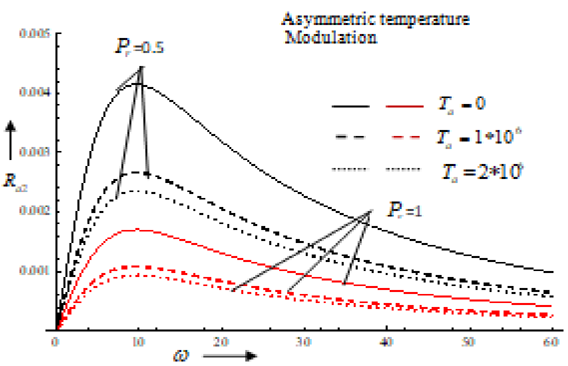

Fig. 5. R_{a2} versus ω for various T_a and P_r with fixed values of $D_a = 10^{-3}$, $M_a = 50$ and $R_e = 6*10^4$.

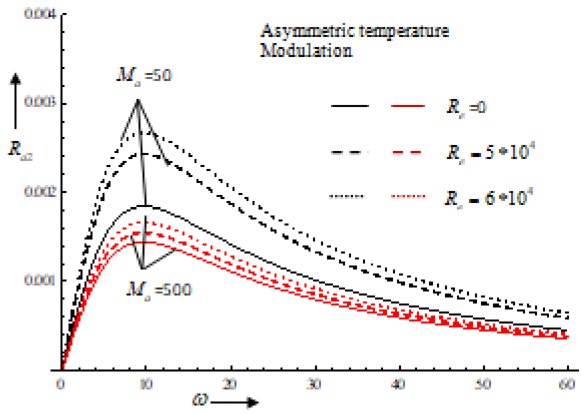

Fig. 6. Variation of R_{a2} versus ω for several estimations of R_e and M_a with fixed $D_a = 10^{-3}$, $P_r = 0.5$ and $T_a = 1*10^6$.

Fig. 6 illustrates that the correction of R_{a2} with ω for several values of R_e and M_a with $D_a = 10^{-3}$, $P_r = 0.5$ and $T_a = 1*10^6$ are fixed when the time- periodic boundary temperature is asymmetric. It is found that from this figure for given ω, R_{a2} increases as R_e increases. Therefore the impact of R_e is to delay the Magnetoelectroconvection.

Further, we also found that for frequency ω, the value of R_{a2} decreases as M_a increases. Hence the Hartmann number on the asymmetric modulated porous layer is to augment Magnetoelectroconvection.

Fig. 7 shows the plot of R_{a2} versus ω for different D_a with $P_r = 0.5$, $T_a = 1*10^6$, $R_e = 6*10^4$, $M_a = 50$ are fixed when the time- periodic boundary temperature is asymmetric. From this figure we note that R_{a2} increases as D_a increases. Thus D_a is to delay the effect on the system of Magnetoelectroconvection for the given frequency ω.

Fig.8 the variation of R_{a2} with ω for various estimations of T_a and P_r for fixed values of $D_a = 10^{-3}$, $M_a = 50$ and $R_e = 6*10^4$ is exhibited when the lower wall temperature is modulated. This Figure clear that for ω, as R_{a2} decreases, T_a increases, Hence rotation on the lower wall temperature modulated porous layer is to advance the Magnetoelectroconvection. However, T_a disappears for fairly large values of ω, in which $R_{a2} \to 0$. Also, it is observed that the estimation of R_{a2} decreases with increasing P_r. Hence P_r reduces the stabilizing effect on the system of modulated lower wall temperature.

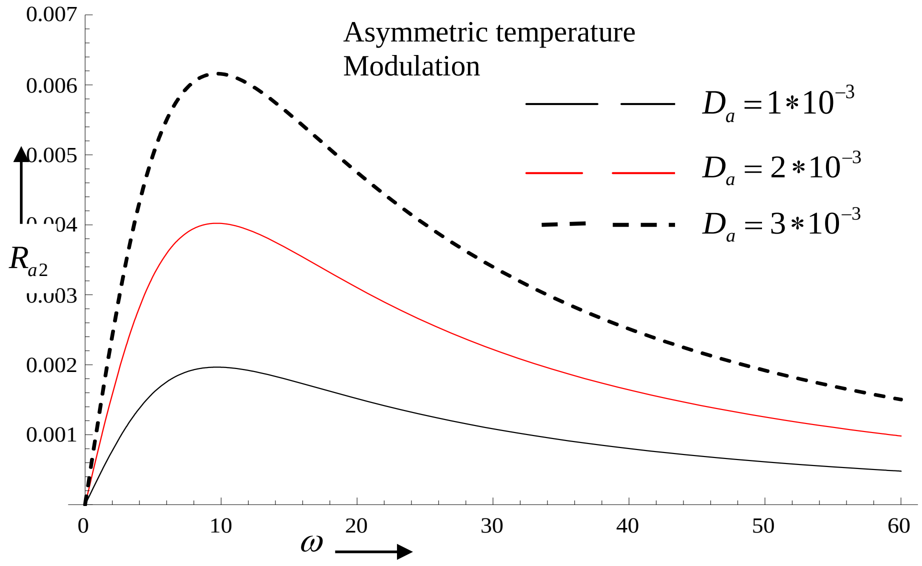

Fig. 7. plot of R_{a2} with ω for different D_a and $P_r = 0.5$, $T_a = 1*10^6$, $R_e = 6*10^4$, $M_a = 50$ are fixed

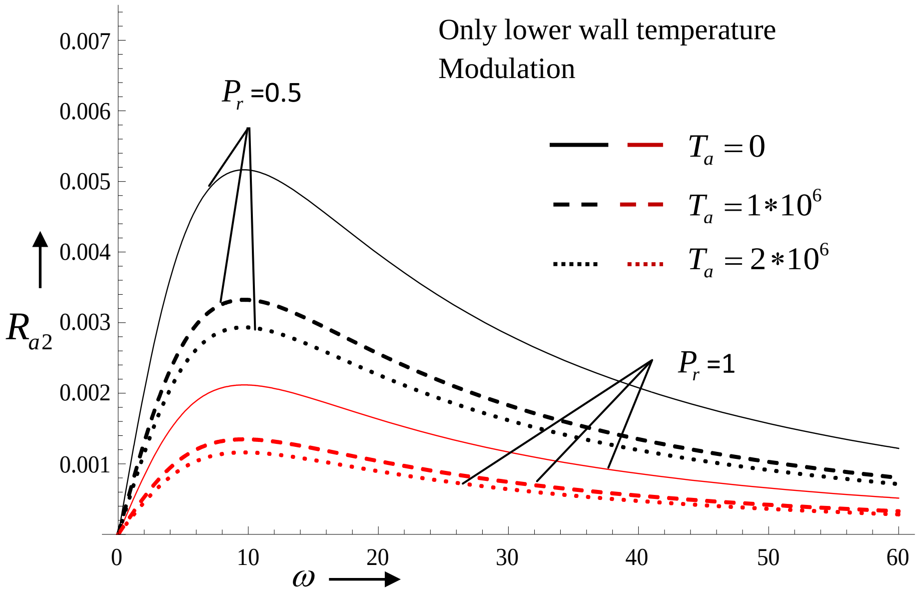

Fig. 8. R_{a2} versus ω for various values of T_a and P_r with $D_a = 10^{-3}$, $M_a = 50$ and $R_e = 6*10^4$ are fixed

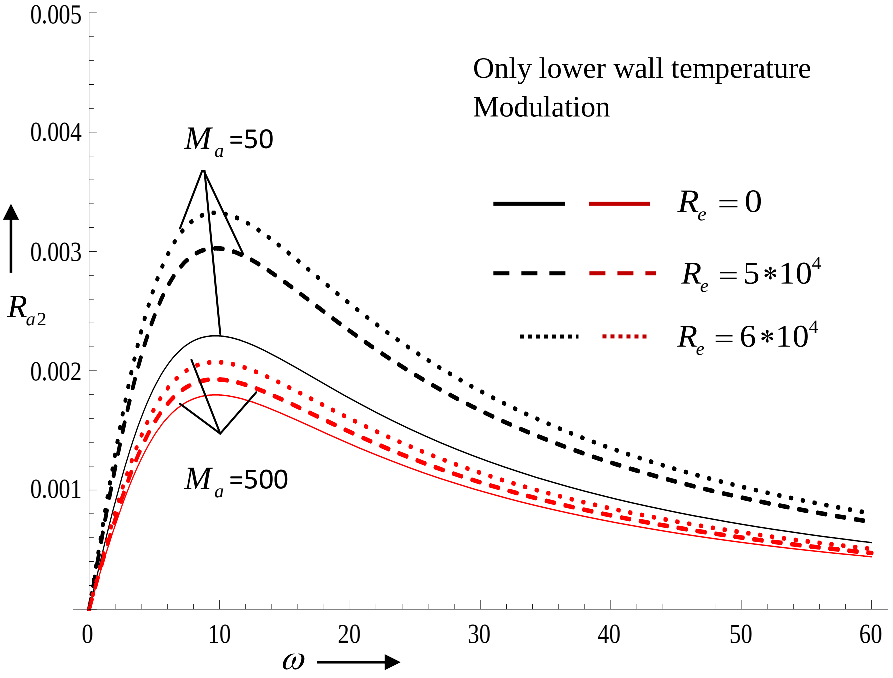

Fig. 9. Fluctuation of R_{a2} versus ω for various of M_a and R_e with $D_a = 10^{-3}$, $P_r = 0.5$ and $T_a = 1*10^6$ are fixed

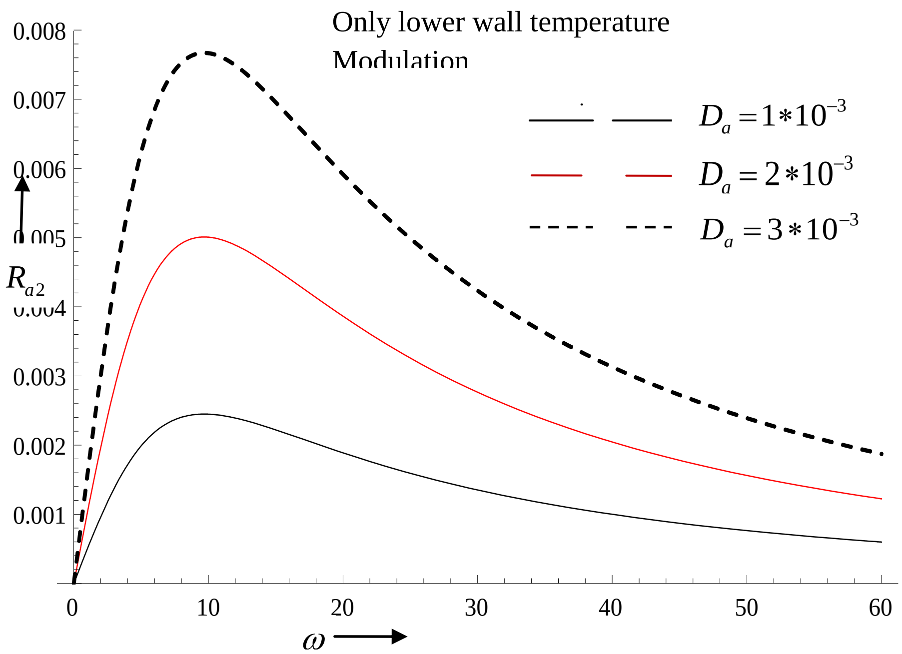

Fig. 10. R_{a2} versus ω for various estimations of D_a with $P_r = 0.5$, $T_a = 1*10^6$, $R_e = 6*10^4$ and $M_a = 50$ are fixed

The variation of R_{a2} versus ω for different R_e and M_a for fixed $D_a = 10^{-3}$, $P_r = 0.5$ and $T_a = 1*10^6$ is exhibited from Fig. 9 for a given frequency ω, R_{a2} increases as R_e increases. Therefore the effect of R_e is to delay the Magnetoelectroconvection when the lower wall temperature is modulated. This figure also indicates that for a given ω, R_{a2} decreases with increase in M_a, indicating that the effect of magnetic field on the lower wall temperature modulated porous layer is to augment Magnetoelectroconvection.

Fig. 10 illustrates that R_{a2} is plotted against frequency ω for various estimations of D_a and $P_r = 0.5$ $T_a = 1*10^6$, $M_a = 50$ are fixed. These curves indicate that for ω, R_{a2} increases as D_a increases. Hence D_a is to delay the effect on the system of modulated lower wall temperature.

5 Conclusions

Finally we summarize the following conclusions.

1. The effect of magnetic field augments Magnetoelectroconvection on Oscillating wall temperature filed is symmetric, asymmetric and only lower wall temperature modulation.
2. The effect of rotation is to advance the onset of Magnetoelectroconvection irrespective of the type of thermal modulation.
3. The Prandtl number P_r is to reduce the stabilizing effect in all the three cases.
4. The effect of electric Rayleighnumber R_e is to delay the onset of Magnetoelectroconvection in all the three cases
5. The effect of Darcy number D_a on symmetric, asymmetric and lower wall temperature modulation is to delay the Magnetoelectroconvection
6. The effect of Taylor number T_a disappears for large frequencies irrespective of the type of thermal modulation.
7. The effect of thermal Rayleigh number R_{a2} is to stabilize the system at small frequency modulation from 0 to 10 and is to destabilize the system at moderate frequency modulation from 10 to 60. Further the effect of R_{a2} disappears for large frequencies irrespective of the type of thermal modulation.

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Appendix

$$D_a = \frac{k}{h^2}, \quad R_a = \frac{\alpha g \Delta T h^3}{\nu\chi}, R_e = \frac{4\eta^2 \varepsilon_m \Delta T^2 (\Delta\phi)^2}{\rho_0 \nu \chi}, P_r = \frac{\nu}{\chi}, \; M_a = B_0 h\sqrt{\frac{\sigma}{\rho_0 \nu}}, \quad T_a = \frac{4\overline{\Omega}^2 h^4}{\delta^2 \nu^2},$$

$$\vec{J} = \sigma\left(\vec{q} \times \vec{B}\right)$$

$$L = \begin{bmatrix} \left(\frac{1}{\delta A P_r}\frac{\partial}{\partial t} + \frac{1}{D_a}\right)^2 \left(\frac{\partial}{\partial t} - \left(D^2 - \mathrm{k}^2\right)\right)\left(D^2 - \mathrm{k}^2\right)^2 + T_a\left(\frac{\partial}{\partial t} - \left(D^2 - \mathrm{k}^2\right)\right)\left(D^2 - \mathrm{k}^2\right)D^2 + R_{a0}\mathrm{k}^2\left(\frac{1}{\delta A P_r}\frac{\partial}{\partial t} + \frac{1}{D_a}\right)\left(D^2 - \mathrm{k}^2\right) \\ -R_e \mathrm{k}^4\left(\frac{1}{\delta A P_r}\frac{\partial}{\partial t} + \frac{1}{D_a}\right) - M_a \mathrm{k}^2\left(\frac{1}{\delta A P_r}\frac{\partial}{\partial t} + \frac{1}{D_a}\right)\left(\frac{\partial}{\partial t} - \left(D^2 - \mathrm{k}^2\right)\right)\left(D^2 - \mathrm{k}^2\right) \end{bmatrix}$$

$$f = \mathrm{Re}\left[\left\{A(\lambda)e^{\lambda z} + A(-\lambda)e^{-\lambda z}\right\}e^{-i\omega t}\right], \; A(\lambda) = \frac{\lambda}{2}\left(\frac{e^{-i\overline{\varphi}} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}}\right), \quad \lambda = (1-i)\left(\frac{\omega}{2}\right)^{1/2}$$

$$B_n(\lambda) = A(\lambda)g_{n1}(\lambda) + A(-\lambda)g_{n1}(-\lambda)$$

$$\mathrm{P} = \frac{p}{\rho_0} - E^2 \frac{\partial \varepsilon}{\partial \rho}, \qquad A = \frac{(\rho C_p)_m}{(\rho C_p)_f}, \qquad g_{nm}(\lambda) = -\frac{4nm\pi^2 \lambda \left[1 + (-1)^{n+m+1} e^{\lambda}\right]}{\left[\lambda^2 + (n-m)^2 \pi^2\right]\left[\lambda^2 + (n+m)^2 \pi^2\right]}$$

$$D1 = \begin{bmatrix} \left(\frac{-\omega^2}{\delta^2 A^2 P_r^{\,2}} + \frac{1}{D_a^{\,2}}\right)(n^2\pi^2 + \mathrm{k}^2)^3 - \frac{2\omega^2 (n^2\pi^2 + \mathrm{k}^2)^2}{\delta A P_r D_a} + T_a n^2\pi^2 (n^2\pi^2 + \mathrm{k}^2)^2 - \frac{R_{a0} k^2}{D_a}(n^2\pi^2 + \mathrm{k}^2) \\ -\frac{R_e \mathrm{k}^4}{D_a} R_e \mathrm{k}^4 + M_a \mathrm{k}^2 \left(\frac{-\omega^2}{\delta A P_r} + \frac{(n^2\pi^2 + \mathrm{k}^2)}{D_a}\right)(n^2\pi^2 + \mathrm{k}^2) \end{bmatrix}$$

$$D2 = \begin{bmatrix} \left(\frac{-\omega^2}{\delta^2 A^2 P_r^{\,2}} + \frac{1}{D_a^{\,2}}\right)(n^2\pi^2 + \mathrm{k}^2)^2 + \frac{2(n^2\pi^2 + \mathrm{k}^2)^3}{\delta A P_r D_a} - \frac{R_{a0} \mathrm{k}^2}{\delta A P_r}(n^2\pi^2 + \mathrm{k}^2) + T_a n^2\pi^2 (n^2\pi^2 + \mathrm{k}^2) \\ -\frac{R_e \mathrm{k}^4}{\delta A P_r} + M_a \mathrm{k}^2 \left(\frac{(n^2\pi^2 + \mathrm{k}^2)}{\delta A P_r} + \frac{1}{D_a}\right)(n^2\pi^2 + \mathrm{k}^2) \end{bmatrix}$$

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