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(T,S) INTUITIONISTIC FUZZY H-IDEALS IN BCK-ALGEBRAS

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ABSTRACT
Using triangular norms, we present a new classification of fuzzy subalgebras and ideals in BCK/BCI-algebras.
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1. INTRODUCTION

BCK/BCI-algebras are an important class of logical algebras introduced by Imai and Iseki [2], and was extensively investigated by several researches. BCK/BCI-algebras generalize, on the one hand, the notion of the algebra of sets with the set subtraction as the only fundemental non-nullary operation and, on the other hand, the notion of the implication algebra. In 1986, Atassanov [2] introduced the notion of intıitionistic fuzzy sets and in 1991, Xi [7] applied this notion to BCK/BCI-algebras. In 2018, modifying Xi's idea, Kutukcu and Tuna [5] introduced anti structures in BCK/BCI-algebras.

In the present paper, we introduce the notions of H-ideals and closed H-ideals of BCK/BCI-algebras with respect to arbitrary t-conorms and t-norms. We prove that our definitions are more general than the classical ones. We also prove that an if-subset of a BCK/BCI-algebra is a H-ideal if and only if the complement of this if-subset is a H-ideal. We also discuss some relationships between such notions. Next, let us recall some basic notions.

Definition 1.1. A BCK-algebra is a non-empty set X with a binary operation • and a constant 0 satisfying the following axioms:

(1) $(\alpha \bullet \beta) \bullet (\alpha \bullet \phi) \leq (\phi \bullet \beta)$,

(2) $\alpha \bullet (\alpha \bullet \beta) \leq \beta$,

(3) $\alpha \leq \alpha$,

(4) $\alpha \leq \beta, \beta \leq \alpha \Rightarrow \alpha = \beta$,

(5) $0 \leq \alpha$, where $\alpha \leq \beta$ is defined by $\alpha \bullet \beta = 0$.

Example 1.2. Let be $X = \{0,1,2,3,4\}$. • process should be defined as follows

•	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0
3	3	3	3	0	0
4	4	3	4	1	0

Then, (X, •, 0) is BCK-algebras.

Definition 1.3. An intuitionistic fuzzy set (if-set for short) A in a non-emty set X is an object having the form A={ $(\alpha, \mu_A(\alpha), \lambda_A(\alpha)) : \alpha \in X$ }, where the function $\mu_A: X \to [0,1]$ and $\lambda_A : X \to [0,1]$ denoted the degree of membership (namely $\mu_A(\alpha)$) and the degree of non membership (namely $\lambda_A(\alpha)$) of each element $\alpha \in X$ to the set A respectively, and $0 \leq \mu_A(\alpha) + \mu_A(\alpha) \leq 1$ for all $\alpha \in X$.

Definition 1.4. An intuitionistic fuzzy set $A = (X, \mu_A, \lambda_A)$ in X is called an intuitionistic fuzzy ideal of X, if it satisfies the following axioms:

(IF1) $\mu_A(0) \geq \mu_A(\alpha)$ and $\lambda_A(0) \leq \lambda_A(\alpha)$,

(IF2) $\mu_A(\alpha) \geq \min \{ \mu_A(\alpha \bullet \beta), \mu_A(\beta) \}$,

(IF3) $\lambda_A(\alpha) \leq \max\{ \lambda_A(\alpha \bullet \beta), \lambda_A(\beta) \}$, for all $\alpha,\beta \in X$.

Definition 1.5. An intuitionistic fuzzy set $A = (X, \mu_A, \lambda_A)$ in X called an intionistic fuzzy closed ideal of X, if it satisfies (IF2), (IF3) and the following:

(IF4) $\mu_A(0 \bullet \alpha) \geq \mu_A(\alpha)$ and $\lambda_A(0 \bullet \alpha) \leq \lambda_A(\alpha)$, for all $\alpha \in X$.

Definition 1.6. An intuitionistic fuzzy set $A = (X, \mu_A, \lambda_A)$ in X is called an intuitionistic fuzzy H-ideal of X, if

1. $\mu_A(0) \geq \mu_A(\alpha)$, $\lambda_A(0) \leq \lambda_A \leq \lambda_A(\alpha)$,
2. $\mu_A(\alpha \bullet \phi) \geq \min\{ \mu_A(\alpha \bullet (\beta \bullet \phi)), \mu_A(\beta) \}$
3. $\lambda_A(\alpha \bullet \phi) \leq \max\{ \lambda_A(\alpha \bullet (\beta \bullet \phi)), \lambda_A(\beta) \}$, for all $\alpha,\beta,\phi \in X$.

Definition 1.7. Let $A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy set in X. Then

i) $\neg A = (X, \mu_A, \overline{\mu}_A)$,

ii) $\Diamond A = (X, \overline{\lambda}_A, \lambda_A)$.

Definition 1.8. A triangular norm (t-norm for short) is a binary operation T on the unit interval [0,1], i.e., a function $T : [0,1]^2 \to [0,1]$, such that for all $\alpha,\beta,\phi \in [0,1]$ the following four axioms are satisfied:

(T1) $T(\alpha,\beta) = T(\beta,\alpha)$, (commutativity)
(T2) $T(\alpha, T(\beta,\phi)) = T(T(\alpha,\beta),\phi)$, (associativity)
(T3) $T(\alpha,\beta) \le T(\alpha,\phi)$ whenever $\beta \le \phi$, (monotonicity)
(T4) $T(\alpha, 1) = \alpha$. (boundary condition)

Some basic t-norms are $T_M(\alpha,\beta) = \min(\alpha,\beta)$, $T_P(\alpha,\beta) = \alpha.\beta$ and $T_L(\alpha,\beta) = \max(\alpha+\beta-1, 0)$.

Definition 1.9. A triangular conorm (t-conorm for short) is a binary operation A on the unit interval [0,1], i.e., a function S : $[0,1]^2 \to [0,1]$, which, for all $\alpha,\beta,\phi \in [0,1]$, satisfies (T1) – (T3) and (S4) $S(\alpha, 0) = \alpha$.

Some basic t-conorms are $S_M(\alpha,\beta) = \max(\alpha,\beta)$, $S_P(\alpha,\beta) = \alpha+\beta-\alpha.\beta$ and $S_L(\alpha,\beta) = \min(\alpha+\beta, 1)$.

2. (T,S) INTUITIONISTIC FUZZY H-IDEAL

Next, we will introduce notions of intuitionistic fuzzy H-ideals and intuitionistic fuzzy closed H-ideals with arbitrary t-norms and t-conorms, then, exemine some relationships between them.

Definition 2.1. An intuitionistic fuzzy set A = (X, μ_A, λ_A) in a BCK algebra X is called an (T,S) intuitionistic fuzzy H-ideal of X, if
(IFH 1) $\mu_A(0) \ge \mu_A(\alpha)$ and $\lambda_A(0) \le \lambda_A(\alpha)$,
(IFH 2) $\mu_A(\alpha \bullet \phi) \ge T(\mu_A(\alpha \bullet (\beta \bullet \phi)), \mu_A(\beta))$,
(IFH 3) $\lambda_A(\alpha \bullet \phi) \le S(\lambda_A(\alpha \bullet (\beta \bullet \phi)), \lambda_A(\beta))$, for all $\alpha,\beta,\phi \in X$.

Definition 2.2 An intuitionistic fuzzy set A = (X, μ_A, λ_A) in a BCK algebra X is called a (T,S) intuitionistic fuzzy closed H-ideal of X, if it satisfies (IFH 2),(IFH 3) and the following:
(IFH 4) $\mu_A(0 \bullet \alpha) \ge \mu_A(x)$ and $\lambda_A(0 \bullet \alpha) \le \lambda_A(\alpha)$, for all $\alpha \in X$.

Definition 2.3 Let A = (X, μ_A, λ_A) be a (T,S) intuitionistic fuzzy set in a BCK algebra X. The set $U(\mu_A; s) = \{\alpha \in X : \mu_A(\alpha) \ge s\}$ is called upper s-level of μ_A and the set $L(\lambda_A;t) = \{\alpha \in X : \lambda_A(\alpha) \le t\}$ is called lower t-level of λ_A.

Lemma 2.4 If A = (X, μ_A, λ_A) is an intuitionistic fuzzy H-ideal of a BCK algebra X, then we have the following $\alpha \le a \Rightarrow \mu_A(\alpha) \ge \mu_A(a)$ and $\lambda_A(\alpha) \le \lambda_A(a)$, for all α, a ∈ X.

Proof. Let α, a ∈ X such that $\alpha \le a \Rightarrow \alpha \bullet a = 0$. Consider $\mu_A(\alpha) = \mu_A(\alpha \bullet 0) \ge T(\mu_A(\alpha \bullet (a \bullet 0)), \mu_A(a)) = T(\mu_A(\alpha \bullet a), \mu_A(a)\} = \mu_A(a)$ and $\lambda_A(\alpha) = \lambda_A(\alpha \bullet 0) \le S(\lambda_A(\alpha \bullet (a \bullet 0)), \lambda_A(a)) = S(\lambda_A(\alpha \bullet a), \lambda_A(a)) = \lambda_A(a)$.

Theorem 2.5 Let A = (X, μ_A, λ_A) be an intuitionistic fuzzy H-ideal of a BCK-algebra X. Then so is $\neg A = (X, \mu_A, \overline{\mu}_A)$.

Proof. We have
$\mu_A(0) \ge \mu_A(\alpha) \Rightarrow 1 - \overline{\mu}_A(0) \ge 1 - \overline{\mu}_A(\alpha) \Rightarrow \overline{\mu}_A(0) \le \overline{\mu}_A(\alpha)$,
for any $\alpha \in X$. Consider, for any $\alpha,\beta,\phi \in X$,

$$\mu_A(\alpha \bullet \phi) \geq T(\mu_A(\alpha \bullet (\beta \bullet \phi)), \mu_A(\beta))$$
$$\Rightarrow 1 - \overline{\mu}_A (\alpha \bullet \phi) \geq T(1 - \overline{\mu}_A (\alpha \bullet (\beta \bullet \phi)), 1 - \overline{\mu}_A (\beta))$$
$$\Rightarrow \overline{\mu}_A (\alpha \bullet \phi) \leq 1 - T(1 - \overline{\mu}_A (\alpha \bullet (\beta \bullet \phi)), 1 - \overline{\mu}_A (\beta))$$
$$\Rightarrow \overline{\mu}_A (\alpha \bullet \phi) \leq S(\overline{\mu}_A (\alpha \bullet (\beta \bullet \phi)), \overline{\mu}_A (\beta)).$$

Hence $\neg A = (X, \mu_A, \overline{\mu}_A)$ is an IFH-ideal of X.

Theorem 2.6 Let $A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy H-ideal of a BCK-algebra X. Then so is $\Diamond A = (X, \overline{\lambda}_A, \lambda_A)$.

Proof. We have

$\lambda_A(0) \leq \lambda_A(\alpha) \Rightarrow 1 - \overline{\lambda}_A (0) \leq 1 - \overline{\lambda}_A (\alpha) \Rightarrow \overline{\lambda}_A (0) \geq \overline{\lambda}_A (\alpha)$,

for any $\alpha \in X$. Consider, for any $\alpha, \beta, \phi \in X$,

$$\lambda_A(\alpha \bullet \phi) \leq S(\lambda_A(\alpha \bullet (\beta \bullet \phi)), \lambda_A(\beta))$$
$$\Rightarrow 1 - \overline{\lambda}_A (\alpha \bullet \phi) \leq S(1 - \overline{\lambda}_A (\alpha \bullet (\beta \bullet \phi)), 1 - \overline{\lambda}_A (\beta))$$
$$\Rightarrow \overline{\lambda}_A (\alpha \bullet \phi) \geq 1 - S(1 - \overline{\lambda}_A (\alpha \bullet (\beta \bullet \phi)), 1 - \overline{\lambda}_A (\beta))$$
$$\Rightarrow \overline{\lambda}_A (\alpha \bullet \phi) \geq T(\overline{\lambda}_A (\alpha \bullet (\beta \bullet \phi)), \overline{\lambda}_A (\beta)).$$

Hence $\Diamond A = (X, \overline{\lambda}_A, \lambda_A)$ is an IFH-ideal of X.

Corollary 2.7 $A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy H-ideal of a BCKalgebra X if and only if $\neg A = (X, \mu_A, \overline{\mu}_A)$ and $\Diamond A = (X, \overline{\lambda}_A, \lambda_A)$ are intuitionistic fuzzy H-ideals of a BCK-algebra X.

Theorem 2.8 If $A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy closed H-ideal of a BCK-algebra X, then so is $\neg A = (X, \mu_A, \overline{\mu}_A)$.

Proof. For any $\alpha \in X$, we have

$$\mu_A(0 \bullet \alpha) \geq \mu_A(\alpha) \Rightarrow 1 - \overline{\mu}_A (0 \bullet \alpha) \geq 1 - \overline{\mu}_A (\alpha) \Rightarrow \overline{\mu}_A (0 \bullet \alpha) \leq \overline{\mu}_A (\alpha).$$

Hence $\neg A = (X, \mu_A, \overline{\mu}_A)$ is closed H-ideal of X.

Theorem 2.9 If $A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy closed H-ideal of a BCK-algebra X, then so is $\Diamond A = (X, \overline{\lambda}_A, \lambda_A)$.

Proof. For any $\alpha \in X$, we have

$$\lambda_A(0 \bullet \alpha) \leq \lambda_A(\alpha) \Rightarrow 1 - \overline{\lambda}_A (0 \bullet \alpha) \leq 1 - \overline{\lambda}_A (\alpha) \Rightarrow \overline{\lambda}_A (0 \bullet \alpha) \geq \overline{\lambda}_A (\alpha).$$

Hence, $\Diamond A = (X, \overline{\lambda}_A, \lambda_A)$ is an intuitionistic fuzzy closed H-ideal of X.

Corollary 2.10 $A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy closed H-ideal of a BCK-algebra X if and only if $\neg A = (X, \mu_A, \overline{\mu}_A)$ and $\Diamond A = (X, \overline{\lambda}_A, \lambda_A)$ are intuitionistic fuzzy closed H-ideals of BCK-algebra X.

Theorem 2.11 $A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy H-ideal of a BCK-algebra X if and only if the non-empty upper s-level cut $U(\mu_A; s)$ and the non-empty lower t-level cut $L(\lambda_A;t)$ are H-ideals of X, for any $s, t \in [0, 1]$.

Proof. Suppose $A = (X, \mu_A, \lambda_A)$ is an IFH-ideal of a BCK-algebra X. For any $s, t \in [0, 1]$, define the sets $U(\mu_A; s) = \{\alpha \in X : \mu_A(\alpha) \geq s\}$ and $L(\lambda_A;t) = \{\alpha \in X : \lambda_A(\alpha) \leq t\}$. Since $L(\lambda_A;t) = \phi$, for $\alpha \in L(\lambda_A;t) \Rightarrow \lambda_A(\alpha) \leq t$

$\Rightarrow \lambda_A(0) \leq t \Rightarrow 0 \in L(\lambda_A;t)$. Let $\alpha \bullet (\beta \bullet \phi) \in L(\lambda_A;t)$ and $\beta \in L(\lambda_A;t)$ implies $\lambda_A(\alpha \bullet (\beta \bullet \phi)) \leq t$ and $\lambda_A(\beta) \leq t$. Since, for all $\alpha, \beta, \phi \in X$, $\lambda_A(\alpha \bullet \phi) \leq S(\lambda_A(\alpha \bullet (\beta \bullet \phi)), \lambda_A(\beta)) \leq S(t, t\} = t \Rightarrow \lambda_A(\alpha \bullet \phi) \leq t$. Therefore $\alpha \bullet \phi \in L(\lambda_A;t)$, for all $\alpha,\beta,\phi \in X$. Hence $L(\lambda_A;t)$ is an H-ideal of X. Similarly, we can prove $U(\mu_A; s)$ is an H-ideal of X. Conversly, suppose that $U(\mu_A; s)$ and $L(\lambda_A;t)$ are H-ideal of X, for any $s, t \in [0, 1]$. If possible, assume $\alpha_0, \beta_0 \in X$ such that $\mu_A(0) < \mu_A(x_0)$ and $\lambda_A(0) > \lambda_A(y_0)$. Put

$s_0 = 1/2\ [\mu_A(0) + \mu_A(\alpha_0)] \Rightarrow s_0 < \mu_A(\alpha_0),\ 0 \leq \mu_A(0) < s_0 < 1 \Rightarrow \alpha_0 \in U(\mu_A; s_0)$.

Since $U(\mu_A; s_0)$ is an H-ideal of X, we have $0 \in U(\mu_A; s_0) \Rightarrow \mu_A(0) \geq s_0$, which is contradiction. Therefore $\mu_A(0) \geq \mu_A(\alpha)$, for all $\alpha \in X$. Similarly by taking $t_0 = 1/2\ [\lambda_A(0) + \lambda_A(\beta_0)]$, we can show $\lambda_A(0) \leq \lambda_A(\beta)$, for any $\beta \in X$. If possible assume $\alpha_0, \beta_0, \phi_0 \in X$ such that $\mu_A(\alpha_0 \bullet \phi_0) < T(\mu_A(\alpha_0 \bullet (\beta_0 \bullet \phi_0)), \mu_A(\beta_0))$.
Put $s_0 = 1/2[\mu_A(\alpha_0 \bullet \phi_0) + T(\mu_A(\alpha_0 \bullet (\beta_0 \bullet \phi_0)), \mu_A(\beta_0))]$
$\Rightarrow s_0 > \mu_A(\alpha_0 \bullet \phi_0)$ and $s_0 < T(\mu_A(\alpha_0 \bullet (\beta_0 \bullet \phi_0)), \mu_A(\beta_0))$
$\Rightarrow s_0 > \mu_A(\alpha_0 \bullet \phi_0)$, $s_0 < \mu_A(\alpha_0 \bullet (\beta_0 \bullet \phi_0))$ and $s_0 < \mu_A(\beta_0)$
$\Rightarrow \alpha_0 \bullet \phi_0 \in U(\mu_A; s_0)$, $\alpha_0 \bullet (\beta_0 \bullet \phi_0) \in U(\mu_A; s_0)$ and $\beta_0 \in U(\mu_A; s_0)$,
which is contradiction to H-ideal $U(\mu_A; s_0)$.
Therefore $\mu_A(\alpha \bullet \phi) \geq T(\mu_A(\alpha \bullet (\beta \bullet \phi)), \mu_A(\beta))$, for any $\alpha,\beta,\phi \in X$. Similarly we can prove $\lambda_A(\alpha \bullet \phi) \leq S(\lambda_A(\alpha \bullet (\beta \bullet \phi)), \lambda_A(\beta))$, for any $\alpha, \beta, \phi \in X$. Hence $A = (X, \mu_A, \lambda_A)$ is an intuitionistic fuzzy H-ideal of a BCK-algebra X.

Theorem 2.12 $A = (X, \mu_A, \lambda_A)$ is an intuitionistic fuzzy closed H-ideal of a BCK-algebra X if and only if the non-empty upper s-level cut $U(\mu_A; s)$ and the non-empty lower t-level cut $L(\lambda_A;t)$ are closed H-ideal of X, for any $s, t \in [0, 1]$.

Proof. Suppose $A = (X, \mu_A, \lambda_A)$ is an intuitionistic fuzzy closed H-ideal of a BCK-algebra X. We have $\mu_A(0 \bullet \alpha) \geq \mu_A(\alpha)$ and $\lambda_A(0 \bullet \alpha) \leq \lambda_A(\alpha)$, for any $\alpha \in X$.
For $\alpha \in U(\mu_A; s) \Rightarrow \alpha \in X$ and $\mu_A(\alpha) \geq s \Rightarrow \mu_A(0 \bullet \alpha) \geq s \Rightarrow 0 \bullet \alpha \in U(\mu_A; s)$. And $\alpha \in L(\lambda_A;t) \Rightarrow \alpha \in X$ and $\lambda_A(\alpha) \leq t \Rightarrow \lambda_A(0 \bullet \alpha) \leq t \Rightarrow 0 \bullet \alpha \in L(\lambda_A;t)$. Therefore $U(\mu_A; s)$ and $L(\lambda_A;t)$ are closed H-ideals of X. Converse, it is enough to show that $\mu_A(0 \bullet \alpha) \geq \mu_A(\alpha)$ and $\lambda_A(0 \bullet \alpha) \leq \lambda_A(\alpha)$, for any $\alpha \in X$. If possible, assume $\alpha_0 \in X$ such that $\mu_A(0 \bullet \alpha_0) < \mu_A(\alpha_0)$. Take $s_0 = 1/2\ [\mu_A(0 \bullet \alpha)+\mu_A(\alpha_0)] \Rightarrow \mu_A(0 \bullet \alpha_0) < s_0 < \mu_A(\alpha_0)$ $\Rightarrow \alpha_0 \in U(\mu_A; s_0)$, but $0 \bullet \alpha_0 \in U(\mu_A; s_0)$, which is contradiction to closed H-ideal. Hence $\mu_A(0 \bullet \alpha) \geq \mu_A(\alpha)$, for any $\alpha \in X$. Similarly we can prove that $\lambda_A(0 \bullet \alpha) \leq \lambda_A(\alpha)$, for any $\alpha \in X$.

Corollary 2.13 If $A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy closed H-ideal of X, then the sets $J = \{\alpha \in X : \mu_A(\alpha) = \mu_A(0)\}$ and $K = \{\alpha \in X : \lambda_A(\alpha) = \lambda_A(0)\}$ are H-ideal of X.

Proof. Since $0 \in X$, $\mu_A(0) = \mu_A(0)$ and $\lambda_A(0) = \lambda_A(0)$ implies $0 \in J$ and $0 \in K$, So $J = \Phi$ and $K = \Phi$. Let $\alpha \bullet (\beta \bullet \phi) \in J$ and $\beta \in J \Rightarrow \mu_A(\alpha \bullet (\beta \bullet \phi)) = \mu_A(0)$ and $\mu_A(\beta) = \mu_A(0)$. Since $\mu_A(\alpha \bullet \phi) \geq T(\mu_A(\alpha \bullet(\beta \bullet \phi)), \mu_A(\beta)) = \mu_A(0) \Rightarrow \mu_A(\alpha \bullet \phi) \geq \mu_A(0)$, but $\mu_A(0) \geq \mu_A(\alpha \bullet \phi)$. It follows that $\alpha \bullet \phi \in J$, for all $\alpha,\beta,\phi \in X$. Hence J is H-ideal of X. Similarly we can prove K is H-ideal of X.

Definition 2.14 Let f be a mapping on a set X and $A = (X, \mu_A, \lambda_A)$ an intuitionistic fuzzy set in X. Then the fuzzy sets u and v on f(X) defined by $u(y) = \sup_{\alpha \in f^{-1}(y)} \mu_A(x)$ and $v(\beta) = \inf_{\alpha \in f^{-1}(y)} \lambda_A(\alpha)$ for all y ∈

f(X), is called image of A under f. If u, v are fuzzy sets in f(X) then the fuzzy sets μ_A = uof and λ_A = vof is called the pre-image of u and v under f.

Theorem 2.15 Let f : X → X be an onto homomorphism of BCK algebras. If A = (X, u, v) is an intuitionistic fuzzy H-ideal of X , then the pre-image of A under f is an intuitionistic fuzzy H-ideal of X.

Proof. Let A = (X, μ_A, λ_A), where μ_A = uof and λ_A = vof is the pre-image of A = (X , u, v) under f. Since A = (X, u, v) is an intuitionistic fuzzy H ideal of X , we have $u(0) \geq u(f(\alpha)) = \mu_A(\alpha)$ and $v(0) \leq v(f(\alpha)) = \lambda_A(\alpha)$. On other hand $u(0) = u(f(0)) = \mu_A(0)$ and $v(0) = v(f(0)) = \lambda_A(0)$. Therefore $\mu_A(0) \geq \mu_A(\alpha)$ and $\lambda_A(0) \leq \lambda_A(\alpha)$, for all $\alpha \in X$. Now we show that

(1). $\mu_A(\alpha \bullet \phi) \geq T(\mu_A(\alpha \bullet (\beta \bullet \phi)), \mu_A(\beta))$,

(2). $\lambda_A(\alpha \bullet \phi) \leq S(\lambda_A(\alpha \bullet (\beta \bullet \phi)), \lambda_A(\beta))$, for any $\alpha,\beta,\phi \in X$.

We have

$\mu_A(\alpha \bullet \phi) = u(f(\alpha \bullet \phi)) = u(f(\alpha) \bullet f(\phi)) \geq T(u(f(\alpha) \bullet (\beta \bullet f(\phi))), u(\beta))$, for $\beta \in X$. Since f is onto homomorphism, there is $\beta \in X$ such that $f(\beta) = \beta$. Thus

$$\begin{aligned}\mu_A(\alpha \bullet \phi) &\geq T(u(f(\alpha) \bullet (\beta \bullet f(\phi))), u(\beta))\\ &= T(u(f(\alpha) \bullet (f(\beta) \bullet f(\phi))), u(f(\beta)))\\ &= T(u(f(\alpha \bullet (\beta \bullet \phi)), u(f(\beta)))\\ &= T(\mu_A((\alpha \bullet (\beta \bullet \phi)), \mu_A(\beta)),\end{aligned}$$

for all $\alpha,\beta,\phi \in X$. Therefore, the result $\mu_A(\alpha \bullet \phi) \geq T(\mu_A(\alpha \bullet (\beta \bullet \phi)), \mu_A(\beta))$, is true for all $\alpha,\beta,\phi \in X$, because β is an arbitrary element of X and f is onto mapping. Similarly, we can prove $\lambda_A(\alpha \bullet \phi) \leq S(\lambda_A(\alpha \bullet (\beta \bullet \phi)), \lambda_A(\beta))$, for any $\alpha, \beta, \phi \in X$. Hence the pre-image A = (X, μ_A, λ_A), of A is an intuitionistic H-ideal of X.

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