Vol.8.Issue.2.2020 (April-June)

http://www.bomsr.com
Email:editorbomsr@gmail.com

RESEARCH ARTICLE

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

AN EFFICIENT PRODUCT-TYPE EXPONENTIAL ESTIMATOR IN POST-STRATIFICATION

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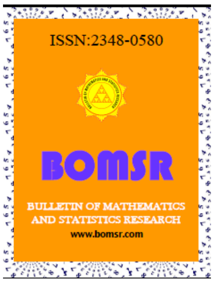
DOI: 10.33329/bomsr.8.2.1



ABSTRACT
Following Tailor & Tailor (2012) and Panda & Sahoo (2020), we propose a product-type exponential estimator in post-stratified sampling using variable transformation. The proposed estimator, under certain conditions are shown to be more efficient than the usual unbiased estimator in case of post-stratification and some existing estimators. The efficiency of the newly proposed estimator has shown through numerical illustration and graphical representation.
Keywords – Auxiliary Information, Variable Transformation, Post-Stratified sampling, Bias, Mean Square Error and Bar Diagram.

1. INTRODUCTION

Stratification is a commonly used techniques in sample survey. Stratified sampling presumes the knowledge that the strata size and sampling frame for each stratum are known. But in many situations, it happens that the overall population size and also the percentage of unit that falls in the strata are known but the sampling frame for the stratum are may not be available or it quit time-consumable for preparing. In such situations we can not be supposed to use stratified random sampling. So, to overcome from this kind of problem we use post-stratification technique in which sample of size 'n' is drawn by simple random sampling without replacement, these sample is stratified in different strata and used as stratified samples.

Let us consider the population size N that stratified into L strata of size $N_1, N_2, N_3 \dots\dots\dots N_L$ such that $\sum_{h=1}^{L} N_h = N$. Let n_h be the sample size falling in the h^{th} stratum such that $\sum_{h=1}^{L} n_h = n$.

Now consider x as the auxiliary variable which is negatively correlated with the study variable y. Let y_{hi} be the observation on i^{th} unit of h^{th} stratum for the study variable y and x_{hi} be the observation on i^{th} unit of h^{th} stratum for the auxiliary variable x, then

$\bar{Y}_h = \frac{1}{N_h}\sum_{h=1}^{N_n} y_{hi}$, $\bar{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{h=1}^{N_h} y_{hi} = \sum_{h=1}^{L} W_h \bar{y}_h$, $\bar{X}_h = \frac{1}{N_h}\sum_{h=1}^{N_n} x_{hi}$,

$\bar{X} = \frac{1}{N}\sum_{h=1}^{L}\sum_{h=1}^{N_h} x_{hi} = \sum_{h=1}^{L} W_h \bar{x}_h$

The usual unbiased estimator of population mean $\bar{Y}$ in Post-stratification is defined as

$$\bar{y}_{PS} = \sum_{h=1}^{L} W_h \bar{y}_h \tag{1.1}$$

where $W_h = \frac{N_h}{N}$, and $\bar{y}_h$is the mean of n_h sample units.

Then

$$V(\bar{y}_{PS}) = \frac{N-n}{Nn}\sum_{h=1}^{L} W_h S_{yh}^2 \tag{1.2}$$

Ige and Tripathy (1989) have proposed a product-type estimator by using the information on population mean $\bar{X}$ of auxiliary variate x for population mean in post-stratification as,

$$\bar{y}_{PPS} = \bar{y}_{PS}\left(\frac{\bar{x}_{PS}}{\bar{X}}\right) \tag{1.3}$$

where $\bar{y}_{PS} = \sum_{h=1}^{L} W_h \bar{y}_h$ and $\bar{x}_{PS} = \sum_{h=1}^{L} W_h \bar{x}_h$ are unbiased estimators of population mean $\bar{Y} = \sum_{h=1}^{L} W_h \bar{Y}_h$ and $\bar{X} = \sum_{h=1}^{L} W_h \bar{X}_h$ in post-stratified random sampling, and $\bar{x}_h$ is the mean of the sample size n_h that falls in h^{th} stratum.

The bias and mean-square error of the Ige and Tripathy (1989) estimator is,

$$B(\bar{y}_{PPS}) = \left(\frac{1}{n} - \frac{1}{N}\right)\frac{1}{\bar{X}}\sum_{h=1}^{L} W_h S_{yxh} \tag{1.4}$$

$$MSE(\bar{y}_{PPS}) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\left(S_{yh}^2 + R^2 S_{xh}^2 - 2RS_{yxh}\right) \tag{1.5}$$

where $= \frac{\bar{Y}}{\bar{X}}$, $S_{xh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(x_{hi} - \bar{X}_h)^2$ and $S_{yxh} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}(y_{hi} - \bar{Y}_h)(x_{hi} - \bar{X}_h)$.

R. Tailor, R. Tailor and S. Chouhan (2017) suggested a new product-type exponential estimator for population mean in post-stratification as,

$$\hat{\bar{y}}_{Pe}^{PS} = \bar{y}_{PS} exp\left(\frac{\bar{x}_{PS} - \bar{X}_h}{\bar{x}_{PS} + \bar{X}_h}\right) = \bar{y}_{PS} exp\left[\frac{\sum_{h=1}^{L} W_h(\bar{x}_h - \bar{X}_h)}{\sum_{h=1}^{L} W_h(\bar{x}_h + \bar{X}_h)}\right] \tag{1.6}$$

Bias and mean-square error of the R. Tailor etal. (2017) estimator is expressed as,

$$B\left(\hat{\bar{y}}_{Pe}^{PS}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\frac{R(4K-1)}{8\bar{X}}\sum_{h=1}^{L} W_h S_{xh}^2 \tag{1.7}$$

$$MSE\left(\hat{\bar{y}}_{Pe}^{PS}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\left[\sum_{h=1}^{L} W_h S_{yh}^2 + \frac{1}{4}(1+4K)R^2\sum_{h=1}^{L} W_h S_{xh}^2\right] \tag{1.8}$$

where $= \frac{\sum_{h=1}^{L} W_h S_{yxh}}{R\sum_{h=1}^{L} W_h S_{xh}^2}$.

Following the R. Tailor etal. (2017), Tailor & Tailor (2012) and Panda & Sahoo (2020) a new estimator is proposed in case of post-stratification.

2. PROPOSED ESTIMATOR

Motivated from Tailor & Tailor (2012) and Panda & Sahoo (2020), We proposed a product- type exponential estimator in case of post-stratification sampling for population mean $\bar{Y}$:

$$\bar{y}_{PS}^{**Pe} = \bar{y}_{PS}\left[\left\{exp\left(\frac{\bar{X} - \bar{x}_{PS}^*}{\bar{X} + \bar{x}_{PS}^*}\right)\right\}\left(\frac{\bar{X}}{\bar{x}_{PS}^*}\right)^{\alpha}\right], \tag{2.1}$$

where α is a scalar quantity and $\bar{x}^{*}_{PS} = \frac{N\bar{X} - n\bar{x}_{PS}}{N-n}$.

To find the bias and mean square error of the proposed estimator $\bar{y}_{PS}^{**Pe}$ we have defined the following quantities:

$\bar{y}_{PS} = \bar{Y}(1+e_0), \bar{x}_{PS} = \bar{X}(1+e_1)$, such that $E(e_0) = E(e_1) = 0$,

$E({e_0}^2) = \left(\frac{1}{n} - \frac{1}{N}\right)\frac{1}{\bar{Y}^2}\sum_{h=1}^{L} W_h S_{yh}^2,\ \ E({e_1}^2) = \left(\frac{1}{n} - \frac{1}{N}\right)\frac{1}{\bar{X}^2}\sum_{h=1}^{L} W_h S_{xh}^2$

$E(e_0 e_1) = \left(\frac{1}{n} - \frac{1}{N}\right)\frac{1}{\bar{Y}\bar{X}}\sum_{h=1}^{L} W_h S_{yxh}$.

By expressing the (2.1) in terms of e's we find,

$$\bar{y}_{PS}^{**Pe} = \bar{Y}\left[1 + e_0 + \left(\alpha + \frac{1}{2}\right)\frac{ne_1}{(N-n)} + \left(\alpha + \frac{1}{2}\right)\frac{ne_0 e_1}{(N-n)} + \left(\frac{4\alpha^2 + 8\alpha + 3}{8}\right)\frac{n^2 {e_1}^2}{(N-n)^2}\right] \quad (2.2)$$

From (2.2) we rise to,

$$B\left(\bar{y}_{PS}^{**Pe}\right) = \bar{Y}\left(\frac{1}{n} - \frac{1}{N}\right)\left[\left(\alpha + \frac{1}{2}\right)\frac{n}{(N-n)}\frac{1}{\bar{X}\bar{Y}}\sum_{h=1}^{L} W_h S_{yxh} + \left\{\left(\frac{4\alpha^2 + 8\alpha + 3}{8}\right)\frac{n^2}{(N-n)^2}\right\}\frac{1}{\bar{X}^2}\sum_{h=1}^{L} W_h S_{xh}^2\right] \quad (2.3)$$

MSE$\left(\bar{y}_{PS}^{**Pe}\right) =$

$$\left(\frac{1}{n} - \frac{1}{N}\right)\left[\sum_{h=1}^{L} W_h {S_{yh}}^2 + \left(\alpha + \frac{1}{2}\right)^2 \frac{n^2}{(N-n)^2} R^2 \sum_{h=1}^{L} W_h {S_{xh}}^2 + 2\left(\alpha + \frac{1}{2}\right)\frac{n}{N-n} R \sum_{h=1}^{L} W_h S_{yxh}\right]. \quad (2.4)$$

Above expression of mean square error is the first order approximation i.e. to $0(n^{-1})$.

3. OPTIMUM VALUE OF α

Differentiating (2.4) with respect to α and equating it to zero, we get the optimum value of α as,

$$\alpha_{opt.} = \frac{N-n}{nR}\left(\frac{-\sum_{h=1}^{L} W_h S_{yxh}}{\sum_{h=1}^{L} W_h S_{xh}^2}\right) - \frac{1}{2} \quad (3.1)$$

By using (3.1) we can find that, the value of MSE$\left(\bar{y}_{PS}^{**Pe}\right)$ to the first order approximation will be,

$$\text{MSE}\left(\bar{y}_{PS}^{**Pe}\right)_{opt.} = \left(\frac{1}{n} - \frac{1}{N}\right)\left[\sum_{h=1}^{L} W_h S_{yh}^2 - \frac{\left(\sum_{h=1}^{L} W_h S_{yxh}\right)^2}{\sum_{h=1}^{L} W_h S_{xh}^2}\right]. \quad (3.2)$$

4. EFFICIENCY COMPARISION

By comparing (3.2) with (1.2), (1.5) and (1.8) , we found that the proposed estimator $\bar{y}_{PS}^{**Pe}$ would be more efficient if,

(i) $MSE\left(\bar{y}_{PS}^{**Pe}\right)_{opt.} < MSE(\bar{y}_{PS})$

$$\Rightarrow \frac{\left(\sum_{h=1}^{L} W_h S_{yxh}\right)^2}{\sum_{h=1}^{L} W_h S_{xh}^2} > 0 \quad (4.1)$$

(ii) $MSE\left(\bar{y}_{PS}^{**Pe}\right)_{opt.} < MSE(\bar{y}_{PPS})$

$$\Rightarrow \frac{\left(\sum_{h=1}^{L} W_h S_{yxh}\right)^2}{\sum_{h=1}^{L} W_h S_{xh}^2} > 2R\sum_{h=1}^{L} W_h\, S_{yxh} - R^2 \sum_{h=1}^{L} W_h S_{xh}^2 \quad (4.2)$$

(iii) $MSE\left(\bar{y}_{PS}^{**Pe}\right)_{opt.} < MSE\left(\hat{\bar{y}}_{Pe}^{PS}\right)$

$$\Rightarrow \frac{\left(\sum_{h=1}^{L} W_h S_{yxh}\right)^2}{\sum_{h=1}^{L} W_h S_{xh}^2} > R\sum_{h=1}^{L} W_h S_{yxh} - \frac{1}{4}R^2 \sum_{h=1}^{L} W_h S_{xh}^2 \quad (4.3)$$

5. NUMERICAL ILLUSTRATION

The proposed estimator is more efficient than the existing estimators i.e. $\bar{y}_{PS}, \bar{y}_{PPS}$ and $\hat{\bar{y}}_{Pe}^{PS}$ with respect to the numerical data. In this section we have computed the percent relative efficiencies (PRE) of $\bar{y}_{PS}, \bar{y}_{PPS}$, $\hat{\bar{y}}_{Pe}^{PS}$ and $\bar{y}_{PS}^{**Pe}$ with respect to $\bar{y}_{PS}$ using the respective formulae given below,

$$PRE(\bar{y}_{PS}, \bar{y}_{PS}) = \frac{MSE(\bar{y}_{PS})}{MSE(\bar{y}_{PS})} \times 100 \tag{5.1}$$

$$PRE(\bar{y}_{PPS}, \bar{y}_{PS}) = \frac{MSE(\bar{y}_{PS})}{MSE(\bar{y}_{PPS})} \times 100 \tag{5.2}$$

$$PRE\left(\hat{\bar{y}}_{Pe}^{PS}, \bar{y}_{PS}\right) = \frac{MSE(\bar{y}_{PS})}{MSE(\hat{\bar{y}}_{Pe}^{PS})} \times 100 \tag{5.3}$$

$$PRE\left(\bar{y}_{PS}^{**Pe}, \bar{y}_{PS}\right) = \frac{MSE(\bar{y}_{PS})}{MSE(\bar{y}_{PS}^{**Pe})} \times 100 \tag{5.4}$$

To examine the performance of the proposed estimator with respect to the other estimator , one population data set is considered. The details of the population are given below:

Population [Source: Japan metrological society]

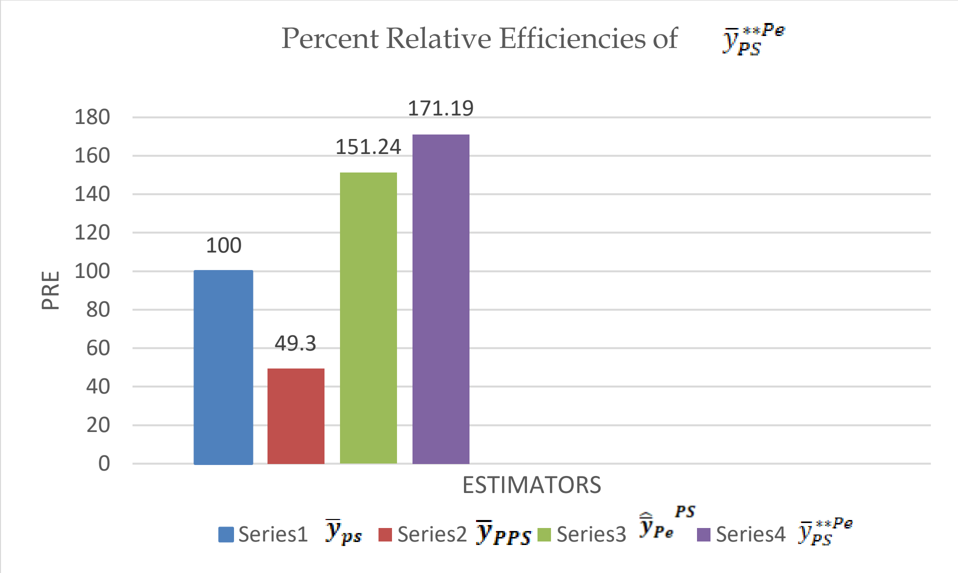
Constants	n_h	N_h	$\bar{X}_h$	$\bar{Y}_h$	S_{xh}^2	S_{yh}^2	S_{yxh}
Stratum I	4	10	149.7	1629.99	181.41	10439.63	-1072.8
Stratum II	4	10	102.6	2035.96	158.84	10663.53	-655.25

Table: Percent relative efficiencies of $\bar{y}_{PS}, \bar{y}_{PPS}$, $\hat{\bar{y}}_{Pe}^{PS}$ and $\bar{y}_{PS}^{**Pe}$ with respect to $\bar{y}_{PS}$

Estimator	$\bar{y}_{PS}$	$\bar{y}_{PPS}$	$\hat{\bar{y}}_{Pe}^{PS}$	$\bar{y}_{PS}^{**Pe}$
Population	100	49.3	151.24	171.19

6. GRAPHICAL STUDY

The percent relative efficiencies of the estimator $\bar{y}_{PS}^{**Pe}$ is more efficient than the existing estimators have shown through the given bar diagram.



7. CONCLUSION

From above Table and graphical study, we conclude that the proposed estimator $\bar{y}_{PS}^{**Pe}$ is more efficient than the existing estimators i.e., $\bar{y}_{PS}, \bar{y}_{PPS}$, $\hat{\bar{y}}_{Pe}^{PS}$.

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