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IMPROVED ESTIMATORS FOR POPULATION MEAN USING AUXILIARY ATTRIBUTE IN THE PRESENCE OF NON-RESPONSE

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ABSTRACT

In this paper, improved estimators for population mean using known population proportion of auxiliary attribute and known coefficient of variation of study character in the presence of non-response have been proposed. The properties of the proposed estimators are studied and the conditions are obtained in which proposed estimators are better than the relevant estimators. The empirical studies are also carried out to judge the performance of the proposed estimators in comparison to the relevant estimators.

Keywords: Study character, auxiliary attribute, known coefficient of variation of study character, non-response, mean square error.

1. INTRODUCTION

The information on auxiliary attribute has an important contribution in increasing the efficiency of the estimates of the population mean. For instance, milk production, crop production and income depend on the breed type, seed type and ownership of a house respectively. In this context, a lot of research works for estimation of population mean have been done by Naik and Gupta (1996), Jhajj et al. (2006), Shabbir and Gupta (2007, 2010), Singh et.al (2007), Abd-Elfattah et al. (2010), Singh and Solanki (2012) and Solanki and Singh (2013).

By using sampling techniques we can estimate the population mean. But sometimes information on all the units selected in the sample is not obtained. In such situation, Hansen and Hurwitz (1946) first suggested a technique of sub sampling from non-respondent in mail surveys. Using the known population mean of auxiliary character, the research work of Hansen and Hurwitz (1946) was extended by Cochran (1977), Rao (1986, 1987), Khare and Srivastava (1996, 1997) and Singh et al. (2010). Further using the known population proportion of auxiliary attribute, some research works are also have been done by Kumar and Kumar (2019). The research works for estimation of population mean using known coefficient of variation of study character have been done by Searls (1964, 1967) and Sen (1978). Further by using known coefficient of variation of study character, some estimators for population mean in the presence of non-response have been suggested by Kumar and Kumar (2017).

Let ($\bar{Y}$, P) denote the population mean and population proportion of study character Y_j and auxiliary attribute ϕ_j : j =1…M respectively. Here ϕ_j will take two values 1 and 0 if it possesses and does not possess the attribute respectively. Let M_1 and M_2 are responding and non-responding units in a population of size M such that $M = M_1 + M_2$. Let P be the proportion of the units in the population possessing the attribute ϕ and (P_1, P_2) be the proportion of the units possessing the attribute ϕ in responding and non-responding part of the population. Let a sample of size m is drawn from the population of size M by using simple random sampling without replacement (SRSWOR) method. After getting the information on m units we found that m_1 units respond and m_2 units do not respond in a sample of size m for the study character y and auxiliary attribute ϕ. Further a subsample of size $r = (m_2 / k, k > 1)$ is drawn from m_2 non-responding units by using SRSWOR method of sampling and collects the information on study character y and on auxiliary attribute ϕ by personal interview. Hence by using Hansen and Hurwitz (1946) techniques, we define the estimators $\bar{y}^*$ and p^* for the population mean $\bar{Y}$ and for the population proportion P based on $(m_1 + r)$ units as:

$$\bar{y}^* = \theta_1 \bar{y}_1 + \theta_2 \bar{y}_2' \tag{1.1}$$

and

$$p^* = \theta_1 p_1 + \theta_2 p_2' , \tag{1.2}$$

where $\theta_1 = m_1 / m$, $\theta_2 = m_2 / m$ and $(\bar{y}_1, \bar{y}_2')$ denote the means of study character y based on m_1 responding units and r subsample units and (p_1, p_2') are the proportions of the units possessing the attribute ϕ in m_1 responding units and in r subsample units respectively.

The mean square error of the estimators $\bar{y}^*$ and p^* are given as:

$$MSE(\bar{y}^*) = A_1 S_y^2 + A_2 S_y'^2 \tag{1.3}$$

and

$$MSE(p^*) = A_1 S_\phi^2 + A_2 S_\phi'^2 , \tag{1.4}$$

where $A_1 = \frac{1}{m} - \frac{1}{M}$, $A_2 = \frac{W_2(k-1)}{m}$, $W_2 = \frac{M_2}{M}$, $(S_y^2, S_y'^2)$ and $(S_\phi^2, S_\phi'^2)$ are population mean squares of study character y and auxiliary attribute ϕ for responding and non-responding parts of the population.

Using the known value of the coefficient of variation of study character, Searls (1964) first defined an estimator for population mean which is given as follows:

$$\bar{y}_s = d_0 \bar{y}\,, \tag{1.5}$$

where $\bar{y} = \frac{1}{m}\sum_{j=1}^{m} y_j$ and d_0 is constant.

By minimizing the mean square error of the estimator $\bar{y}_s$, the optimum value of d_0 is obtained as follows:

$$d_{0(opt)} = \left(1 + A_1 V_y^2\right)^{-1}, \tag{1.6}$$

where $V_y = S_y / \bar{Y}$ is the known coefficient of variation of study character y.

Further using Searls (1964) estimator, Khare and Kumar (2009) proposed the estimator for population mean in the presence of non-response which is given as:

$$\bar{y}^{**} = d_1 \bar{y}^*\,, \tag{1.7}$$

where d_1 is constant.

By minimizing the mean square error of the estimator $\bar{y}^{**}$, the optimum value of d_1 is obtained as:

$$d_{1(opt)} = \left(1 + A_1 \frac{S_y^2}{\bar{Y}^2} + A_2 \frac{S_y'^2}{\bar{Y}^2}\right)^{-1} \tag{1.8}$$

If we take $\frac{S_y^2}{\bar{Y}^2} = \frac{S_y'^2}{\bar{Y}^2} = V_y^2$ and neglecting the term of order $1/M$, we get

$$d_{1(opt)} = \left[1 + \frac{V_y^2}{m}\left\{1 + \frac{M_2}{M}(k-1)\right\}\right]^{-1}. \tag{1.9}$$

Hence the mean square error of the estimator $\bar{y}^{**}$ is obtained as:

$$MSE(\bar{y}^{**}) = (1-2L)\left\{A_1 S_y^2 + A_2 S_y'^2\right\}, \tag{1.10}$$

where $L = \frac{V_y^2}{m}\left\{1 + \frac{M_2}{M}(k-1)\right\}$.

In situation when non-response occurs both on study character y and on auxiliary attribute ϕ, the conventional generalized ratio estimator for population mean of study character in the presence of non-response is defined as:

$$T_1 = \bar{y}^*\left(\frac{P}{p^*}\right)^a, \tag{1.11}$$

where a is constant.

For $a = 1, T_1$ reduces to $t_1 = \bar{y}^* \frac{P}{p^*}$ and for $a = -1, T_1$ reduces to $t_2 = \bar{y}^* \frac{p^*}{P}$.

In situation when non-response occurs only on study character y , the alternative generalized ratio estimator for population mean of study character in the presence of non-response is defined as:

$$T_2 = \bar{y}^* \left(\frac{P}{p} \right)^b , \tag{1.12}$$

where b is constant and p is the proportion of the units possessing the attribute ϕ in the sample of size m units.

For $b = 1, T_2$ reduces to $t_3 = \bar{y}^* \frac{P}{p}$ and for $b = -1, T_2$ reduces to $t_4 = \bar{y}^* \frac{p}{P}$.

2. THE PROPOSED ESTIMATORS

In situation when non-response occurs both on study character y and on auxiliary attribute ϕ , we propose the conventional generalized ratio estimator for population mean using known population proportion of auxiliary attribute ϕ and know coefficient of variation of study character y in the presence of non-response, which is given as follows:

$$T_{k1} = \bar{y}^{**} \left(\frac{P}{p^*} \right)^\alpha , \tag{2.1}$$

where α is constant.

For $\alpha = 1, T_{k1}$ reduces to $t_{k1} = \bar{y}^{**} \frac{P}{p^*}$ and for $\alpha = -1, T_{k1}$ reduces to $t_{k2} = \bar{y}^{**} \frac{p^*}{P}$.

In situation when non-response occurs only on study character y , we propose the alternative generalized ratio estimator for population mean using known population proportion of auxiliary attribute ϕ and know coefficient of variation of study character y in the presence of non-response, which is given as follows:

$$T_{k2} = \bar{y}^{**} \left(\frac{P}{p} \right)^\beta , \tag{2.2}$$

where β is constant.

For $\beta = 1, T_{k2}$ reduces to $t_{k3} = \bar{y}^{**} \frac{P}{p}$ and for $\beta = -1, T_{k2}$ reduces to $t_{k4} = \bar{y}^{**} \frac{p}{P}$.

3. THE MEAN SQUARE ERROR (MSE) OF THE PROPOSED ESTIMATORS

In order to derive the expressions for the mean square error of the proposed estimators T_{k1} and T_{k2} .

Let $\bar{y}^* = \bar{Y}(1 + e_0)$, $p^* = P(1 + e_1)$, $p = P(1 + e_2)$ such that $E(e_0) = E(e_1) = E(e_2) = 0$.

By using simple random sampling without replacement method, we get

$E(e_0^2) = \frac{1}{\bar{Y}^2} V(\bar{y}^*) = \frac{1}{\bar{Y}^2}\left[A_1 S_y^2 + A_2 S_y'^2\right],$

$E(e_1^2) = \frac{1}{P^2} V(p^*) = \frac{1}{P^2}\left[A_1 S_\phi^2 + A_2 S_\phi'^2\right],$

$E(e_2^2) = \frac{1}{P^2} V(p) = A_1 \frac{S_\phi^2}{P^2},\ E(e_0 e_1) = \frac{1}{\bar{Y}P} COV(\bar{y}^*, p^*) = \frac{1}{\bar{Y}P}\left[A_1 S_{y\phi} + A_2 S_{y\phi}'\right],$

$$E(e_0 e_2) = \frac{1}{\bar{Y}P} COV(\bar{y}^*, p) = \frac{1}{\bar{Y}P} A_1 S_{y\phi}, \tag{3.1}$$

where $S_{y\phi} = \rho_{y\phi} S_y S_\phi$, $S'_{y\phi} = \rho'_{y\phi} S'_y S'_\phi$ and $(\rho_{y\phi}, \rho'_{y\phi})$ are point bi-serial correlation coefficients between study character y and auxiliary attribute ϕ for responding and not responding part of the population.

The expressions for the mean square error of the proposed estimators T_{k1} and T_{k2} up to the terms of order m^{-1} are obtained as follows:

$$MSE(T_{k1}) = MSE(\bar{y}^{**}) + \bar{Y}^2\left[\left(\frac{1}{m} - \frac{1}{M}\right)\left\{\alpha(\alpha - (3\alpha+1)L)C_p^2 - 2\alpha(1-3L)C_{yp}\right\} + \frac{W_2(k-1)}{m}\left\{\alpha(\alpha - (3\alpha+1)L)C_p'^2 - 2\alpha(1-3L)C'_{yp}\right\}\right] \tag{3.2}$$

and

$$MSE(T_{k2}) = MSE(\bar{y}^{**}) + \bar{Y}^2\left[\left(\frac{1}{m} - \frac{1}{M}\right)\left\{\beta(\beta - (3\beta+1)L)C_p^2 - 2\beta(1-3L)C_{yp}\right\}\right] \tag{3.3}$$

The expressions for the optimum values of α and β which minimize the $MSE(T_{k1})$ and $MSE(T_{k2})$ respectively, are obtained as:

$$\alpha_{opt} = \frac{\left\{A_1 C_{yp} + A_2 C'_{yp}\right\}}{\left\{A_1 C_p^2 + A_2 C_p'^2\right\}} + \frac{L}{2(1-3L)} \text{ and } \beta_{opt} = \frac{C_{yp}}{C_p^2} + \frac{L}{2(1-3L)} \tag{3.4}$$

The expressions for the minimum mean square error of the estimators T_{k1} and T_{k2} are obtained as:

$$MSE(T_{k1})_{\min} = MSE(\bar{y}^{**}) - \bar{Y}^2\left[(1-3L)\frac{\left\{A_1 C_{yp} + A_2 C'_{yp}\right\}^2}{\left\{A_1 C_p^2 + A_2 C_p'^2\right\}} + \frac{L^2}{4}\left\{A_1 C_p^2 + A_2 C_p'^2\right\} + L\left\{A_1 C_{yp} + A_2 C'_{yp}\right\}\right] \tag{3.5}$$

and

$$MSE(T_{k2})_{\min} = MSE(\bar{y}^{**}) - \bar{Y}^2 A_1\left[(1-3L)\frac{C_{yp}^2}{C_p^2} + \frac{L^2}{4} C_p^2 + L\, C_{yp}\right] \tag{3.6}$$

The expressions for the mean square error of the estimators T_1 and T_2 up to the terms of order m^{-1} are obtained as:

$$MSE(T_1) = MSE(\bar{y}^*) + \bar{Y}^2\left[\left(\frac{1}{m}-\frac{1}{M}\right)\left\{a^2C_p^2 - 2aC_{yp}\right\} + \frac{W_2(k-1)}{m}\left\{a^2C_p'^2 - 2aC_{yp}'\right\}\right], \quad (3.7)$$

and

$$MSE(T_2) = MSE(\bar{y}^*) + \bar{Y}^2\left[\left(\frac{1}{m}-\frac{1}{M}\right)\left\{b^2C_p^2 - 2bC_{yp}\right\}\right]. \quad (3.8)$$

The expressions for the optimum values of a and b which minimize the $MSE(T_1)$ and $MSE(T_2)$ respectively, are obtained as:

$$a_{opt} = \frac{\{A_1C_{yp} + A_2C_{yp}'\}}{\{A_1C_p^2 + A_2C_p'^2\}} \quad \text{and} \quad b_{opt} = \frac{C_{yp}}{C_p^2} \quad (3.9)$$

The expressions for the minimum mean square error of the estimators T_1 and T_2 are obtain as:

$$MSE(T_1)_{\min} = MSE(\bar{y}^*) - \bar{Y}^2\left[\frac{\{A_1C_{yp} + A_2C_{yp}'\}^2}{\{A_1C_p^2 + A_2C_p'^2\}}\right] \quad (3.10)$$

and

$$MSE(T_2)_{\min} = MSE(\bar{y}^*) - \bar{Y}^2A_1\rho_{y\phi}^2C_y^2 \quad (3.11)$$

where $C_{yp} = \rho_{y\phi}C_yC_p$, $C_{yp}' = \rho_{y\phi}'C_y'C_p'$, $C_y = \frac{S_y}{\bar{Y}}$, $C_p = \frac{S_\phi}{P}$, $C_y' = \frac{S_y'}{\bar{Y}}$, $C_p' = \frac{S_\phi'}{P}$,

$S_y^2 = \frac{1}{M-1}\sum_{j=1}^{M}(Y_j - \bar{Y})^2$, $S_\phi^2 = \frac{1}{M-1}\sum_{j=1}^{M}(\phi_j - P)^2$, $S_y'^2 = \frac{1}{M_2-1}\sum_{j=1}^{M_2}(Y_j - \bar{Y}_2)^2$,

$S_\phi'^2 = \frac{1}{M_2-1}\sum_{j=1}^{M_2}(\phi_j - P_2)^2$ and $(\bar{Y}_2, P_2)$ are population mean and population proportion of study character y and auxiliary attribute ϕ for the responding and non-responding parts of the population.

4 COMPARISON OF THE PROPOSED ESTIMATORS WITH RELEVANT ESTIMATORS

Comparing the proposed conventional estimator T_{k1} with relevant estimator $\bar{y}^*$ and corresponding conventional estimator T_1.

$$MSE(T_{k1}) < MSE(\bar{y}^*) \quad \text{If } \rho_{y\phi} > \frac{\left[\alpha\{\alpha - (3\alpha+1)L\}C_p^2 - 2LC_y^2\right]}{2\alpha(1-3L)C_yC_p}$$

and

$$\rho_{y\phi}' > \frac{\left[\alpha\{\alpha - (3\alpha+1)L\}C_p'^2 - 2LC_y'^2\right]}{2\alpha(1-3L)C_y'C_p'}$$

and

$$MSE(T_{k1}) < MSE(T_1) \quad \text{If } \rho_{y\phi} > \frac{\left[\{\alpha(\alpha - (3\alpha+1)L) - a^2\}C_p^2 - 2LC_y^2\right]}{\{2\alpha(1-3L) - 2a\}C_yC_p}$$

and $$\rho'_{y\phi} > \frac{\left[\left\{\alpha(\alpha-(3\alpha+1)L)-a^2\right\}C_p'^2 - 2LC_y'^2\right]}{\left\{2\alpha(1-3L)-2a\right\}C'_y C'_p}$$

Comparing the proposed alternative estimator T_{k2} with relevant estimator $\bar{y}^*$ and corresponding alternative estimator T_2.

$$MSE(T_{k2}) < MSE(\bar{y}^*) \text{ If } \rho_{y\phi} > \frac{\left[\beta\{\beta-(3\beta+1)L\}C_p^2 - 2LC_y^2\right]}{2\beta(1-3L)C_y C_p}$$

and

$$MSE(T_{k2}) < MSE(T_2) \text{ If } \rho_{y\phi} > \frac{\left[\left\{\beta(\beta-(3\beta+1)L)-b^2\right\}C_p^2 - 2LC_y^2\right]}{\left\{2\beta(1-3L)-2b\right\}C_y C_p}$$

5. EMPIRICAL STUDIES

5.1 Data Set I [Source: Sinha (2014)]

One hundred nine village population of urban area under police station-Baria, Tahsil – Champua, Orissa has been taken under consideration from district Census Handbook, 1981, Orissa, published by Govt. of India. The first 25% villages (i.e. 27 villages) have been considered as non-responding group of the population. In this data set, we have considered the study character y and auxiliary attribute ϕ as y - number of literate persons in the village and ϕ-number of literate persons greater than equal to 150. The values of parameters are as follows:

$\bar{Y} = 145.30$, $P = 0.3211$, $S_y = 111.3835$, $S_\phi = 0.4691$, $C_y = 0.7666$,
$C_p = 1.4608$, $\bar{Y}_2 = 189.52$, $P_2 = 0.5185$, $S'_y = 121.204$, $S'_\phi = 0.5092$,
$C'_y = 0.6395$, $C'_p = 0.9820$, $\rho_{y\phi} = 0.742$, $\rho'_{y\phi} = 0.672$, $V_y = 0.7$.

5.2 Data Set II [Source: Sinha (2014)]

A list of 70 villages in a tehsil of India along with their population in 1981 and cultivated area in the same year is given. The first 30% villages (i.e. 21 villages) have been considered as non-responding group of the population. In this data set, we have considered the study character y and auxiliary attribute ϕ as y - cultivated areas (in acres) in the village and ϕ-total population of village greater than equal to 1100. The values of parameters are as follows:

$\bar{Y} = 981.29$, $P = 0.6000$, $S_y = 613.3558$, $S_\phi = 0.4934$, $C_y = 0.6253$,
$C_p = 0.8224$, $\bar{Y}_2 = 1216.8$, $P_2 = 0.7619$, $S'_y = 430.3177$, $S'_\phi = 0.4364$,
$C'_y = 0.3536$, $C'_p = 0.5728$ $\rho_{y\phi} = 0.596$, $\rho'_{y\phi} = 0.690$, $V_y = 0.6$.

Table 1: Optimum values of constants and relative efficiency (RE) of the proposed estimators with respect to $\bar{y}^*$ (M =109, m=25)

Estimators	$1/k$		
	1/4	1/3	1/2
$\bar{y}^*$	100.00 (819.1013)	100.00 (673.5449)	100.00 (527.9886)
T_1	165.94 (493.6191)	174.29 (386.4415)	189.11 (279.1997)

a_{opt}	0.4040266	0.4002454	0.395505
T_2 b_{opt}	134.60 (608.5347) 0.3893875	145.48 (462.9784) 0.3893875	166.34 (317.422) 0.3893875
T_{k1} α_{opt}	184.97 (442.8385) 0.42306	192.03 (350.7566) 0.4163133	206.25 (255.9883) 0.4087006
T_{k2} β_{opt}	147.53 (555.2206) 0.4084209	158.17 (425.8334) 0.4054554	179.83 (293.5978) 0.4025831

Figures in parenthesis give mean square error (.).

Table 2: Optimum values of constants and relative efficiency (RE) of the proposed estimators with respect to $\bar{y}^*$ (M =70, m =25)

Estimators	$1/k$		
	1/4	1/3	1/2
$\bar{y}^*$	100.00 (16340.09)	100.00 (14118.01)	100.00 (11895.93)
T_1 a_{opt}	150.70 (10842.93) 0.4421545	151.68 (9307.501) 0.4446797	153.07 (7771.648) 0.4481371
T_2 b_{opt}	126.65 (12901.32) 0.45316	132.20 (10679.24) 0.45316	140.66 (8457.161) 0.45316
T_{k1} α_{opt}	162.52 (10054.12) 0.4570578	161.59 (8736.733) 0.4570551	161.11 (7383.694) 0.4580541
T_{k2} β_{opt}	135.28 (12078.41) 0.4680633	139.93 (10089.15) 0.4655354	147.57 (8061.289) 0.463077

Figures in parenthesis give mean square error (.).

6. CONCLUSION

From table 1 and 2 it has been observed that using known coefficient of variation V_y of study character y, the proposed estimators T_{k1} and T_{k2} are more efficient in comparison to the usual estimator $\bar{y}^*$ for the optimum values of α and β. The proposed estimators T_{k1} and T_{k2} are also more efficient than the corresponding estimators T_1 and T_2 for the optimum values of α and β. The values of mean square error of all estimators decrease as the value of k decreases. Hence from practical point of view, the proposed estimators T_{k1} and T_{k2} are preferable than T_1 and T_2 because it is showing higher efficiency by using known coefficient of variation of study character y as compared with others.

REFERENCES

[1]. Abd-Elfattah, A.M., El-Sherpieny, E.A., Mohamed, S.M. and Abdou, O.F. (2010). Improvement in estimating the population mean in simple random sampling using information on auxiliary attribute. *Appl. Math. Comp*. 215, 4198-4202.

[2]. Cochran, W.G. (1977). Sampling techniques, 3rd edn. Wiley, New York.

[3]. Hansen, M.H. and Hurwitz, W.N. (1946). The problems of non-response in sample surveys. *J. Amer. Stat. Assoc*. 41, 517–529.

[4]. Jhajj, H.S., Sharma, M.K. and Grover, L.K. (2006). A family of estimators of population mean using information on auxiliary attribute. *Pak. Jour. Stat.* 22, 43–50.

[5]. Khare, B.B. and Kumar, S. (2009). Utilization of coefficient of variation in the estimation of population mean using auxiliary character in the presence of non-response. *Natl. Acad. Sci. Lett., India* 32, 235-241.

[6]. Khare, B.B. and Srivastava, S. (1996). Transformed product type estimators for population mean in presence of soft-core observations. *Proc. Math. Soci. B.H.U.* 12, 29-34.

[7]. Khare, B.B. and Srivastava, S. (1997). Transformed ratio type estimators for the population mean in the presence of non-response. *Communi. Stat. Theo. Meth.* 26, 1779–1791.

[8]. Kumar, K. and Kumar, A. (2019). Estimation of population mean using auxiliary attribute in the presence of non-response. *Int. J. Comp. Theo. Stat.* 1, 43-49.

[9]. Kumar, K. and Kumar, M. (2017). Improved exponential ratio and product type estimators for population mean in the presence of non-response. *Bullen. Math. Stats. Rese.* 5, 68-76.

[10]. Naik, V.D. and Gupta, P.C. (1996). A note on estimating of mean with known population of an auxiliary character. *Jour. Ind. Soc. Ag. Stat.* 48, 151–158.

[11]. Rao, P.S.R.S. (1986). Ratio estimation with sub sampling the non-respondents. *Survey Methodology* 12, 217–230.

[12]. Rao, P.S.R.S. (1987). Ratio and regression estimates with sub sampling the non- respondents. Paper presented at a special contributed session of the International Statistical Association Meeting, Sept. 2–16, Tokyo**,** Japan.

[13]. Searls, D.T. (1964). The utilization of a known coefficient of variation in the estimation procedure. *J. Amer. Stat. Assoc.* 59, 1225-1226.

[14]. Searls, D.T. (1967). A note on the use of an approximately known coefficient of variation. *J. Amer. Stat. Assoc.* 21, 20-21.

[15]. Sen, A.R., (1978). Estimation of the population mean when the coefficient of variation is known. *Commun. Stats. Theo. Meth.* A7 (7), 657-672.

[16]. Shabbir, J. and Gupta S. (2007). On estimating the finite population mean with known population proportion of an auxiliary variable. *Pakistan Journal of Statistics* 23, 1-9.

[17]. Shabbir, J. and Gupta, S. (2010). Estimation of the finite population mean in two phase sampling when auxiliary variables are attributes. *Hacett. Jour. Math. Stat.* 39, 121–129.

[18]. Singh, H.P. and Solanki, R.S. (2012). Improved estimation of population mean in simple random sampling using information on auxiliary attribute. *Appl. Math. Comp*. 218, 7798–7812.

[19]. Singh, R., Chouhan, P., Sawan, N. and Smarandache, F. (2007). Ratio-product Type Exponential Estimator for Estimating Finite Population Mean Using Information on Auxiliary Attribute. *Renaissance High Press, USA,* 18-32.

[20]. Singh, R., Kumar, M., Chaudhary, M.K. and Smarandache, F. (2010). Estimation of population mean in presence of non-response using exponential estimator. *Paper published in the Book Multispace & Multistructure Neutrosophic Transdisciplinarity* 4, 758-768.

[21]. Sinha, R.R. (2014). Estimation of ratio of two population means using auxiliary attributes in presence of non-response. *Int. Jour. Rec. Tren. Life Sci. Math.* 1(2), 01-07.

[22]. Solanki, R.S. and Singh, H.P. (2013). Improved estimation of population mean using population proportion of an auxiliary character. *Chil. J. Stats*. 4(1), 3-17.