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RESEARCH ARTICLE

A COMPARATIVE STUDY AMONG THE SOLUTIONS OF AN ADVECTION DIFFUSION EQUATION, OBSERVED DATA AND STATISTICAL RESULTS

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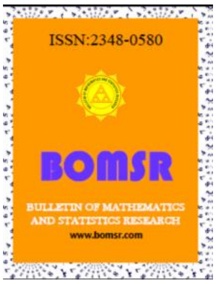


ABSTRACT

In this paper, the analytical solutions of an advection diffusion equation describing the dispersion of air pollutants released from a continuous point source in a finite atmospheric boundary layer formulated by assuming the wind velocity and eddy diffusivity as the power law of vertical height as well as the constants are compared with statistical method and observed the better agreement for the observed data collected at Inshas. It is observed that there is a good agreement between observed and predicted concentration given by variables wind speed and eddy diffusivity than predicted concentration given by constants wind speed and eddy diffusivity.
Keywords: Advection-diffusion equation, Concentration of pollutants, Analytical solution, Statistical method, Wind velocity.

1. Introduction

Air pollution is a global problem. The scientists and researcher in the field of pollutions for study of atmospheric dispersion of air pollutants from point, line and area sources have done a lot of contributions in the past few decades. Various authors have been put different solutions of the advection diffusion equations under different conditions.

Moreira et al. have solved the advection-diffusion equation assuming eddy diffusivity depends up on x and z by taking the x-domain into sub-domains by using the Laplace transform method [10].

Marrouf et al. have given an analytical solution of advection diffusion equation by using variable separable method taking the wind velocity as the linear function of vertical height and eddy diffusivity depends on vertical height as well as both constants. They also compared predicted and the observed concentration collected from the nine experiments conducted at Cairo [9].

Naresh and Nath have given a solution an advection- diffusion equation to examine the characteristics of steady state diffusion transport of pollutants emitted from ground based area sources.[11].Ermark has given a model for the air pollutants emitted from an elevated point sources[5].

Bhandari has given the solution of an advection diffusion equation taking wind velocity and eddy diffusivity as variables and graphically compared with observed data collected at Inshas Cairo[2].

Sharan and Kumar have forwarded an analytical model for the crosswind integrated concentrations emitted from a continuous source by supposing the wind velocity as a power law profile of vertical height above the ground and eddy diffusivity as an explicit function of downwind distance from the source and vertical height [12].

In this field, the different solutions of the advection diffusion equation under different conditions of wind velocity and eddy diffusivity have given by different authors, Demuth [4], Verma,V.S.[14],Verma,V.S.,Srivastava,U.and Bhandari,P.S.[15],Verma,V.S.,Srivastava,U. and Bhandari, P.S.[16], Kumar, P. and Sharan, M.[8], Agarwal, M,Verma,V.S. and Srivastava, S.[1], Wortmann et al. [17], Sharan and Modani [13], John, M.S.[7], Goncalves, G. A. et al. [6], Verma,V.S. and Bhandari, P.S.[3] .

In this paper, the analytical solutions of the model for the dispersion of air pollutants released from a continuous point source in a finite atmospheric boundary layer which is obtained by assuming the wind velocity and eddy diffusivity as the power law of vertical height as well as the constants are compared with statistical method and observed the better agreement for the observed data. For this, we have used the observed data and meteorological parameters found in the nine experiments conducted at Inshas, Cairo, Egypt.

2 Mathematical Model

The dispersion of air pollutants in the steady state condition in the atmosphere can be described by

$$v\frac{\partial C}{\partial x} = \frac{\partial}{\partial y}(k_y\frac{\partial C}{\partial y}) + \frac{\partial}{\partial z}(k_z\frac{\partial C}{\partial z}) \quad \textbf{(1)}$$

where C is the mean concentration of a pollutant, k_y *and* k_z are the eddy diffusivity in y and z directions respectively , x-axis is oriented in the direction of mean wind v and v much greater than the wind speed in y- direction. Source pollutants are ignored and we neglect the diffusion term in x-direction as the advection term in x- direction is larger than the diffusion in x- direction.

For the solution of solution of (1), we take boundary conditions:

$$\frac{\partial C(x,z)}{\partial z} = 0 \text{ at points z=0, z=H} \quad \textbf{(2)}$$

$$c(x,y,z) = 0 \text{ at point z= H} \quad \textbf{(3)}$$

where H is source height.

$$v\mathrm{C} = Q\delta(z - z_s) \;\; at \;\; point \; \mathrm{x} = 0 \quad \textbf{(4)}$$

here δ be the Dirac's delta function and Q a point source strength.

$$c(x,y,z) = 0 \; as \; the \; variables \; x,y,z \; \rightarrow \infty \quad \textbf{(5)}$$

3 Solution of Model

The solutions of the model are obtained in two cases by using the variables separable method:

If eddy diffusivity k_z and wind velocity v are supposed variables as

$$v = uz^n, z \neq 0 \; and \; v = v_0 \; at \; z = 0 \quad \textbf{(6)}$$

and $k_z = u_1 z^n$ **(7)**

where u is the friction velocity and u_1 is the turbulence intensity, then the solution is given by[3]

$$c_y(x,z) = \frac{2Q_p z^{-1/2} z_s^{3/2} J_{1/2}(\eta_\beta z_s)}{uH^2[J_{1/2+1}(\eta_\beta H)]^2}[\sum_{\alpha=1}^{\infty} J_{1/2}(\eta_\alpha z)]e^{-\lambda^2 x} \quad \textbf{(8)}$$

where $J_{1/2}$ is the Bessel's function of first kind of order $1/2$, λ^2 $is\ a\ constant.$

in which $\eta_\beta H$ is given as $J_{1/2}(\eta_\beta H) = 0$ **(9)**

And if the wind velocity v and eddy diffusivity k_z are supposed as constants, then the solution of the model (1) is given by [3]

$$c_y(x,z) = \frac{Q}{v_{z_s}} e^{-\lambda^2 x} cos(\lambda\sqrt{\frac{v}{K}})z \; sec(\lambda\sqrt{\frac{v}{K}})z_s \quad \textbf{(10)}$$

5 Results, Statistical Method and Discussion

The concentration of pollutants was computed using the data collected at vertical distance of a 30 m multi-level micrometeorological tower. Table 1 gives the computed concentrations using the analytical solution equations (8) and (10) and the observed concentrations for nine runs as below:

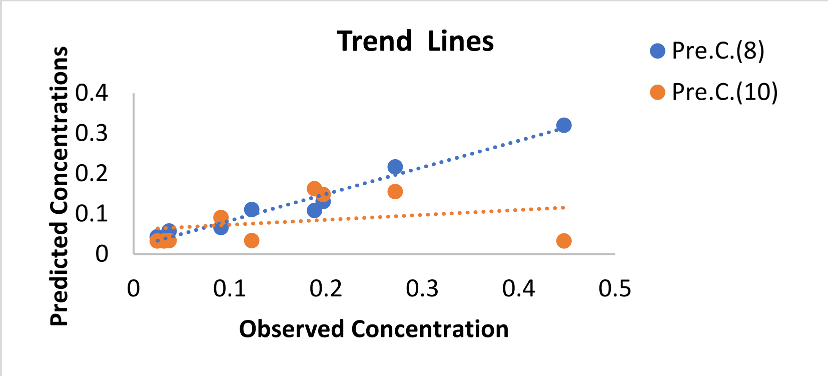
Table 1 Predicted concentrations from analytical equations (8) and (10) and observed (in run nine experiments conducted at Inshas, Cairo):

Test Numbers	Downwind distance	Vertical distance	Observed concentration	Predicted concentration equation(8)	Predicted concentration equation(10)
1	100	5	.025	.043	.032
2	98	10	.037	.057	.033
3	115	5	.091	.066	.090
4	135	5	.197	.130	.148
5	99	2	.272	.216	.155
6	184	11	.188	.108	.162
7	165	12	.447	.320	.032
8	134	7.5	.123	.110	.033
9	96	5	.032	.037	.032

The variation of predicted and observed concentrations of nine typical tests with downwind distance is graphically shown in the paper of Bhandari [3]. He has found that there is a good agreement between observed and predicted concentration given by equation (8) than the predicted concentration given by the equation (10).

Also there is not good agreement between predicted concentration from equation (10) and observed concentration.

Using the trend lines we have also observed the predicted concentrations given by (8) and (10) with observed data. The figure below shows the trend of the concentrations.



Using the Statistical method, we compare among the analytical, observed and statistical results. We follow the following standard statistical performance measures that characterize the agreement between predictions (c_p) and observations (c_o) :

i) Normalized mean square error (NMSE): It is defined as NMSE= $\frac{\overline{(c_o-c_p)^2}}{\overline{c_o}\ \overline{c_p}}$. This formula estimates the overall deviations between predicted and observed concentrations. Smaller values of NMSE indicate a better model performance.

ii) Fractional bias (FB): It is defined as FB = $\frac{(\overline{c_o}-\ \overline{c_p})}{0.5(\overline{c_o}-\ \overline{c_p})}$. It informs the tendency of the model to overestimate or underestimate the observed concentrations. Its value lies between -2 and +2. For an idea model, FB is zero.

iii) Correlation Coefficient(R): It is defined as R= $\frac{\overline{(c_o-\overline{c_o})((c_p-\overline{c_p})}}{\sigma_o\sigma_p}$. It measures the degree of association between predicted and observed concentrations.

iv) Fraction within a factor of two (FAC2): It is defined as the fraction of the data for which the following relation holds:$0.5\leq \frac{c_p}{c_o}\leq 2$.

In the above performance measures formulas, σ_o and σ_p are the standard deviations of c_o and c_p respectively and the over bars represent average over all measurements. If the performance measures NMSE=FB=0 and COR=FAC2=1, then the model is supposed a perfect model.

The performance results from the standard statistical performance measures are shown in table 2. For this two predicted concentration equations (8) and (10) are used with Inshas observed data.

Table 2 Performance comparisons between predicted and observed data.

Statistical functions	NMSE	FB	COR	FAC2
Predicted concentration equation(8)	0.18	0.25	0.98	0.96
Predicted concentration equation(10)	1.75	0.65	0.29	0.74

Table 2 shows that the predicted concentrations equations (8) and (10) lie inside factor of 2 with observed data. From the measures NMSE, FB, COR and FAC2, we summarize that the predicted concentration equation (8) is better than that of predicted concentration equation (10) with the observed data.

7. Conclusion

In this paper, a mathematical model for dispersion of air pollutants with wind velocity and eddy diffusivity as variables and constants which has been already formulated is used. The analytical solutions are compared with observed data collected from nine experiments conducted at Inshas, Cairo, Egypt using statistical tools. Using the Statistical method, we also found that there is a good agreement between observed and predicted concentration given by variables wind speed and eddy diffusivity equation (8) than predicted concentration given by constants wind speed and eddy diffusivity equation (10) .

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