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ON HOMOGENEOUS CUBIC EQUATION WITH THREE UNKNOWNS $x^2 - y^2 + z^2 = 2kxyz$

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ABSTRACT

The homogeneous cubic equation with three unknowns represented by the diophantine equation $x^2 - y^2 + z^2 = 2kxyz$ is analyzed for its patterns of non – zero integral solutions. A few interesting properties among the solutions are presented.

Key words: Cubic equation with three unknowns, integral solutions

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INTRODUCTION

The Diophantine equations offer an unlimited field for research due to their variety [1-3].In particular, one may refer [4-19] for cubic equations with three unknowns. This communication concerns with yet another interesting equation $x^2 - y^2 + z^2 = 2kxyz$ representing homogeneous cubic equation with three unknowns for determining its infinitely many non-zero integral points. Also, a few interesting relations among the solutions are presented.

METHOD OF ANALYSIS

The Diophantine equation representing the cubic equation with three unknowns under consideration is $x^2 - y^2 + z^2 = 2kxyz$ (1)

The substitution of linear transformations

$$x = u + v, y = 2k, z = u - v \tag{2}$$

in (1) leads to

$$(2k^2 + 1)v^2 - (2k^2 - 1)u^2 = 2k^2 \tag{3}$$

The assumption

$$u = X + (2k^2 + 1)T, v = X + (2k^2 - 1)T \tag{4}$$

in (3) implies

$$X^2 = (4k^2 - 1)T^2 + k^2$$

which is satisfied by

$$X_n = \frac{1}{2k^{n-1}}[(2k^3 + k\sqrt{4k^4 - 1})^n + (2k^3 - k\sqrt{4k^4 - 1})^n]$$

$$T_n = \frac{1}{2k^{n-1}\sqrt{4k^4 - 1}}[(2k^3 + k\sqrt{4k^4 - 1})^n - (2k^3 - k\sqrt{4k^4 - 1})^n] \quad (5)$$

In view of (5),(4),(3) and (2), the correspomding non-zero integral solutions of (1) are given by

$$x_n = \frac{f_n}{k^{n-1}} + \frac{2g_n}{k^{n-3}\sqrt{4k^4 - 1}}$$

$$y_n = 2k \qquad \text{n=1,2,3,……}$$

$$z_n = \frac{g_n}{k^{n-1}\sqrt{4k^4 - 1}}$$

Where

$$f_n = [(2k^3 + k\sqrt{4k^4 - 1})^n + (2k^3 - k\sqrt{4k^4 - 1})^n]$$

$$g_n = [(2k^3 + k\sqrt{4k^4 - 1})^n - (2k^3 - k\sqrt{4k^4 - 1})^n]$$

A few interesting properties observed are as follows

- $x_n = z_{n+1}$
- $x_{n+2} - 4k^2 x_{n+1} + x_n = 0$
- $x_n - 2k^2 z_n \equiv 0 \pmod{k}$
- $x_n - 2k^2 z_n + y_n \equiv 0 \pmod{k}$
- Each of the following properties represents a nasty number

  (i) $6k\{x_{2n+1} - 2k^2 z_{2n+1} + 2k\}$

  (ii) $6k(x_{2n+1}) - 12k^3 z_{2n+1} + 6ky_{2n+1}$
- $k^2 x_{3n+2} - 2k^4 z_{3n+2} + 3k^2(x_n - 2k^2 z_n)$ is a cubical integer
- $k^3\{(x_{4n+3} - 2k^2 z_{4n+3}) + 4(x_{2n+1} - 2k^2 z_{2n+1} + y_{2n+1}) + 2\}$

  is a bi-quadratic integer

NOTE: Instead of (4), one may also consider $u = X - (2k^2 + 1)T, v = X - (2k^2 - 1)T$.

For this choice, the corresponding integer solutions are found to be

$$x_n = \frac{f_n}{k^{n-1}} - \frac{2g_n}{k^{n-3}\sqrt{4k^4 - 1}}$$

$$y_n = 2k$$

$$z_n = \frac{-g_n}{k^{n-1}\sqrt{4k^4 - 1}}$$

CONCLUSION

To conclude, one may search for other patterns of solutions and their corresponding properties.

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