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A NOTE ON ANTI Q-FUZZY KU-SUBALGEBRAS AND HOMOMORPHISM OF KU- ALGEBRAS

P.M.SITHAR SELVAM[1] ,T.PRIYA[1], K.T.NAGALAKSHMI[2] ,T.RAMACHANDRAN[3]

[1] Department of Mathematics, PSNA College of Engineering and Technology, Dindigul-624 622, Tamilnadu , India.

[2]Department of Mathematics, K L N College of Information and Technology, pottapalayam- 630611, Sivagangai District, Tamilnadu , India.

[3]Department of Mathematics, M.V.Muthiah Government Arts College for Women, Dindigul-624001, Tamilnadu , India.

*** P.M.Sithar Selvam**

Author for Correspondence

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ABSTRACT

In this paper, we introduce the concept of Anti Q-fuzzy KU-subalgebras of KU-algebras. Also we discussed few results of KU-ideal of KU-algebra under homomorphism and anti homomorphism and some of its properties. we proved that if μ and δ are anti Q- fuzzy KU- ideals in KU – algebra X, then μ x δ is an anti Q- fuzzy KU-ideal in X x X and few more results in Cartesian product.

Keywords

KU-algebra, Anti Q- fuzzy sub algebra, fuzzy KU- ideal, Anti Q-fuzzy KU-ideal, Anti homomorphism, Cartesian product.

AMS Subject Classification (2000): 20N25, 03E72, 03F055 , 06F35, 03G25.

INTRODUCTION

Y.Imai and K.Iseki[6,7] introduced two classes of abstract algebras : BCK-algebras and BCI –algebras. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. Q.P.Hu and X .Li[4,5] introduced a wide class of abstract BCH-algebras. They have shown that the class of BCI-algebras. J.Neggers, S.S.Ahn and H.S.Kim[1] introduced Q-algebras which is generalization of BCK / BCI algebras and obtained several results. C.Prabpayak and U.Leerawat [8] introduced a new algebraic structure which is called KU-algebras and investigated some properties. Samy M.Mostafa and Mokthar A. Abdel Naby[13] introduced fuzzy KU-ideals in KU-algebras. R.Biswas introduced the concept of Anti fuzzy subgroups of groups. Modifying his idea ,P.M.sithar Selvam, T.Priya and T.Ramachandran introduced the concept of Anti Q-fuzzy KU-ideals in KU-algebras. In this paper we apply the notion of Anti Q-fuzzy KU-ideals of KU-algebras in homomorphism, Anti homomorphism, Cartesian products and subalgebras and investigate some of its results.

Preliminaries

In this section we site the fundamental definitions that will be used in the sequel.

Definition 2.1[13]

A non empty set X with a constant 0 and a binary operation * is called a KU-algebra if it satisfies the following axioms.

1. (x * y) *[(y * z) * (x * z)] = 0
2. 0 * x = x
3. x * 0 = 0
4. x * y = 0 = y * x implies x = y , for all x, y , z ∈ X

In X we can define a binary operation ≤ by x ≤ y if and only if y * x = 0. Then (X,*,0) is a KU-algebra if and only if it satisfies that

i. (y * z) * (x * z) ≤ (x * y)
ii. 0 ≤ x
iii. x ≤ y , y ≤ x implies x = y.
iv. x ≤ y if and only if y * x = 0, for all x, y , z ∈ X.

In a KU-algebra,the following identities are true[ ]:

1. z * z = 0.
2. z * (x * z) = 0
3. x ≤ y ⇒ y * z ≤ x * z
4. z * (y * x) = y * (z * x)
5. y * [(y * x) * x] =0 , for all x, y , z ∈ X.

Example 2.1

Let X = {0, 1, 2, 3} be a set with a binary operation * defined by the following table

*	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	1	0	3
3	0	0	2	0

Then Clearly (X , * , 0) is a KU-algebra.

Definition 2.2[13]

Let (X , * , 0) be a KU-algebra. A non empty subset I of X is called KU ideal of X if it satisfies the following conditions

(1) 0 ∈ I

(2) x * (y * z) ∈ I and y ∈ I ⇒ x * z ∈ I for all x , y, z ∈ X .

Remark: From Example 2.1, It is clear that I_1 = {0, a} and I_2 = {0, a, b} are KU-ideals of X.

Definition 2.3[15]

Let X be a non-empty set**.** A fuzzy subset μ of the set X is a mapping μ : X →[0, 1].

Definition 2.4[9]

Let Q and G be any two sets. A mapping β: G x Q →[0, 1] is called a Q –fuzzy set in G.

Definition 2.5[9]

A Q- fuzzy set μ in X is called a Q-fuzzy KU- ideal of X if

(i) μ(0,q) ≥ μ(x,q)

(ii) μ(x* z ,q) ≥ min{μ(x *(y * z),q), μ(y,q)},for all x, y, z∈ X and q ∈ Q.

Definition 2.6[9]

A Q-fuzzy set μ of a KU-algebra X is called an anti Q-fuzzy KU-ideal of X, if

(i) μ(0,q) ≤ μ(x,q)

(ii) μ(x *z,q) ≤ max { μ ((x * (y * z)) , q), μ(y , q)}, for all x,y,z∈ X and q ∈ Q.

Example 2.2

Let X = { 0 , 1 , 2, 3,4 } be a set with a binary operation * defined by the following table

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	2	4
2	0	0	0	1	4
3	0	0	0	0	4
4	0	0	0	1	0

Then Clearly (X , * , 0) is a KU-algebra.

Let $t_0, t_1, t_2 \in [0, 1]$ be such that $t_0 < t_1 < t_2$ .Define a Q-fuzzy set $\mu : X \times Q \to [0, 1]$ by $\mu(0,q) = t_0$, $\mu(1,q) = t_1 = \mu(2,q)$, $\mu(3,q) = t_2 = \mu(4,q)$,routine calculations μ is an anti Q-fuzzy KU- ideal of X and q∈Q.

Definition 2.7[9]

If μ is a Q-fuzzy set in set X , then the complement denoted by μ^c is the Q-fuzzy subset of X given by $\mu^c(x,q) = 1 - \mu(x,q)$, for all x,y∈ X and q ∈ Q.

3. Anti Q- Fuzzy KU-Subalgebras of KU- algebra

Definition 3.1[12]

A fuzzy set μ in a KU-algebra X is called a fuzzy KU-Subalgebra of X if

$\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$,for all x,y∈ X.

Definition 3.2[9]

A fuzzy set μ in a KU- algebra X is called an Anti fuzzy KU- subalgebra of X if

$\mu(x * y) \leq \max\{\mu(x), \mu(y)\}$, for all x,y∈ X.

Definition 3.3[9]

A Q-fuzzy set μ in a KU- algebra X is called an Anti Q-fuzzy KU-subalgebra of X if

$\mu(x * y, q) \leq \max\{\mu(x,q), \mu(y,q)\}$, for all x,y∈ X and q∈Q.

Theorem 3.1

If μ is an anti Q-fuzzy subalgebra of a KU-algebra X, then $\mu(0,q) \leq \mu(x,q)$, for any x ∈ X and q∈Q

Proof

Since x * x = 0 for any x ∈ X and q ∈ Q , then

$$\mu(0,q) = \mu(x * x, q) \leq \max\{\mu(x,q), \mu(x,q)\} = \mu(x,q)$$

$$\mu(0,q) \leq \mu(x,q).$$

Definition 3.3[11]

Let μ be a Q-fuzzy set of X. For a fixed t∈[0,1], the set $\mu^t = \{x \in X \mid \mu(x,q) \leq t \text{ for all } q \in Q\}$ is called the lower level subset of μ. Clearly $\mu^t \cup \mu_t = X$ for t∈[0,1] if $t_1 < t_2$,then $\mu^{t1} \subseteq \mu^{t2}$.

Theorem 3.2

A Q-fuzzy set μ of a KU – algebra X is an anti Q- fuzzy KU- subalgebra if and only if for every t∈ [0,1] , μ^t is either empty or a KU-sub algebra of X.

Proof:

Assume that μ is an anti Q-fuzzy subalgebra of X and $\mu^t \neq \phi$. Then for any $x,y \in \mu^t$ and q ∈ Q, we have

$$\mu(x * y,q) \leq \max\{\mu(x,q), \mu(y,q)\} \leq t.$$

Therefore $x*y \in \mu^t$. Hence μ^t is a KU-sub algebra of X.

Now Let x,y∈ X and q ∈ Q.

Take t = max {μ (x,q), μ (y,q)}. Then by assumption μ^t is a subalgebra of X implies $x * y \in \mu^t$.

Therefore $\mu(x * y,q) \leq t = \max\{\mu(x,q), \mu(y,q)\}$.

Hence μ is an Anti Q-fuzzy KU- subalgebra of X.

Theorem 3.3

Any KU-subalgebra of a KU– algebra X can be realized as a level KU-subalgebra of some Anti Q-fuzzy KU-subalgebra of X.

Proof:

Let A be a KU-sub algebra of a given KU – algebra X and let μ be a Q- fuzzy set in X defined by

$$\mu(x,q) = \begin{cases} t, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases}$$

Where t∈ [0,1] is fixed and q∈Q. It is clear that μ^t =A.

Now we prove such defined μ is an anti Q- fuzzy KU-subalgebra of X.

Let x,y∈ X. If x,y ∈ A, then x * y ∈ A.

Hence μ(x,q) = μ(y,q)= μ (x*y,q) = t and

μ(x*y,q) ≤ max{ μ(x,q), μ(y,q)}

If x,y ∉A,then μ(x,q) = μ(y,q) = 0 and

μ(x * y,q) ≤ max {μ (x,q),μ(y,q)}= 0.

If at most one of x,y ∈ A, then at least one of μ (x,q) and μ (y,q) is equal to 0.

Therefore, max{μ (x,q), μ (y,q)}= 0 so that μ(x * y,q) ≤ 0, which completes the proof.

Theorem 3.4

Two level KU- subalgebras μ^s, μ^t (s < t) of an anti Q-fuzzy KU-subalgebra are equal iff there is no x∈X such that s ≤ μ(x,q) < t.

Proof

Let $\mu^s = \mu^t$ for some s < t. If there exist x∈X and q∈Q such that s ≤ μ(x,q) < t , then μ^t is a proper subset of μ^s, which is a contradiction.

Conversely, assume that there is no x∈X such that s ≤ μ(x,q) < t.

If x∈ μ^s, then μ(x,q) ≤ s and μ(x,q) ≤ t,

Since μ(x,q) does not lie between s and t. Thus x ∈ μ^t , which gives

$\mu^s \subseteq \mu^t$, Also $\mu^t \subseteq \mu^s$.

Therefore $\mu^s = \mu^t$.

4. Homomorphism and Anti Homomorphism on Anti Q-fuzzy KU- algebras

In this section, we discussed about ideals in KU-algebra under homomorphism and anti homomorphism and some of its properties.

Definition 4.1[13]

Let (X,*,0) and (Y ,Δ ,0`) be KU– algebras. A mapping f: X → Y is said to be a homomorphism if f(x * y) = f(x) Δ f(y) for all x,y ∈ X.

Definition 4.2 [10]

Let (X,*,0) and (Y ,Δ ,0`) be KU–algebras. A mapping f: X → Y is said to be an anti homomorphism if f(x * y) = f(y) Δ f(x) for all x,y ∈ X.

Definition 4.3 [10]

Let f: X → X be an endomorphism and μ be a fuzzy set in X. We define a new fuzzy set in X by μ_f in X as μ_f (x) = μ (f(x)) for all x in X.

Definition 4.4 [10]

For any homomorphism f: X → Y, the set {x∈X / f(x) = 0'} is called the kernel of f, denoted by Ker(f) and the set { f(x) / x∈X} is called the image of f, denoted by Im(f).

Theorem 4.1

Let f be an endomorphism of a KU- algebra X. If μ is an anti Q- fuzzy KU-ideal of X, then so is μ_f .

Proof:

Let μ be an anti Q-fuzzy KU-ideal of X.

Now, $\mu_f(0,q) = \mu (f (0,q))$

$\leq \mu (f(x,q)) = \mu_f(x,q)$, for all $x,y \in X$ and $q \in Q$.

Let $x,y,z \in X$ and $q \in Q$.

Then $\mu_f(x *z ,q) = \mu (f(x* z,q))$

$= \mu (f(x, q)) * f (z ,q))$

$\leq \max \{ \mu (f(x, q) * (f(y, q) * f(z, q))), \mu(f (y, q)) \}$

$= \max\{ \mu (f(x, q) * f(y * z ,q)), \mu(f (y, q))\}$

$= \max \{ \mu (f(x *(y * z), q) , \mu(f (y, q))\}$

$= \max \{ \mu_f (x *(y * z), q) , \mu_f(y, q)\}$

$\therefore \mu_f(x *z ,q) \leq \max \{ \mu_f (x *(y * z), q) , \mu_f(y, q)\}$

Hence μ_f is an anti Q -fuzzy KU-ideal of X.

Theorem 4.2

Let f: $X \to Y$ be an epimorphism of KU- algebra. If μ_f is an anti Q-fuzzy KU-ideal of X, then μ is an anti Q-fuzzy KU-ideal of Y.

Proof:

Let μ_f be an anti Q-fuzzy KU-ideal of X.

Let $y \in Y$ and $q \in Q$. Then there exists $x \in X$ such that $f(x, q) = (y, q)$.

Now,

$\mu (0,q) = \mu ((y_1,q) \Delta (0,q))$

$= \mu (f (x_1,q) \Delta f(0,q))$

$= \mu (f ((x_1,q) * (0,q)))$

$= \mu_f ((x_1,q) * (0,q))$

$= \mu_f (0, q)$

$\leq \mu_f (x_1,q) = \mu (f(x_1,q)) = \mu (y_1,q)$

$\therefore \mu (0,q) \leq \mu (y_1,q)$

Let $y_1,y_2 \in Y$ and $q \in Q$.

$\mu ((y_1,q) \Delta (y_2,q)) = \mu (f (x_1,q) \Delta f(x_2,q))$

$= \mu (f ((x_1,q) * (x_2,q)))$

$= \mu_f((x_1,q) * (x_2,q))$

$\leq \max \{ \mu_f ((x_1,q) * ((x_3,q) * (x_2,q)), \mu_f (x_3,q)\}$

$= \max \{ \mu [f ((x_1,q) * ((x_3,q) * (x_2,q)))], \mu (f(x_3,q))\}$

$= \max \{ \mu[f (x_1,q) \Delta f((x_3,q) * (x_2,q))] , \mu (f(x_3,q))\}$

$= \max \{ \mu[f (x_1,q) \Delta (f (x_3,q) \Delta f (x_2,q))] , \mu (f(x_3,q))\}$

$= \max \{ \mu[(y_1,q) \Delta ((y_3,q) \Delta (y_2,q))] , \mu (y_3,q)\}$

$\therefore \mu ((y_1,q) \Delta (y_2,q)) \leq \max \{ \mu[(y_1,q) \Delta ((y_3,q) \Delta (y_2,q))] , \mu (y_3,q)\}$

$\Rightarrow$ μ is an anti Q-fuzzy KU-ideal of Y.

Theorem 4.3

Let f: $X \to Y$ be a homomorphism of KU- algebra. If μ is an anti Q-fuzzy KU-ideal of Y then μ_f is an anti Q-fuzzy KU-ideal of X.

Proof:

Let $x,y \in X$ and $q \in Q$.

$\mu_f (0,q) = \mu [f(0,q)]$

$\leq \mu [f(x,q)]$

$= \mu_f (x,q)$

$\Rightarrow \mu_f (0,q) \leq \mu_f (x,q)$.

$\mu_f (x * z , q) = \mu (f ((x * z),q)$

$= \mu (f(x,q) \Delta f (z,q))$

$\leq \max\{ \mu [f(x,q) \Delta (f(y,q) \Delta f(z,q))] , \mu (f (y,q)) \}$

$= \max\{ \mu [f(x,q) \ \Delta (f((y * z),q)] , \mu (f (y,q)) \}$
$= \max\{ \mu [f(x * (y * z),q)] , \mu (f (y,q)) \}$
$= \max\{ \mu_f (x * (y * z),q) , \mu_f (y,q)\}$

$\therefore \mu_f (x * z , q) \le \max\{ \mu_f (x * (y * z),q) , \mu_f (y,q)\}$

Hence μ_f is an anti Q-fuzzy KU-ideal of X.

5. Cartesian Product of Anti Q-Fuzzy KU-ideals of KU – algebras

In this section, we introduce the concept of Cartesian product of anti Q-fuzzy KU-ideals of KU-algebra.

Definition 5.1 [13]

Let μ and δ be the fuzzy sets in X. The Cartesian product $\mu \times \delta : X \times X \to [0,1]$ is defined by $(\mu \times \delta)(x, y) = \min\{\mu(x),\delta(y)\}$, for all $x, y \in X$.

Definition 5.2 [9]

Let μ and δ be the anti fuzzy sets in X. The Cartesian product $\mu \times \delta : X \times X \to [0,1]$ is defined by $(\mu \times \delta)(x, y) = \max\{\mu(x), \delta(y)\}$, for all $x, y \in X$.

Definition 5.3 [9]

Let μ and δ be the anti Q-fuzzy sets in X. The Cartesian product $\mu \times \delta : X \times X \to [0,1]$ is defined by $(\mu \times \delta)((x, y),q) = \max\{\mu(x, q), \delta(y, q)\}$, for all $x, y \in X$ and $q \in Q$.

Theorem 5.1

If μ and δ are anti Q-fuzzy KU-ideals in a KU – algebra X, then $\mu \times \delta$ is an anti Q-fuzzy KU-ideal in X x X.

Proof:

Let $(x_1, x_2) \in X \times X$ and $q \in Q$.

$(\mu \times \delta)((0,0),q) = \max\{ \mu (0,q), \delta (0,q) \}$
$\le \max\{ \mu (x_1,q) , \delta (x_2,q) \}$
$= (\mu \times \delta) ((x_1,x_2),q)$

Let $(x_1, x_2) , (y_1, y_2) , (z_1, z_2) \in X \times X$ and $q \in Q$.

Now,

$(\mu \times \delta) (((x_1 , x_2),q) * ((z_1 , z_2),q))$
$= (\mu \times \delta) ((x_1 *z_1,q) , (x_2 * z_2,q))$
$= \max\{ \mu (x_1*z_1,q), \delta (x_2 *z_2,q)\}$
$\le \max\{\max\{\mu(x_1 *(y_1* z_1),q), \mu(y_1,q)\},\max\{\delta (x_2 * (y_2 * z_2),q), \delta (y_2,q)\}\}$
$= \max\{\max\{\mu(x_1 *(y_1* z_1),q),\delta (x_2 * (y_2 * z_2),q)\}, \max\{ \mu(y_1,q), \delta(y_2,q)\}\}$
$= \max\{(\mu \times \delta)(((x_1 , x_2),q) * (((y_1 * y_2),q) * ((z_1, z_2),q))), (\mu \times \delta) ((y_1 , y_2),q)\}.$

$\therefore (\mu\times\delta)(((x_1, x_2),q)*((z_1, z_2),q)) \le \max\{(\mu \times \delta)(((x_1 , x_2),q) * (((y_1 * y_2),q) * ((z_1, z_2),q))), (\mu \times \delta) ((y_1 , y_2),q)\}.$

Hence, $\mu \times \delta$ is an anti Q-fuzzy KU- ideal in X x X.

Theorem 5.2:

Let μ & δ be fuzzy sets in a KU-algebra X such that $\mu \times \delta$ is an Anti Q-fuzzy KU-ideal of X x X. Then

(i) Either $\mu(0,q) \le \mu(x, q)$ (or) $\delta(0,q) \le \delta(x,q)$ for all $x \in X$ and $q \in Q$.

(ii) If $\mu(0,q) \le \mu(x,q)$ for all $x \in X$ and $q \in Q$, then either $\delta(0,q) \le \mu(x, q)$ (or) $\delta(0,q) \le \delta(x, q)$

(iii) If $\delta(0,q) \le \delta(x,q)$ for all $x \in X$ and $q \in Q$, then either $\mu(0,q) \le \mu(x,q)$ (or)
$\mu(0,q) \le \delta(x,q)$.

Proof:

Let $\mu \times \delta$ be an anti Q- fuzzy KU-ideal of X x X .

(i) Suppose that $\mu(0,q) > \mu(x,q)$ and $\delta(0,q) > \delta(x,q)$ for some $x, y \in X$ and $q \in Q$.

Then $(\mu \times \delta) ((x ,y),q) = \max\{ \mu(x,q) , \delta (y,q) \}$
$< \max\{ \mu (0,q) , \delta(0,q) \}$
$= (\mu \times \delta) ((0,0),q)$, Which is a contradiction.

Therefore $\mu(0,q) \le \mu(x,q)$ or $\delta(0,q) \le \delta(x,q)$ for all $x \in X$ and $q \in Q$.

(ii) Assume that there exists $x ,y \in X$ and $q \in Q$ such that

$\delta(0,q) > \mu(x,q)$ and $\delta(0,q) > \delta(x,q)$.

Then $(\mu \times \delta)((0,0),q) = \max\{\mu(0,q), \delta(0,q)\} = \delta(0,q)$ and hence

$(\mu \times \delta)((x, y),q) = \max\{\mu(x,q),\delta(y,q)\} < \delta(0,q) = (\mu \times \delta)((0,0),q)$ Which is a contradiction**.**

Hence,if $\mu(0,q) \le \mu(x,q)$ for all $x \in X$ and $q\in Q$, then either $\delta(0,q) \le \mu(x,q)$ (or) $\delta(0,q) \le \delta(x,q)$.

Similarly, we can prove that if $\delta(0,q) \le \delta(x,q)$ for all $x \in X$ and $q\in Q$,then either $\mu(0,q) \le \mu(x,q)$ (or) $\mu(0,q) \le \delta(y,q)$, which yields (iii) .

Theorem 5.3:

Let μ & δ be fuzzy sets in a KU-algebra X such that $\mu \times \delta$ is an Anti Q-fuzzy KU-ideal of X x X. Then either μ or δ is an anti Q-fuzzy KU-ideal of X.

Proof:

First we prove that δ is an anti Q- fuzzy KU-ideal of X.

Since by 4.2(i) either $\mu(0,q) \le \mu(x,q)$ or $\delta(0,q) \le \delta(x,q)$ for all $x\in X$ and $q\in Q$.

Assume that $\delta(0,q) \le \delta(x,q)$ for all $x\in X$ and $q\in Q$. It follows from 4.2(iii) that either $\mu(0,q) \le \mu(x,q)$ (or) $\mu(0,q) \le \delta(x,q)$.

If $\mu(0,q) \le \delta(x,q)$, for any $x\in X$ and $q\in Q$,then $\delta(x,q) = \max\{\mu(0,q), \delta(x,q)\}$

$= (\mu \times \delta)((0, x),q)$

$\delta((x * z),q) = \max\{\mu(0,q) , \delta((x * z),q)\}$.

$= (\mu \times \delta)((0, x*z),q)$

$= (\mu \times \delta)(((0*0),q),((x*z),q))$

$= (\mu \times \delta)(((0,x),q) * ((0,z),q))$

$\le \max\{(\mu \times \delta)[((0, x),q) * (((0,y),q) * ((0,z),q))], (\mu \times \delta)((0, y),q)\}$

$= \max\{(\mu \times \delta)[((0, x),q) * ((0*0,y*z),q)], (\mu \times \delta)((0, y),q)\}$

$= \max\{(\mu \times \delta)[(0* (0*0),x *(y*z)),q)], (\mu \times \delta)((0, y),q)\}$

$= \max\{(\mu \times \delta)[((0,x *(y*z)),q)], (\mu \times \delta)((0, y),q)\}$

$= \max\{\delta(x *(y*z),q), \delta(y,q)\}$

Hence δ is an anti Q- fuzzy KU-ideal of X.

Next we will prove that μ is an anti Q- fuzzy KU-ideal of X.

Let $\mu(0,q) \le \mu(x, q)$

Since by theorem 4.2(ii), either $\delta(0,q) \le \mu(x, q)$ or $\delta(0,q) \le \delta(x, q)$.

Assume that $\delta(0,q) \le \mu(x,q)$,then

$\mu(x,q) = \max\{\mu(x, q) , \delta(0,q)\} = (\mu \times \delta)((x,0),q)$

$\mu(x * z, q) = \max\{\mu(x * z,q), \delta(0,q)\}$

$= (\mu \times \delta)((x * z,0), q)$

$= (\mu \times \delta)((x * z ,q),(0 * 0,q))$

$= (\mu \times \delta)(((x,0),q) * ((z,0),q))$

$\le \max\{(\mu \times \delta)[((x,0),q) * (((y,0),q) * ((z,0),q))], (\mu \times \delta)((y,0),q)\}$

$= \max\{(\mu \times \delta)[((x,0),q) * ((y*z,0*0),q)], (\mu \times \delta)((y,0),q)\}$

$= \max\{(\mu \times \delta)[((x *(y*z),0*(0*0)),q)], (\mu \times \delta)((y,0),q)\}$

$= \max\{(\mu \times \delta)[((x *(y*z),0),q)], (\mu \times \delta)((y,0),q)\}$

$= \max\{\mu(x *(y*z),q), \mu(y,q)\}$

Hence μ is an anti Q- fuzzy KU-ideal of X.

CONCLUSION

In this article we have discussed anti Q-fuzzy KU-ideal, Anti Q-fuzzy of KU-algebras under homomorphism and Anti homomorphism, anti Q-fuzzy KU-subalgebras, Cartesian Products. It has been observed that the KU-algebra as a generalization of BE-algebra. Interestingly, Q-fuzzy concept adds an another dimension to the defined KU-algebras. This concept can further be generalized to Intuitionistic fuzzy set, interval valued fuzzy sets, Lie algebra for new results.

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