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INTEGRAL POINTS ON THE HYPERBOLA $x^2 - 4xy + y^2 + 11x = 0$

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ABSTRACT

This paper concerns with the problem of obtaining infinitely many non-zero distinct integer solutions of the binary quadratic Diophantine equation representing hyperbola given by $x^2 - 4xy + y^2 + 11x = 0$. Employing the lemma of Brahmagupta, infinitely many integral solutions of the above equation are obtained. The recurrence relations on the solutions are presented. A few interesting relations among the solutions are also given.

Key words: Binary quadratic, Hyperbola, Pell equation, Integer solutions

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INTRODUCTION

The binary quadratic Diophantine equations offer an unlimited field for research because of their variety [1,2]. For an extensive review of various problems one may refer [3-21]. This communication concerns with yet another interesting binary quadratic equation $x^2 - 4xy + y^2 + 11x = 0$ representing hyperbola for determining its infinitely many non zero integral solutions. Also, a few interesting relations among the solutions are presented.

Method of analysis

The Diophantine equation to be solved for its non-zero distinct integral solution is

$$x^2 - 4xy + y^2 + 11x = 0 \qquad (1)$$

Treating (1) as a quadratic in y, we get

$$y = 2x \pm \sqrt{3x^2 - 11x} \qquad (2)$$

Let $\alpha^2 = 3x^2 - 11x$ (3)

Using (3) in (2) we have

$$X^2 = 12\alpha^2 + 11^2 \qquad (4)$$

where $X = 6x - 11$ (4a)

The initial solution of (4) is

$$\alpha_0 = 2 \quad \& \quad X_0 = 13$$

Now consider the Pell equation

$$X^2 = 12\alpha^2 + 1 \tag{5}$$

whose fundamental solution is $(\widetilde{\alpha_0}, \widetilde{X_0})$=(2,7). The other solutions of (4) can be derived from the relations

$$\widetilde{X_n} = \frac{f_n}{2} \text{ and } \widetilde{\alpha_n} = \frac{g_n}{4\sqrt{3}}$$

where $f_n = \left[(7+4\sqrt{3})^{n+1} + (7-4\sqrt{3})^{n+1}\right]$

$$g_n = \left[(7+4\sqrt{3})^{n+1} - (7-4\sqrt{3})^{n+1}\right], \qquad n=0,1,2,3.......$$

Applying the lemma of Brahmagupta between (α_0, X_0) and $(\widetilde{\alpha_n}, \widetilde{X_n})$, the other solutions of (4) can be obtained from the relations

$$\left.\begin{aligned} \alpha_{n+1} &= f_n + \frac{13}{4\sqrt{3}} g_n \\ X_{n+1} &= 13\frac{f_n}{2} + \frac{6}{\sqrt{3}} g_n \end{aligned}\right\} \tag{6}$$

Taking the positive sign in the RHS of (2) and using (4a) and (6), the non-zero distinct integer solutions of the hyperbola (1) are represented by

$$\left.\begin{aligned} x_{n+1} &= \frac{1}{12}[13f_n + 4\sqrt{3}g_n + 22] \\ y_{n+1} &= \frac{1}{12}[38f_n + 21\sqrt{3}g_n + 44] \end{aligned}\right\} \tag{7}$$

where n= 0,1,2,3.......

Some numerical examples are presented below:

n	x_{n+1}	y_{n+1}
0	25	90
1	324	1206
2	4489	16750
3	62500	233250

The recurrence relations satisfied by the solutions of (1) are given by

$$y_{n+3} - 14y_{n+2} + y_{n+1} = -44$$

$$x_{n+3} - 14x_{n+2} + x_{n+1} = -22$$

A few interesting relations among the solutions are as follows:

1. $1452x_{n+3} + 21780x_{n+1} - 81312y_{n+1} = -255552$
2. $1452x_{n+2} + 1452x_{n+1} - 5808y_{n+1} = -15972$
3. $1452y_{n+2} + 5808x_{n+1} - 21780y_{n+1} = -63888$
4. $1452y_{n+3} + 81312x_{n+1} - 303468y_{n+1} = -958320$
5. $6[252x_{2n+2} - 48y_{2n+2} - 44]$ is a Nasty number.
6. $11[252x_{3n+3} - 48y_{3n+3} + 756x_{n+1} - 144y_{n+1} - 1129]$ is a cubical number.
7. $252x_{2n+2} - 48y_{2n+2} - 528$ is perfect square.

Also, taking the negative sign in the R.H.S of (2), the corresponding integer solutions of (1) are given by

$$x_{n+1} = \frac{1}{12}[13f_n + 4\sqrt{3}g_n + 22]$$

$$y_{n+1} = \frac{1}{12}[14f_n - 5\sqrt{3}g_n + 44] \qquad n=0,1,2,3..........$$

PROPERTIES:

1. $1452x_{n+2} - 21780x_{n+1} + 5808y_{n+1} = -15972$
2. $1452x_{n+3} - 303468x_{n+1} + 81312y_{n+1} = -255552$
3. $1452y_{n+2} - 5805x_{n+1} + 1452y_{n+1} = 0$
4. $1452y_{n+3} - 81312x_{n+1} + 21780y_{n+1} = -63888$

CONCLUSION

As the binary quadratic Diophantine equations are rich in variety, one may consider other choices of hyperbolas and search for their patterns of solutions and their corresponding properties.

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