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RESEARCH ARTICLE

REFLECTION AND TRANSMISSION PHENOMENON AT AN IMPERFECT BOUNDARY OF VISCOELASTIC SOLID AND FLUID SATURATED INCOMPRESSIBLE POROUS SOLID

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ABSTRACT

The present investigation is concerned with the reflection and transmission phenomenon at an imperfectly bonded interface between viscoelastic solid half space and fluid saturated porous half space. P-wave or SV-wave incidents on the interface through fluid saturated porous solid half space. The amplitude ratios of various reflected and transmitted waves to that of incident wave are obtained and hence deduced for normal force stiffness, transverse force stiffness and for perfect bonding. These amplitude ratios have been computed numerically for a specific model and results thus obtained are depicted graphically to understand the behaviour of amplitude ratios with angle of incidence of incident wave. It is found that these amplitude ratios depend on angle of incidence of the incident wave and material properties of medium and also these are affected by the stiffness. A special case of empty porous solid and viscoelastic solid reduces to elastic solid is also obtained and discussed from the present study.

Introduction

The state of the deep interior of the earth cannot be explained by assuming the earth to be an elastic solid. Keeping this view in mind several problems of reflection and refraction in a linear viscoelastic solid have been solved by many researchers like Lockett (1962), Cooper (1965,1966,1967), and Borcherdt (1973,1982,1985,1986) etc.

For fluid saturated porous material especially liquid filled pores, it is difficult to describe the mechanical behaviour with the help of classical theory. Because of the solid and liquid phases have different motions and different material properties and the complicated structures of pores, the mechanical behaviour of a fluid saturated porous medium is very complex. So many researchers have tried to overcome this difficulty from time to time.

Bowen (1980) and de Boer and Ehlers (1990a, 1990b) developed a theory for incompressible fluid saturated porous medium based on the work of Fillunger model (1913). Based on this theory, many researchers like de Boer and Didwania (2004),de Boer and Liu(1994,1995),Kumar and Hundal (2003),Tajuddin and Hussaini (2006),Kumar et.al.(2011) etc. studied the problems of wave propagation in fluid saturated porous medium. Imperfect interface considered in this problem means that the stress components are continuous and small displacement field is not continuous. The infinite values of interface parameters imply vanishing of displacement jumps and therefore correspond to perfect interface conditions. On the other hand, zero values of the interface parameters imply vanishing of the corresponding interface tractions which corresponds to complete debonding. Any finite values of the interface parameters define an imperfect interface. The values of the interface parameters depend upon the material properties of the medium. Recently, Chen et.al. (2004), Kumar and Rupender (2009) and Kumar and Chawala (2010) etc. used the imperfect conditions at an interface to study the various types of wave problems.

Using de Boer and Ehlers (1990) theory for fluid saturated porous medium and Borcherdt (1973) theory for viscoelastic solid, the reflection and transmission of longitudinal wave (P-wave) or transverse wave (SV-wave) at an imperfect interface between linear viscoelastic solid half space and fluid saturated porous half space medium is studied. A special case when fluid saturated porous half space reduces to empty porous solid has been deduced and discussed accordingly. Also, A special case when viscoelastic solid half space reduces to elastic solid half space is obtained. Amplitudes ratios for various reflected and transmitted waves are computed for a particular model and depicted graphically. The model considered is assumed to exist in the oceanic crust part of the earth and the propagation of wave through such a model may be of great use in the fields related to earth sciences and seismology.

2. Basic equations

The equations governing the deformation of an incompressible porous medium saturated with non-viscous fluid in the absence of body forces are given by de Boer and Ehlers (1990) as

$$\nabla.(\eta^S\dot{\mathbf{u}}^S + \eta^{\mathbf{F}}\dot{\mathbf{u}}^{\mathbf{F}}) = 0, \tag{1}$$

$$(\lambda^S + \mu^S)\nabla(\nabla.\mathbf{u}^{\mathbf{S}}) + \mu^S\nabla^2\mathbf{u}^{\mathbf{S}} - \eta^S\nabla p - \rho^S\ddot{\mathbf{u}}^{\mathbf{S}} + S_v(\dot{\mathbf{u}}^{\mathbf{F}} - \dot{\mathbf{u}}^S) = 0, \tag{2}$$

$$\eta^{\mathbf{F}}\nabla p + \rho^{\mathbf{F}}\ddot{\mathbf{u}}^{\mathbf{F}} + S_v(\dot{\mathbf{u}}^{\mathbf{F}} - \dot{\mathbf{u}}^S) = 0, \tag{3}$$

$$\mathbf{T}^{\mathbf{S}}_{\mathbf{E}} = 2\mu^S\mathbf{E}^{\mathbf{S}} + \lambda^S(\mathrm{E}^S.\mathbf{I})\mathbf{I}, \tag{4}$$

$$\mathbf{E}^{\mathbf{S}} = \frac{1}{2}(\mathrm{grad}\,\mathbf{u}^{\mathbf{S}} + \mathrm{grad}^{\mathrm{T}}\mathbf{u}^{\mathbf{S}}), \tag{5}$$

where $\mathbf{u}^i, \dot{\mathbf{u}}^i, \ddot{\mathbf{u}}^i, \; i = F, S$ denote the displacement, velocity and acceleration of fluid and solid phases, respectively and p is the effective pore pressure of the incompressible pore fluid. ρ^S and $\rho^{\mathbf{F}}$ are the densities of the solid and fluid constituents, respectively. $\mathbf{T}^{\mathbf{S}}_{\mathbf{E}}$ is the effective stress in the solid phase and $\mathbf{E}^{\mathbf{S}}$ is the linearized langrangian strain tensor. λ^S and μ^S are the macroscopic Lame's parameters of the porous solid and η^S and $\eta^{\mathbf{F}}$ are the volume fractions satisfying

$$\eta^S + \eta^F = 1. \tag{6}$$

In the case of isotropic permeability, the tensor $\mathbf{S}_v$ describing the coupled interaction between the solid and fluid is given by de Boer and Ehlers (1990) as

$$\mathbf{S}_v = \frac{(\eta^F)^2 \gamma^{FR}}{K}\mathbf{I}, \tag{7}$$

where γ^{FR} is the specific weight of the fluid and K is the Darcy's permeability coefficient of the porous medium and **I** stands for unit vector.

Assuming the displacement vector $\mathbf{u}^i$ $(i = F, S)$ as

$$\mathbf{u}^i = (u^i, 0, w^i), \quad \text{where} \quad i = F, S, \tag{8}$$

and hence eqs. (1)- (3) give the equations of motion for fluid saturated incompressible porous medium in the component form as

$$(\lambda^S + \mu^S)\frac{\partial \theta^S}{\partial x} + \mu^S \nabla^2 u^S - \eta^S \frac{\partial p}{\partial x} - \rho^S \frac{\partial^2 u^S}{\partial t^2} + S_v\left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{9}$$

$$(\lambda^S + \mu^S)\frac{\partial \theta^S}{\partial z} + \mu^S \nabla^2 w^S - \eta^S \frac{\partial p}{\partial z} - \rho^S \frac{\partial^2 w^S}{\partial t^2} + S_v\left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \tag{10}$$

$$\eta^F \frac{\partial p}{\partial x} + \rho^F \frac{\partial^2 u^F}{\partial t^2} + S_v\left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{11}$$

$$\eta^F \frac{\partial p}{\partial z} + \rho^F \frac{\partial^2 w^F}{\partial t^2} + S_v\left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \tag{12}$$

$$\eta^S\left[\frac{\partial^2 u^S}{\partial x\,\partial t} + \frac{\partial^2 w^S}{\partial z\,\partial t}\right] + \eta^F\left[\frac{\partial^2 u^F}{\partial x\,\partial t} + \frac{\partial^2 w^F}{\partial z\,\partial t}\right] = 0, \tag{13}$$

where

$$\theta^S = \frac{\partial(u^S)}{\partial x} + \frac{\partial(w^S)}{\partial z}. \tag{14}$$

and

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}. \tag{15}$$

With the help of Helmholtz decomposition of displacement vector, the displacement components u^i and w^i in terms of the potential functions ϕ^i and ψ^i can be written as

$$u^i = \frac{\partial \phi^i}{\partial x} + \frac{\partial \psi^i}{\partial z}, \quad w^i = \frac{\partial \phi^i}{\partial z} - \frac{\partial \psi^i}{\partial x}, \quad i = F, S. \tag{16}$$

Using, (16) in equations (9)- (13), we get

$$\nabla^2\phi^S - \frac{1}{C^2}\frac{\partial^2\phi^S}{\partial t^2} - \frac{S_v}{(\lambda^S+2\mu^S)(\eta^F)^2}\frac{\partial\phi^S}{\partial t} = 0, \tag{17}$$

$$\phi^F = -\frac{\eta^S}{\eta^F}\phi^S, \tag{18}$$

$$\mu^S\nabla^2\psi^S - \rho^S\frac{\partial^2\psi^S}{\partial t^2} + S_v\left[\frac{\partial\psi^F}{\partial t} - \frac{\partial\psi^S}{\partial t}\right] = 0, \tag{19}$$

$$\rho^F\frac{\partial^2\psi^F}{\partial t^2} + S_v\left[\frac{\partial\psi^F}{\partial t} - \frac{\partial\psi^S}{\partial t}\right] = 0, \tag{20}$$

$$(\eta^F)^2 p - \eta^S\rho^F\frac{\partial^2\phi^S}{\partial t^2} - S_v\frac{\partial\phi^S}{\partial t} = 0, \tag{21}$$

where

$$C = \sqrt{\frac{(\eta^F)^2(\lambda^S+2\mu^S)}{(\eta^F)^2\rho^S+(\eta^S)^2\rho^F}}. \tag{22}$$

The normal and tangential stresses in the solid phase take the form as given below

$$t_{zz}{}^S = \lambda^S\left(\frac{\partial^2\phi^S}{\partial x^2} + \frac{\partial^2\phi^S}{\partial z^2}\right) + 2\mu^S\left(\frac{\partial^2\phi^S}{\partial z^2} - \frac{\partial^2\psi^S}{\partial x\,\partial z}\right), \tag{23}$$

$$t_{zx}{}^S = \mu^S\left(2\frac{\partial^2\phi^S}{\partial x\,\partial z} + \frac{\partial^2\psi^S}{\partial z^2} - \frac{\partial^2\psi^S}{\partial x^2}\right). \tag{24}$$

The time harmonic solution of the system of equations (17) - (21) can be taken as

$$(\phi^S, \phi^F, \psi^S, \psi^F, p) = \left(\phi_1{}^S, \phi_1{}^F, \psi_1{}^S, \psi_1{}^F, p_1\right)\exp(i\omega t), \tag{25}$$

where ω is the complex circular frequency.

Using equations (17)-(21) and (25), we get

$$\left[\nabla^2 + \frac{\omega^2}{C_1{}^2} - \frac{i\omega S_v}{(\lambda^S+2\mu^S)(\eta^F)^2}\right]\phi_1{}^S = 0, \tag{26}$$

$$[\mu^S\nabla^2 + \rho^S\omega^2 - i\omega S_v]\psi_1{}^S = -i\omega S_v\psi_1{}^F, \tag{27}$$

$$[-\omega^2\rho^F + i\omega S_v]\psi_1{}^F - i\omega S_v\psi_1{}^S = 0, \tag{28}$$

$$(\eta^F)^2 p_1 + \eta^S\rho^F\omega^2\phi_1{}^S - i\omega S_v\phi_1{}^S = 0, \tag{29}$$

$$\phi_1{}^F = -\frac{\eta^S}{\eta^F}\phi_1{}^S. \tag{30}$$

Equation (26) represents the propagation of a longitudinal wave with velocity V_1, where V_1 is given by

$$V_1{}^2 = \frac{1}{G_1}, \tag{31}$$

and

$$G_1 = \left[\frac{1}{C_1{}^2} - \frac{iS_v}{\omega(\lambda^S+2\mu^S)(\eta^F)^2}\right]. \tag{32}$$

From equations (27) and (28), we get

$$\left[\nabla^2 + \frac{\omega^2}{V_2{}^2}\right]\psi_1{}^S = 0. \tag{33}$$

Equation (33) corresponds to the propagation of a transverse wave with velocity V_2, which is given by

$$V_2{}^2 = \frac{1}{G_2},$$

where

$$G_2 = \left\{\frac{\rho^S}{\mu^S} - \frac{iS_v}{\mu^S\omega} - \frac{S_v{}^2}{\mu^S(-\rho^S\omega^2 + i\omega S_v)}\right\}. \tag{34}$$

For $\mathbf{M_1}$ (linear viscoelastic solid medium)

Following Borcherdt (1973), the governing equation for the small motions in a linear viscoelastic solid can be written as

$$(K' + 4M'/3)\nabla(\nabla.\mathbf{u}') - M'\nabla\times(\nabla\times\mathbf{u}') = \rho_l\ddot{\mathbf{u}}', \tag{35}$$

where symbols K' is the complex bulk modulus, M' is the shear modulus, ρ_l is the density of linear viscoelastic solid and $\mathbf{u}'$ is the displacement vector. Superposed dots on right hand side of equation (35) denote the second partial derivative with respect to time.

The stresses in the linear viscoelastic solid are given by

$$\sigma_{kl}{}' = (K' - 2M'/3)\theta\,\delta_{kl} + 2M'e_{kl}, \tag{36}$$

where

$$e_{kl} = \frac{1}{2}\left(\frac{\partial u_k{}'}{\partial x_l} + \frac{\partial u_l{}'}{\partial x_k}\right), \quad \theta = \nabla.\mathbf{u}', \tag{37}$$

Using Helmholtz's theorem

$$\mathbf{u}' = \nabla\phi' + \nabla\times\boldsymbol{\psi}', \quad \nabla.\boldsymbol{\psi}' = 0, \tag{38}$$

where ϕ' and ψ' satisfy

$$\alpha^2\nabla^2\phi' = \ddot{\phi}^* \text{ and } \beta^2\nabla^2\psi' = \ddot{\psi}', \tag{39}$$

and

$$\alpha^2 = \left(K' + \frac{4M'}{3}\right)/\rho_1, \qquad \beta^2 = M'/\rho_1, \tag{40}$$

Also, $\psi' = -(\boldsymbol{\psi}')_y,$ (41)

$$u' = \frac{\partial \phi'}{\partial x} + \frac{\partial \psi'}{\partial z}, \qquad w' = \frac{\partial \phi'}{\partial z} - \frac{\partial \psi'}{\partial x}. \tag{42}$$

3. Formulation of the problem and its solution. Consider linear viscoelastic solid half space medium $M_2\ [z < 0]$ and fluid saturated porous medium $M_1\ [z > 0]$ (see figure1) being in contact with each other at a plane interface z=0. Take the z-axis pointing into lower half-space. Consider a longitudinal wave (P-wave) or transverse wave (SV-wave) propagating through the fluid saturated porous half space medium M_1 and incident at the plane z=0 and making an angle θ_0 with normal to the surface. Corresponding to each incident wave (P-wave or SV-wave), we get two reflected waves P-wave and SV-wave in the medium M_1 and two transmitted waves P-wave and SV-wave in medium M_2. The Geometry of the problem shows two dimensional problem.

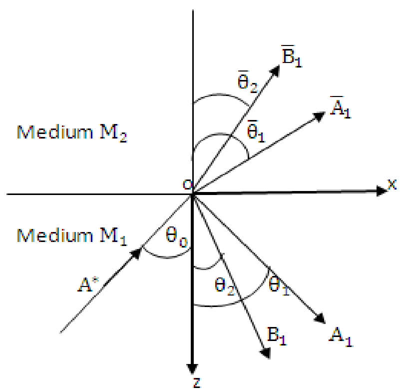


Fig.1 Geometry of the problem

In medium M_1

The potential function that satisfy the equations (17)-(21) can be taken as

$$\begin{aligned}\{\phi^S, \phi^F, p\} = \{1, m_1, m_2\}[&A_{01} \exp\{ik_1(x \sin\theta_0 - z\cos\theta_0) + i\omega_1 t\} \\ &+ A_1 \exp\{ik_1(x\sin\theta_1 + z\cos\theta_1) + i\omega_1 t\}],\end{aligned} \tag{43}$$

$$\begin{aligned}\{\psi^S, \psi^F\} = \{1, m_3\}[&B_{01} \exp\{ik_2(x \sin\theta_0 - z\cos\theta_0) + i\omega_2 t\} \\ &+ B_1 \exp\{ik_2(x\sin\theta_2 + z\cos\theta_2) + i\omega_2 t\}],\end{aligned} \tag{44}$$

where

$$m_1 = -\frac{\eta^S}{\eta^F}, \quad m_2 = -\left[\frac{\eta^S \omega_1{}^2 \rho^F - i\omega_1 S_v}{(\eta^F)^2}\right], \quad m_3 = \frac{i\omega_2 S_v}{i\omega_2 S_v - \omega_2{}^2 \rho^F}, \tag{45}$$

where A_{01} and B_{01} are amplitudes of the incident P-wave and SV-wave, respectively and A_1 , B_1 are amplitudes of the reflected P-wave and SV-wave respectively and to be determined from boundary conditions.

In medium $\mathbf{M_1}$

The potential function satisfying the equation (39) can be taken as

$$\phi' = \bar{A}_1 \exp\{i\bar{k}_1(x \sin\bar{\theta}_1 - z\cos\bar{\theta}_1) + i\bar{\omega}_1 t\}, \tag{46}$$

$$\psi' = \bar{B}_1 \exp\{i\bar{k}_2(x \sin\bar{\theta}_2 - z\cos\bar{\theta}_2) + i\bar{\omega}_2 t\}, \tag{47}$$

where $\bar{k}_1$ and $\bar{k}_2$ are wave numbers of refracted P-wave and SV-wave, respectively. $\bar{A}_1$ and $\bar{B}_1$ are amplitudes of refracted P-wave and SV-wave and are unknown to be determined from boundary conditions.

4. Boundary conditions. The appropriate boundary conditions at the interface z=0 are the continuity of displacement and stresses. Mathematically, these boundary conditions can be expressed as:

$$t_{zz}{}^{S} - p = \sigma_{zz}', \quad t_{zx}{}^{S} = \sigma_{zx}', \quad \sigma_{zz} = K_n(w^S - w'), \quad \sigma_{zx} = K_t(u^S - u'), \tag{48}$$

In order to satisfy the boundary conditions, the extension of the Snell's law will be

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\bar{\theta}_1}{\bar{V}_1} = \frac{\sin\bar{\theta}_2}{\bar{V}_2}, \tag{49}$$

Also

$$k_1V_1 = k_2V_2 = \bar{k}_1\bar{V}_1 = \bar{k}_2\bar{V}_2 = \omega, \quad \text{at } z = 0, \tag{50}$$

For P-wave,

$$V_0 = V_1, \quad \theta_0 = \theta_1, \tag{51}$$

For SV-wave,

$$V_0 = V_2, \quad \theta_0 = \theta_2, \tag{52}$$

For incident longitudinal wave at the interface z=0, putting $B_{01} = 0$ in equation (44) and for incident transverse wave putting $A_{01} = 0$ in equation (43). Substituting the expressions of potentials given by (43)-(44) and(46)-(47) in equations (16),(23)-(24) and (36)-(37) and using equations (48)-(52), we get a system of four non homogeneous which can be written as

$$\sum_{j=0}^{4} a_{ij}\, Z_j = Y_i, \qquad (i = 1,2,3,4) \tag{53}$$

Where

$$Z_1 = \frac{A_1}{A^*}, \quad Z_2 = \frac{A_2}{A^*}, \quad Z_3 = \frac{\bar{A}_1}{A^*}, \qquad Z_4 = \frac{\bar{B}_1}{A^*} \tag{54}$$

Also a_{ij} in non dimensional form can be written as

$$a_{11} = \frac{\lambda^S}{\mu^S} + 2\cos^2\theta_1 + \frac{m_2}{\mu^S k_1^{\,2}}, \quad a_{12} = -2\sin\theta_2\cos\theta_2\frac{k_2^{\,2}}{k_1^{\,2}}, \quad a_{13} = \frac{\bar{k}_1^{\,2}}{k_1^{\,2}\mu^S}\left[-K' + \frac{2}{3}M'(\sin^2\bar{\theta}_1 - 2\cos^2\bar{\theta}_1)\right],$$

$$a_{14} = -\frac{M'\bar{k}_2^{\,2}}{k_1^{\,2}\mu^S}\sin 2\bar{\theta}_2, \quad a_{21} = 2\sin\theta_1\cos\theta_1, \quad a_{22} = \frac{k_2^{\,2}}{k_1^{\,2}}(\cos^2\theta_2 - \sin^2\theta_2), \quad a_{23} = \frac{M'\bar{k}_1^{\,2}}{k_1^{\,2}\mu^S}\sin 2\bar{\theta}_1,$$

$$a_{24} = -\frac{M'\bar{k}_2^{\,2}}{k_1^{\,2}\mu^S}\cos 2\bar{\theta}_2, \quad a_{31} = i\sin\theta_1, \quad a_{32} = \frac{i\,k_2\cos\theta_2}{k_1}, \quad a_{33} = -\frac{i\,\bar{k}_1}{k_1}\sin\bar{\theta}_1 - \frac{M'\,\bar{k}_1^{\,2}\sin 2\bar{\theta}_1}{K_t\,k_1},$$

$$a_{34} = \frac{i\,\bar{k}_2\cos\bar{\theta}_2}{k_1} + \frac{M'\,\bar{k}_2^{\,2}\cos 2\bar{\theta}_2}{K_t\,k_1}, \quad a_{41} = i\cos\theta_1, \quad a_{42} = -\frac{i\,k_2\sin\theta_2}{k_1},$$

$$a_{43} = \frac{\overline{i\,k}_1\cos\bar{\theta}_1}{k_1} - \frac{\bar{k}_1^{\,2}\left[-K' + \frac{2M'}{3}(\sin^2\bar{\theta}_1 - 2\cos^2\bar{\theta}_1)\right]}{K_n\,k_1}, \quad a_{44} = \frac{i\,\bar{k}_2\sin\bar{\theta}_2}{k_1} + \frac{\bar{k}_2^{\,2}M'\sin 2\bar{\theta}_2}{K_n\,k_1}, \tag{55}$$

For incident longitudinal wave:

$$A^* = A_{01}, B_{01} = 0, Y_1 = -a_{11}, Y_2 = a_{21}, Y_3 = -a_{31}, Y_4 = a_{41}, \tag{56}$$

For incident transverse wave:

$$A^* = B_{01}, A_{01} = 0, Y_1 = a_{12}, Y_2 = -a_{22}, Y_3 = a_{32}, Y_4 = -a_{42}, \tag{57}$$

5. Particular cases:

Case I: Normal force stiffness $(K_n \neq 0, K_t \to \infty)$

In this case, we obtain a system of four non homogeneous equations as those given by equation (55) with the changed a_{ij} as follows

$$a_{33} = -\frac{i\,\bar{k}_1}{k_1}\sin\bar{\theta}_1, \qquad a_{34} = \frac{i\,\bar{k}_2\cos\bar{\theta}_2}{k_1}, \tag{58}$$

Case II: Transverse force stiffness $(K_t \neq 0, K_n \to \infty)$

In this case also, a system of four non homogeneous equations as those given by equation (42) is obtained with the changed a_{ij} as given below

$$a_{43} = \frac{\overline{i\,k}_1\cos\bar{\theta}_1}{k_1}, \qquad a_{44} = \frac{i\,\bar{k}_2\sin\bar{\theta}_2}{k_1}, \tag{59}$$

Case III: Welded contact $(K_n \to \infty, K_t \to \infty)$

Again in this case, a system of four non homogeneous equations is obtained as those given by equation (55) with some a_{ij} changed as

$$a_{33} = -\frac{i\,\bar{k}_1}{k_1}\sin\bar{\theta}_1, \qquad a_{34} = \frac{i\,\bar{k}_2\cos\bar{\theta}_2}{k_1}, \qquad a_{43} = \frac{i\,\bar{k}_1\cos\bar{\theta}_1}{k_1}, \qquad a_{44} = \frac{i\,\bar{k}_2\sin\bar{\theta}_2}{k_1}, \tag{60}$$

Special case:-

If pores are absent or gas is filled in the pores then ρ^F is very small as compared to ρ^S and can be neglected, so the relation (22) gives us

$$C = \sqrt{\frac{\lambda^S + 2\mu^S}{\rho^S}}. \tag{61}$$

and the coefficients a_{11} in (55) changes to

$$a_{11} = \frac{\lambda^S}{\mu^S} + 2\cos^2\theta_1, \tag{62}$$

and the remaining coefficients in (55) remain same. In this situation the problem reduces to the problem of viscous liquid half space over empty porous solid half space.

6. Numerical results and discussion

To discuss the theoretical results obtained in above sections, we have computed them numerically for a model by taking the relevant elastic parameters as

In medium M_1, the physical constants for fluid saturated incompressible porous medium are taken from de Boer, Ehlers and Liu (1993) as

$$\eta^s = 0.67, \quad \eta^F = 0.33, \quad \rho^s = 1.34\ \mathrm{Mg/m^3}, \quad \rho^F = 0.33\ \mathrm{Mg/m^3}, \quad \lambda^s = 5.5833\ \mathrm{MN/m^2},$$

$$K^F = 0.01\mathrm{m/s}, \quad \gamma^{FR} = 10.00\mathrm{KN/m^3}, \quad \mu^s = 8.3750\mathrm{N/m^2}, \quad \omega^* = 10/\mathrm{s}, \tag{63}$$

In medium M_2,

Following Silva (1976), the physical parameters representing the crust as a linear viscoelastic solid are as follows

$$Q_P = 100, \quad Q_S = 45, \quad \rho_1 = 2.6\ \mathrm{gm/cm^3}, \quad V_P = 6.1\ \mathrm{km/s}, \quad V_S = 3.5\ \mathrm{km/s} \tag{64}$$

A computer programme in MATLAB has been developed and modulus of various amplitude ratios $|Z_i|, (i = 1,2,3,4,)$ for reflected and transmitted waves are computed. $|Z_1|$ and $|Z_2|$ depict the modulus of amplitude ratios for reflected P and reflected SV-wave respectively. Also, $|Z_3|$ and $|Z_4|$ present the modulus of amplitude ratios for transmitted P and transmitted SV-wave respectively. In all figures (2)-(9) dashed dotted line represents the variations of the amplitude ratios for imperfect boundary, dotted line correspond to transverse force stiffness, dashed line for normal force stiffness and solid line for welded contact w.r.t. angle of incidence of the incident P and SV-wave. Also when medium M_1 reduces to empty porous solid, the variations of the amplitude ratios for this case are represented by figures (10)-(17). In these figures EGEN denotes the curve for imperfect boundary, ENFS denote normal force stiffness, and ETFS denote transverse force stiffness, EWD for welded contact. The variations in all the figures are shown for the range $0^0 \le \theta \le 90^0$. In the figures (18)-(25), VGEN denotes the curve for imperfect boundary, VNFS denote normal force stiffness, and VTFS denote transverse force stiffness, VWD for welded contact in case of reducing the viscoelastic solid to elastic solid half space. The variations in all the figures are shown for the range $0^0 \leq \theta \leq 90^0$.

Incident P-wave

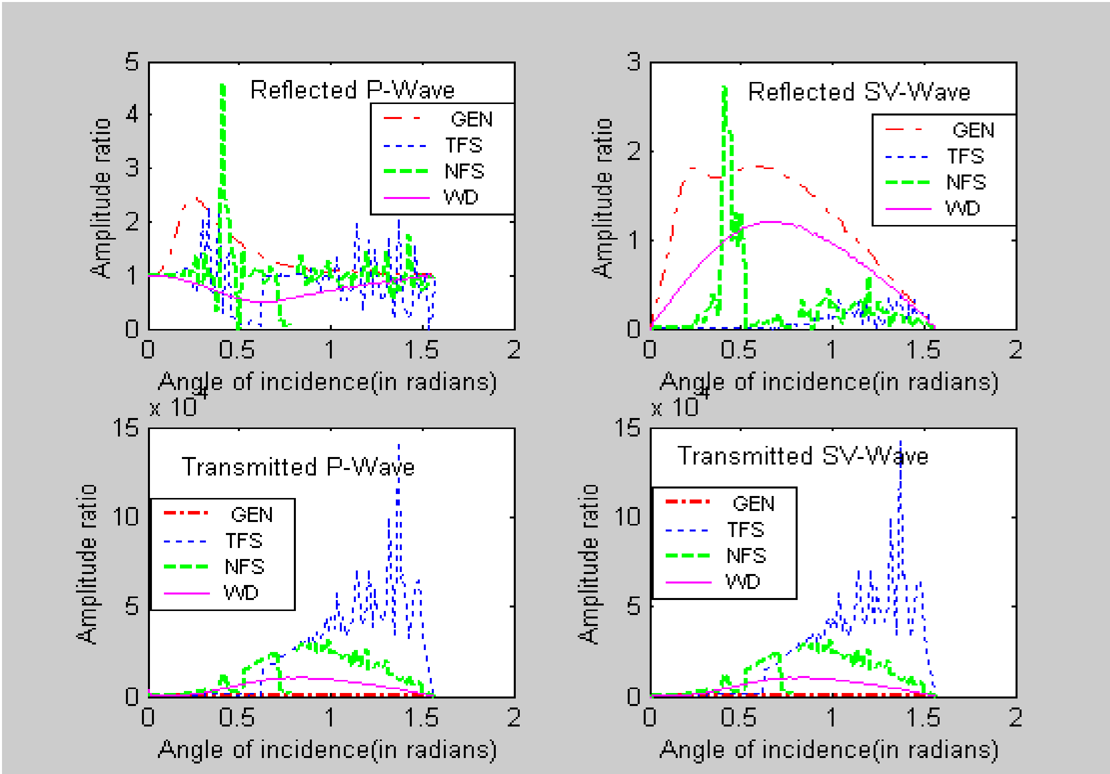
Figures (2)-(5) show the variations of the amplitude ratios $|Z_i|$, $(i = 1,2,3,4,)$ with angle of incidence of the incident P wave. The behaviour of all these distribution curves is similar i.e. increasing from normal incidence to maximum value and then decreasing from maximum value to grazing incidence. Figures (10)-(13) show the variations of the amplitude ratios $|Z_i|$ with angle of incidence of the incident P wave in case of empty porous solid. The effect of fluid filled in the pores of fluid saturated porous medium is clear by comparing the maximum values of corresponding amplitude ratio in figures (2)-(5) and (10)-(13). Figures (18)-(21) represent the variations for imperfect boundary, normal force stiffness, and transverse force stiffness, welded contact for the case when linear viscoelastic solid half space medium M_2 reduces to elastic solid half space. In figures (18)-(21),the modulus of the amplitude ratios for reflected P-wave and reflected SV-wave, are greater for the case of imperfect boundary than all other values .Also, the values of modulus of the amplitude ratios for transmitted P-wave and transmitted SV-wave, are greater for the case of transverse force stiffness than all other values .

Incident SV-wave

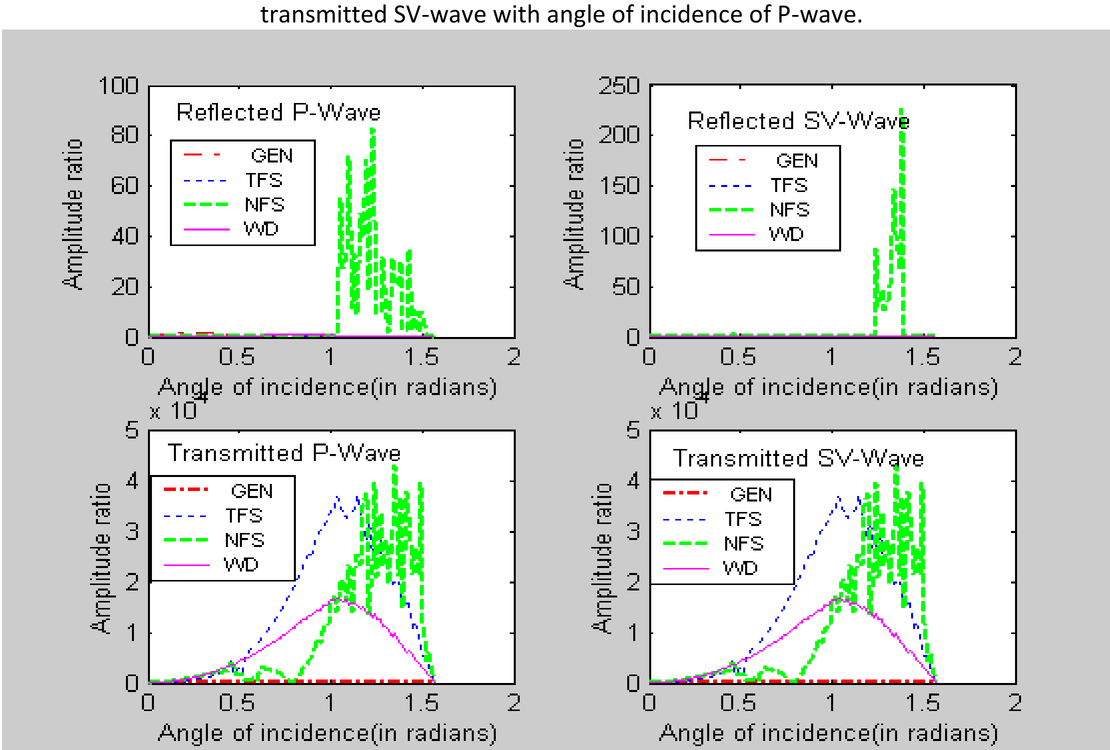
Figures (6)-(9) show the variations of the amplitude ratios $|Z_i|$, $(i = 1,2,3,4,)$ with angle of incidence of the incident SV-wave whereas figures (14)-(17) represent the case of empty porous solid. The behaviour of all these curves in figures (6)-(9) and (14)-(17) is same i.e. they oscillates. In figures (6)-(9), for amplitude ratios $|Z_1|$ and $|Z_2|$, the values for imperfect boundary (GEN) are greater than other stiffness cases whereas in case of $|Z_3|$ and$|Z_4|$, the values for imperfect boundary (GEN) are smaller than the values of other stiffness cases. In figures (14)-(17), the values for $|Z_1|$ are greater than other stiffness cases and the values for imperfect boundary (EGEN) for $|Z_3|$ and $|Z_4|$ are smaller than other stiffness cases. Figures (22)-(25) depict the variations for imperfect boundary, normal force stiffness, and transverse force stiffness, welded contact for the case when linear viscoelastic solid half space medium M_2 reduces to elastic solid half space. The curves in figures (22)-(25) oscillate.

7. Conclusion

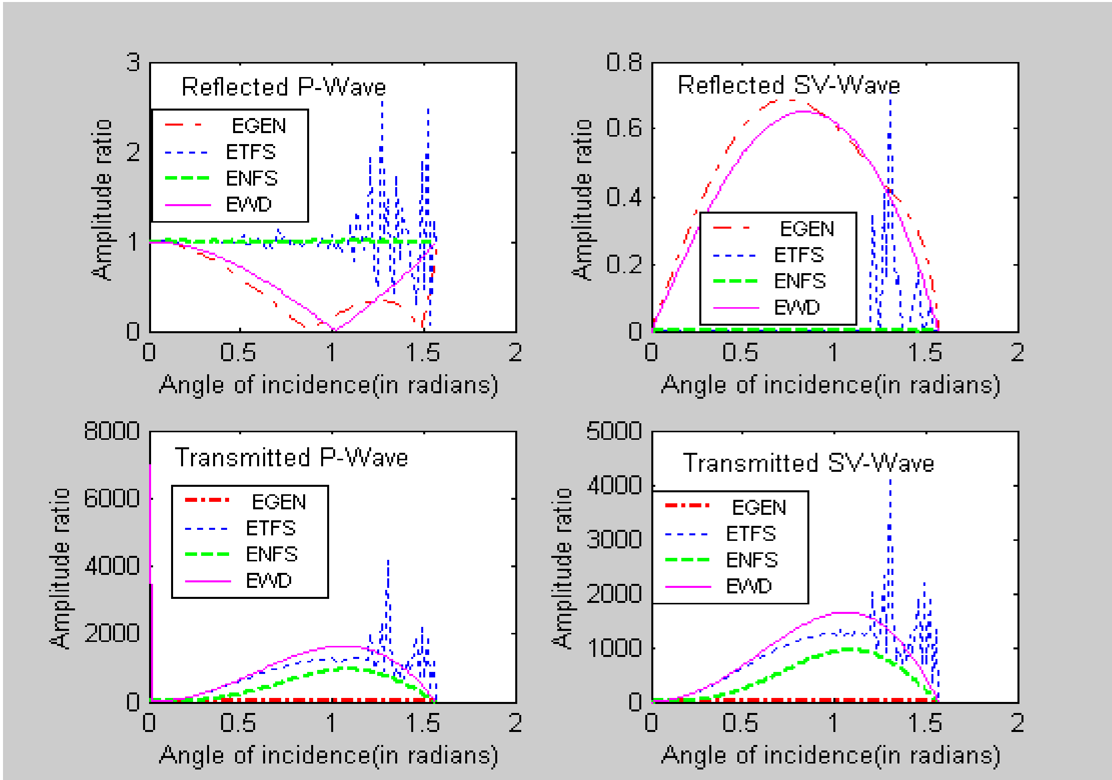
Reflection and transmission phenomenon of incident elastic waves at an imperfect interface between viscoelastic solid half space and fluid saturated porous half space has been studied when P-wave or SV-wave is incident. It is observed that the amplitudes ratios of various reflected and transmitted waves depend on the angle of incidence of the incident wave and material properties. The effect of fluid filled in the pores of incompressible fluid saturated porous medium is significant on amplitudes ratios. Effect of stiffness is observed on amplitude ratios. The research work is supposed to be useful in further studies; both theoretical and observational of wave propagation in more realistic models of fluid saturated porous solid present in the earth’s interior. The problems may be of use in engineering, seismology and geophysics etc.



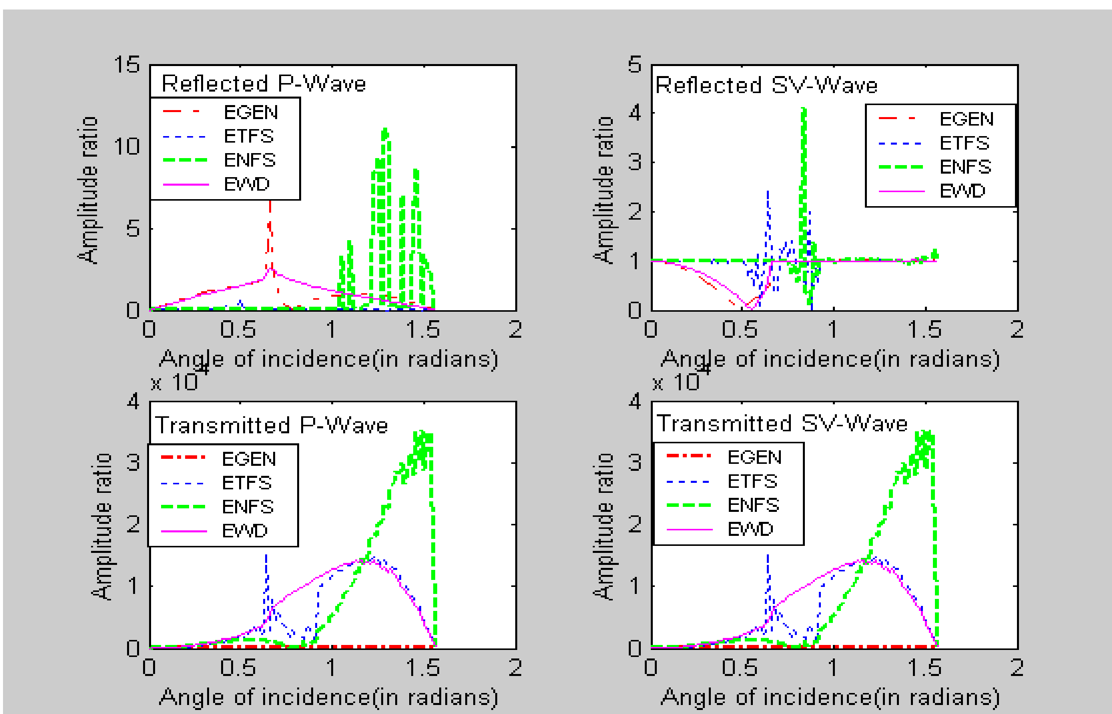
Figures 2-5. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of P-wave.



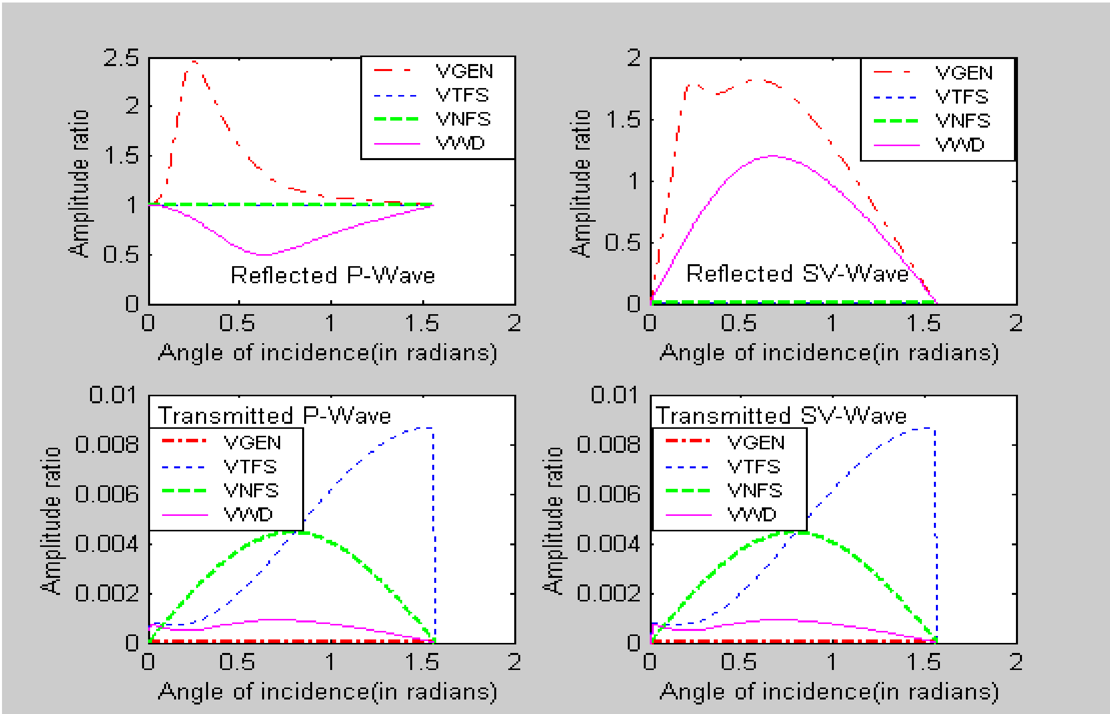
Figures 6-9. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of SV-wave.



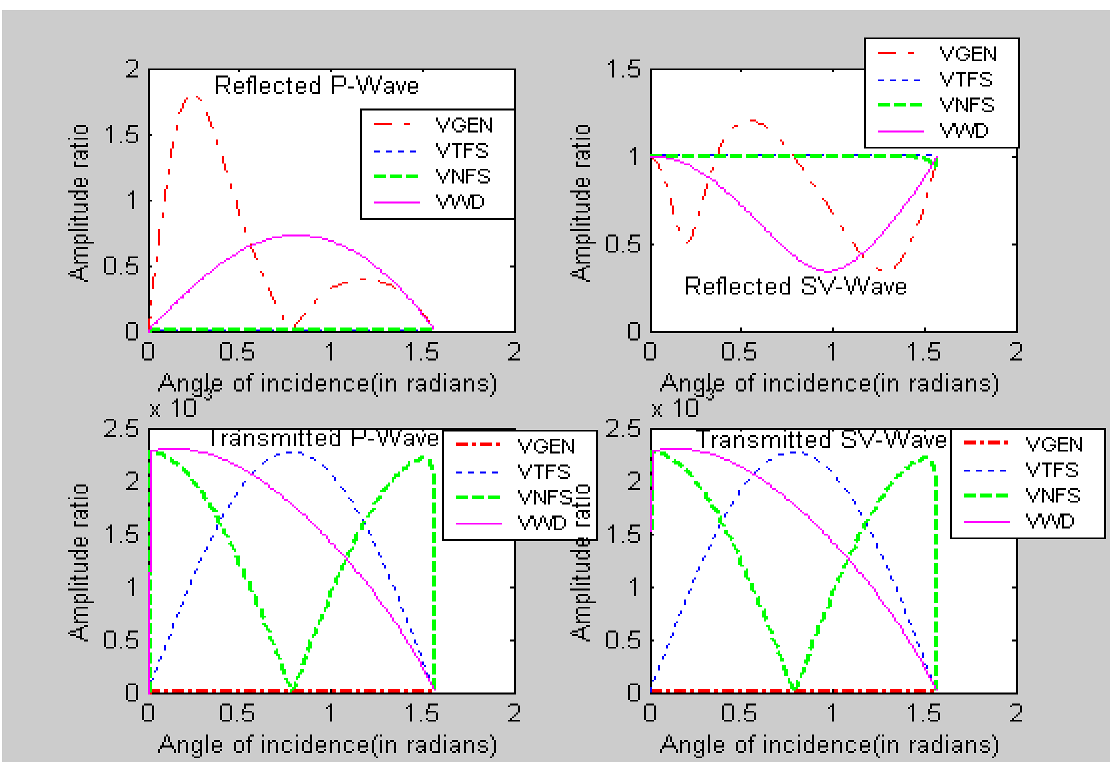
Figures 10-13. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of P-wave in case of empty porous solid.



Figures 14-17. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of P-wave in case of empty porous solid.



Figures 18-21. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of P-wave in case of elastic solid



Figures 22-25. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of P-wave in case of elastic solid

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