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RESEARCH ARTICLE

FUZZY SHORTEST PATH USING CENTROID OF TYPE-1 FUZZY SETS

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ABSTRACT

Finding Shortest path on a graph is a fundamental problem in the area of graph theory. In an application where we cannot exactly determine the weights of edges, fuzzy weights can be used instead of crisp weights, and type-2 fuzzy weights will be more suitable if this uncertainty varies under some conditions. In this paper, shortest path is obtained using type reduction method.

Key words : Type-2 fuzzy set, Centroid of a type-2 fuzzy set, Distance Measure, Extension Principle

INTRODUCTION

In graph theory the shortest path problem is the problem of finding a path between source node to destination node on a network. It has applications in various fields like transportation, Communication, routing and scheduling. In real world problem the arc length of the network may represent the time or cost which is not stable in the entire situation, hence it can be considered to be a fuzzy set.

The fuzzy shortest path problem was first analyzed by Dubois and Prade[2]. Okada and Soper [4] developed an algorithm based on the multiple labeling approach, by which a number of nondominated paths can be generated. Type-2 fuzzy set was introduced by Zadeh[10] as an extension of the concept of an ordinary fuzzy set. The type-2 fuzzy logic has gained much attention recently due to its ability to handle uncertainty and many advances appeared in both theory and applications.

Type reduction is one phase to defuzzify type-2 fuzzy sets. It means that by this method, we can transform a type-2 fuzzy set into a type-1 fuzzy set. Type reduction was proposed by Karnik and Mendel [7],[8],[9]. It is an “extended Version” [10] of type-1 defuzzification methods and is called type reduction because this operation takes us from the type-2 output sets of the FLS to a type-1 set that is called the “type reduced set”. This set may then be defuzzified to obtain a single crisp number; however, in many applications, the type reduced set may be more important than a single crisp number since it conveys a measures of uncertainties that have flown through the type-2 FLS. There exist many kinds of type-reduction, such as centroid, centre-of sets, heights and modified heights, the details of which are given in[7],[8],[9]. In this paper, we focus on the centroid of Gaussian type-2 fuzzy sets for finding fuzzy shortest path.

This paper is organized as follows: In Section 2, we have some basic concepts required for analysis. Section 3, gives an algorithm to find shortest path and shortest path length with type-2 fuzzy number. Section 4 gives the network terminology. To illustrate the proposed algorithm the numerical example is solved in section 5.

1. CONCEPTS

2.1 Type-2 Fuzzy Set:

A Type-2 fuzzy set denoted $\tilde{A}$, is characterized by a Type-2 membership function $\mu_{\tilde{A}}(x,u)$ where x $\in$ X and u $\in J_x \subseteq [0,1]$.

ie., $\tilde{A}$ = { ((x,u), $\mu_{\tilde{A}}(x,u)$) / $\forall$ x $\in$ X, $\forall$ u $\in J_x \subseteq [0,1]$ } in which $0 \le \mu_{\tilde{A}}(x,u) \le 1$. $\tilde{A}$ can be expressed as $\tilde{A} = \int_{x\in X}\int_{u\in J_x} \mu_{\tilde{A}}(x,u)/(x,u)$ $J_x \subseteq [0,1]$, where $\int\int$ denotes union over all admissible x and u. For discrete universe of discourse $\int$ is replaced by $\sum$.

2.2 Type-2 Fuzzy Number:

Let $\tilde{A}$ be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied:

1. $\tilde{A}$ is normal,
2. $\tilde{A}$ is a convex set,
3. The support of $\tilde{A}$ is closed and bounded, then $\tilde{A}$ is called a type-2 fuzzy number.

2.3 Discrete Type-2 Fuzzy Number:

The discrete type-2 fuzzy number $\tilde{A}$ can be defined as follows:

$\tilde{A} = \sum_{x\in X} \mu_{\tilde{A}}(x)/x$ where $\mu_{\tilde{A}}(x) = \sum_{u\in J_x} f_x(u)/u$ where J_x is the primary membership.

2.4 Extension Principle:

Let $A_1, A_2, \ldots, A_r$ be type-1 fuzzy sets in $X_1, X_2, \ldots, X_r$, respectively. Then, Zadeh’s Extension Principle allows us to induce from the type-1 fuzzy sets $A_1, A_2, \ldots, A_r$ a type-1 fuzzy set B on Y, through f, i.e,B = f($A_1, \ldots, A_r$), such that

$$\mu_B(y) = \begin{cases} \sup\limits_{x_1,x_2,\ldots x_n \in f^{-1}(y)} \min\{\mu_{A_1}(x_1),\ldots,\mu_{A_n}(x_n) & iff^{-1}(y) \ne \phi \\ 0, & f^{-1}(y) = \phi \end{cases}$$

2.5 Addition On Type-2 Fuzzy Numbers:

Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number be $\tilde{A} = \sum \mu_{\tilde{A}}(x)/\mathrm{x}$ and $\tilde{B} = \sum \mu_{\tilde{B}}(\mathrm{y})/\mathrm{y}$ where $\mu_{\tilde{A}}(x) = \sum f_x(\mathrm{u})/\mathrm{u}$ and $\mu_{\tilde{B}}(x) = \sum g_y(\mathrm{w})/\mathrm{w}$. The addition of these two types-2 fuzzy numbers $\tilde{A} \oplus \tilde{B}$ is defined as

$$\mu_{\tilde{A}\oplus\tilde{B}}(\mathrm{z}) = \bigcup_{z=x+y} (\mu_{\tilde{A}}(\mathrm{x}) \bigcap \mu_{\tilde{B}}(\mathrm{y}))$$

$$= \bigcup_{z=x+y} ((\sum_i \mathrm{f}_x(\mathrm{u}_i)/\mathrm{u}_i) \bigcap (\sum_j g_y(\mathrm{w}_j)/\mathrm{w}_j))$$

$$\mu_{\tilde{A}\oplus\tilde{B}}(\mathrm{z}) == \bigcup_{z=x+y} ((\sum_{i,j} (\mathrm{f}_x(\mathrm{u}_i) \wedge g_y(\mathrm{w}_j))/(\mathrm{u}_i \wedge \mathrm{w}_j))$$

2.6 Minimum of two discrete type-2 fuzzy number:

Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number then minimum of two type-2 fuzzy sets is denoted as Min($\tilde{A}$, $\tilde{B}$) is given by

$$Min(\tilde{A},\tilde{B})(z) = \sup_{z=Min(x,y)} \left[(f_x(u_i) \wedge g_y(w_j))/(\mathrm{u}_i \wedge w_j) \right]$$

Where $\tilde{A} = \sum f_x(\mathrm{u})/\mathrm{u}/\mathrm{x}$ and $\tilde{B} = \sum g_y(\mathrm{w})/\mathrm{w}/\mathrm{y}$.

2.7 Similarity Measure:

If d is the distance measure between two fuzzy sets A and B on the universe X, then the following measure of similarity is presented respectively.

Kochy similarity Measure :

S(A,B) = $\dfrac{1}{1+d(A,B)}$.

2.8 Distance based similarity measures for fuzzy sets:

Various distance measures are available in literature. Here we are using the following measure for the proposed algorithm.

1. The Hamming distance

$$d_H(A,B) = \sum_{i=1}^{n} \left| A(x_i) - B(x_i) \right|$$

2.9 Centroid of Type-2 Fuzzy Sets

Suppose that $\tilde{A}$ is a type-2 fuzzy set in the discrete caase. The centroid of $\tilde{A}$ can be defined as follows:

$$C_{\tilde{A}} = \int_{\theta_1 \in J_{x_1}} \cdot \int_{\theta_2 \in J_{x_2}} \cdots\cdots \int_{\theta_R \in J_{x_R}} [\, f_{x_1}(\theta_1).f_{x_2}(\theta_2)\ldots\ldots f_{x_R}(\theta_R) \Bigg/ \frac{\sum_{j=1}^{R} x_j \mu_A(x_j)}{\sum_{j=1}^{R} \mu_A(x_j)}$$

Where $\tilde{A} = \sum_{j=1}^{R} \left[\sum_{u \in J_{x_j}} f_{x_j}(\mathrm{u})/\mathrm{u} \right] / \mathrm{x}_j$

ALGORITHM

Algorithm for Finding Shortest Path Length

Step 1 : Form the possible paths from starting node to destination node and compute the corresponding path lengths, $\tilde{L}_i$ i = 1,2,. . . .n for possible n paths.

Step 2 : Find $C_{\tilde{L}_i}$ using def 2.9

Step 3 : Set $C_{\tilde{L}} = C_{\tilde{L}_1}$

Step 4 : Let i=2

Step 5 : Compute $C_{\tilde{L}}$ = Min ($C_{\tilde{L}}$, $C_{\tilde{L}_i}$) using def 2.6.

Step 6 : Set i = i + 1

Step 7 : If i$\leq$ n goto step 5

Step 8 : The shortest path length is $C_{\tilde{L}}$

Step 9 : Let j = 1

Step 10 : Compute $d(C_{\tilde{L}}, C_{\tilde{L}_j}) = \sum_{i=1}^{m} \left| C_{\tilde{L}(x_i)} - C_{\tilde{L}_j(x_i)} \right|$

Step 11 : Compute $s_j(C_{\tilde{L}}, C_{\tilde{L}_j}) = \dfrac{1}{1 + d(C_{\tilde{L}}, C_{\tilde{L}_j})}$

Step 12 : If j = 1, Assign $s\ (C_{\tilde{L}}, C_{\tilde{L}_j}) = s_j(C_{\tilde{L}}, C_{\tilde{L}_j})$

Step 13 : Compute s = Min (S, S_j)

Step 14 : Put j = j + 1

Step 15 : If j$\leq$n goto step 10.

Step 16 : The Least Similarity degree is S and that corresponding path is the Shortest path and $C_{\tilde{L}}$ is the shortest path length.

2. NETWORK TERMINOLOGY

Consider a directed network G(V,E) consisting of a finite set of nodes V = {1,2, . . .n} and a set of m directed edges $E \subseteq VXV$. Each edge is denoted by an ordered pair (i,j), where i,j $\in$ V and i$\neq$ j. In this network, we specify two nodes, denoted by s and t, which are the source node and the destination node, respectively. We define a path P_{ij} as a sequence P_{ij} = {i = i_1, (i_1,i_2),i_2,. . . ., i_{l-1}, (i_{l-1},i_l), i_l = j} of alternating nodes and edges. The existence of at least one path P_{si} in G(V,E) is assumed for every node i$\in$ V – {s}.

$\tilde{d}_{ij}$ denotes a Type-2 Fuzzy Number associated with the edge (i,j), corresponding to the length necessary to transverse (i,j) from i to j. The fuzzy distance along the path P is denoted as $\tilde{d}(P)$ is defined as $\tilde{d}(P) = \sum_{(i,j\in P)} \tilde{d}_{ij}$

3. NUMERICAL EXAMPLE

The problem is to find the shortest path and shortest path length between source node and destination node in the network having 6 vertices and 8 edges with type-2 fuzzy number.

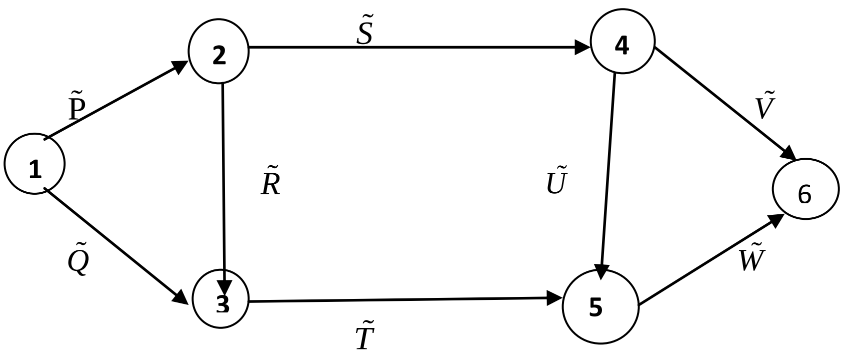

Fig 5.1

Solution:

The edge Lengths are

$\tilde{P}$ = (0.3/0.8+0.2/0.7)/2 + (0.3/0.9)/4

$\tilde{Q}$ = (0.5/0.8 + 0.3/0.6)/3

$\tilde{R}$ = (0.7/0.6)/1 + (0.5/0.7)/2

$\tilde{S}$ = (0.4/0.4 + 0.5/0.5)/2 + (0.2/0.9)/3

$\tilde{T}$ = (0.5/0.7)/2 + (0.7/0.4)/3

$\tilde{U}$ = (0.2/0.6)/1 + (0.3/0.5 + 0.4/0.4)/3

$\tilde{V}$ = (0.6/0.6)/2 +(0.7/0.5 + 0.4/0.4)/3

$\tilde{W}$ = (0.6/0.8)/1 + (0.4/0.5)/3

Algorithm for Finding Shortest Path Length

Step 1 : Form the possible paths from starting node to destination node and compute the corresponding path lengths, $\tilde{L}_i$ i = 1,2,. . . .n for possible n paths.

$\tilde{P}_1$: 1 – 2 – 4 – 6

$\tilde{P}_2$: 1 – 2 – 4 – 5 – 6

$\tilde{P}_3$: 1 – 2 – 3 – 5 – 6

$\tilde{P}_4$: 1 – 3 – 5 – 6

$\tilde{L}_1$ = (0.3/0.4 + 0.3/0.5)/6+ (0.2/0.6)/7 + (0.2/0.5 + 0.2/0.4)/8 + (0.2/0.6)/9 +(0.2/0.5 + 0.2/0.4)/10

$\tilde{L}_2$ = (0.2/0.4 + 0.2/0.5)/6 + (0.2/0.6)/7 + (0.2/0.4+ 0.2/0.5)/8 + (0.2/0.6)/9 + (0.2/0.4 + 0.2/0.5)/10 + (0.2/0.4 + 0.2/0.5)/11 + (0.3/0.4 + 0.3/0.5)/12 + (0.2/0.4 + 0.2/0.5)/13

$\tilde{L}_3$ = (0.3/0.6)/6 + (0.3/0.7)/7 + (0.3/0.6)/8 + (0.3/0.7)/9 + (0.3/0.7)/10 + (0.3/0.5)/11 + (0.3/0.5)/12

$\tilde{L}_4$ = (0.5/0.7 + 0.3/0.6)/6 + (0.5/0.4)/7 + (0.4/0.5)/8 + (0.4/0.4)/9

Step 2 : Find $C_{\tilde{L}_i}$ using def 2.9

$C_{\tilde{L}_1}$ = 0.00048/8.4 + 0.00048/8.1 + 0.00048/8 + 0.00048/7.9 +0.00048/7.8

$C_{\tilde{L}_2}$ = 0.0000038/9.3 + 0.0000038/9.4 +0.0000038/9.5 + 0.0000038/9.6

$C_{\tilde{L}_3}$ = 0.00022/8.9

$C_{\tilde{L}_4}$ = 0.04/7.3 + 0.024/7.4

Step 3 : Set $C_{\tilde{L}} = C_{\tilde{L}_1}$

Step 4 : Let i=2

Step 5 : Compute $C_{\tilde{L}}$ = Min ($C_{\tilde{L}}$, $C_{\tilde{L}_i}$) using def 2.6.

$C_{\tilde{L}}$ = 0.04/7.3 + 0.024/7.4

Step 6 : Set i = i + 1

Step 7 : If i $\leq$ n goto step 5

Step 8 : The shortest path length is $C_{\tilde{L}}$

$C_{\tilde{L}}$ = 0.04/7.3 + 0.024/7.4

Step 9 : Let j = 1

Step 10 : Compute $d(C_{\tilde{L}}, C_{\tilde{L}_j}) = \sum_{i=1}^{m} \left| C_{\tilde{L}(x_i)} - C_{\tilde{L}_j(x_i)} \right|$

$d(C_{\tilde{L}}, C_{\tilde{L}_1}) = 0.0664$

Step 11 : Compute $s_j(C_{\tilde{L}}, C_{\tilde{L}_j}) = \frac{1}{1 + d(C_{\tilde{L}}, C_{\tilde{L}_j})}$

$s_1(C_{\tilde{L}}, C_{\tilde{L}_1}) = 0.938$

Step 12 : If j = 1, Assign $s\ (C_{\tilde{L}}, C_{\tilde{L}_j}) = s_j(C_{\tilde{L}}, C_{\tilde{L}_j})$

$s\ (C_{\tilde{L}}, C_{\tilde{L}_1}) = 0.938$

Step 13 : Compute s = Min (S, S_j)

S = 0.938

Step 14 : Put j = j + 1

j = 2

Step 15 : If j $\leq$ n goto step 10.

2 $\leq$ 4 goto step 10

Step 10 : Compute $d(C_{\tilde{L}}, C_{\tilde{L}_j}) = \sum_{i=1}^{m} \left| C_{\tilde{L}(x_i)} - C_{\tilde{L}_j(x_i)} \right|$

$d(C_{\tilde{L}}, C_{\tilde{L}_2}) = 0.064$

Step 11 : Compute $s_j(C_{\tilde{L}}, C_{\tilde{L}_j}) = \frac{1}{1 + d(C_{\tilde{L}}, C_{\tilde{L}_j})}$

$s_2(C_{\tilde{L}}, C_{\tilde{L}_2}) = 0.939$

Step 13 : Compute s = Min (S, S_j)

S = 0.938

Step 14 : Put j = j + 1

j = 3

Step 15 : If j $\leq$ n goto step 10.

3 $\leq$ 4 goto step 10

Step 10 : Compute $d(C_{\tilde{L}}, C_{\tilde{L}_j}) = \sum_{i=1}^{m} \left| C_{\tilde{L}(x_i)} - C_{\tilde{L}_j(x_i)} \right|$

$d(C_{\tilde{L}}, C_{\tilde{L}_3}) = 0.064$

Step 11 : Compute $s_j(C_{\tilde{L}}, C_{\tilde{L}_j}) = \frac{1}{1 + d(C_{\tilde{L}}, C_{\tilde{L}_j})}$

$s_3(C_{\tilde{L}}, C_{\tilde{L}_3}) = 0.934$

Step 13 : Compute s = Min (S, S_j)

S = 0.934

Step 14 : Put j = j + 1

j = 4

Step 15 : If j $\leq$ n goto step 10.

4 $\leq$ 4 goto step 10

Step 10 : Compute $d(C_{\tilde{L}}, C_{\tilde{L}_j}) = \sum_{i=1}^{m} \left| C_{\tilde{L}(x_i)} - C_{\tilde{L}_j(x_i)} \right|$

$d(C_{\tilde{L}}, C_{\tilde{L}_4}) = 0$

Step 11 : Compute $s_j(C_{\tilde{L}}, C_{\tilde{L}_j}) = \dfrac{1}{1 + d(C_{\tilde{L}}, C_{\tilde{L}_j})}$

$$s_4(C_{\tilde{L}}, C_{\tilde{L}_4}) = 1$$

Step 13 : Compute s = Min (S, S_j)

S = 0.934

Step 14 : Put j = j + 1

Step 15 : If j $\leq$ n goto step 10.

j = 5

Step 16 : The Least Similarity degree is S= 0.934 and that corresponding path 1 – 2 - 3- 5 - 6 is the Shortest path and

$C_{\tilde{L}}$ = 0.04/7.3 + 0.024/7.4

is the shortest path length.

CONCLUSION

In this paper, we have calculated the fuzzy shortest path using type reduction. This type reduction procedure reduces the length of the computational part.

REFERENCES

[1]. V. Anusuya and R, Sathya, " Type-2 fuzzy shortest path", International Journal of fuzzy mathematical Archive, Vol.2, 2013, 36-42.

[2]. D. Dubois and H. Prade, " Theory and Applications:", Fuzzy sets and systems, Man and Cybernetics, Vol. 23, No.2, 1993, pp.403-405.

[3]. N. Karnik and M. Mendel, " Type-2 fuzzy logic systems: Type – reduction", in IEEE Syst., Man, cybern. Conf., San Diego, CA, oct. 1998.

[4]. N. Karnik and M. Mendel, " An introduction to type-2 fuzzy logic systems", USC Report, http://sipi.usc.edu/~mendel/report, oct. 1998.

[5]. N. Karnik and M. Mendel, " Type-2 fuzzy logic systems", IEEE Trans. Fuzzy Syst., vol.7, dec 1999, pp.643-658.

[6]. Nagoorgani, A. and Anusuya, V. Fuzzy Shortest Path by Linear Multiple Objective Programming, Proceedings of the International Conference On Mathematics and Computer Science.

[7]. S. Okada and T. Soper, " A shortest path problem on a network with fuzzy arc lengths", Fuzzy sets and systems, 109, 2000, 129-140.

[8]. Sujatha, L. and Elizabeth, S. Fuzzy Shortest Path Problem based On Similarity degree, Applied mathematical Sciences, Vol.5, 2011, PP: 3263 – 3276.

[9]. Yung – Yuan Kung, Tzung – Nan Chuang and Chia – Tzu Lin, A new dynamic Programming approach for finding the shortest path length and the corresponding shortest path in a discrete fuzzy network, Journal of Intelligent and fuzzy systems, 18, 2007, 117 – 122.

[10]. L.A. Zadeh, " The Concept of a Linguistic Variable and its Application to Approximate Reasoning – 1", Inform.