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SOME COMMON FIXED POINT RESULTS FOR HYBRID PAIRS OF OCCASIONALLY WEAKLY COMPATIBLE MAPPINGS IN MENGER SPACE

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ABSTRACT

The motive of this paper is to obtain some common fixed point theorems for occasionally weakly compatible mappings for hybrid pairs of single valued and multi-valued maps in Menger space.

Keywords: Occasionally weakly compatible mappings, common fixed point theorem, hybrid maps, Menger space.

2000 Mathematics Subject Classification: 47H10; 54H25

INTRODUCTION

K. Menger [9] introduced the notion of probabilistic metric space, which is a generalization of the metric space. The study of this space was done mainly with the pioneering works of Schweizer and Sklar [15, 16] and many of their co-workers. Such a probabilistic generalization of metric spaces appears to be well adapted for the investigation of physiological thresholds and physical quantities. It has importance in probabilistic functional analysis, nonlinear analysis and applications (see [3], [4], [5], [10], [11], [14], [19]). In 1972, Sehgal and Bharucha-Reid[17] initiated the study of contraction maps in probabilistic metric spaces (shortly, PM-spaces) which is an important step in the development of fixed point theorems.

The study of fixed point theorems, involving four self maps, began with the assumption that all of the maps are commuted. Sessa [13] weakened the condition of commutativity to that of pairwise weakly commuting. Jungck generalized the notion of weak commutativity to that of pairwise compatible [6] and then pairwise weakly compatible maps [7]. Jungck and Rhoades [8] introduced the concept of occasionally weakly compatible maps.

Abbas and Rhoades [1] generalized the concept of weak compatibility in the setting of single and multi-valued maps by introducing the notion of occasionally weakly compatible (owc).Also Abbas and Rhoades [2] extended the idea of owc maps to hybrid pairs of single-valued and multi-valued maps using a symmetric δ derived from an ordinary symmetric *d*.

We prove some common fixed point theorems for single-valued and multi-valued owc maps in Menger space.

Preliminary Notes

Definition 2.1[18]

A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t − norm if $*$ is satisfying conditions:

(i) $*$ is commutative and associative;

(ii) $*$ is continuous;

(iii) $a * 1 = a$ for all $a \in [0,1]$;

(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ and $a, b, c, d \in [0,1]$.

Definition 2.2 [18] A mapping $F: \mathcal{R} \to \mathcal{R}^+$ is called a distribution function if it is non decreasing and left continuous with $\inf\{F(t): t \in \mathcal{R}\} = 0$ and $\sup\{F(t): t \in \mathcal{R}\} = 1$.

We shall denote by $\Im$ the set of all distribution functions defined on $[-\infty, \infty]$ while H(t) will always denote the specific distribution function defined by

$$H(t) = \begin{cases} 0, & if\ t \leq 0; \\ 1, & if\ t > 1. \end{cases}$$

If X is a non-empty set, $\mathcal{F}: X \times X \to \Im$ is called a probabilistic distance on X and the value of $\mathcal{F}$ at $(x, y) \in X \times X$ is represented by $F_{x,y}$.

Definition 2.3[18] A PM-space is an ordered pair $(X, \mathcal{F})$, where X is a nonempty set of elements and $\mathcal{F}$ is a probabilistic distance satisfying the following conditions: for all $x, y, z \in X$ and $t, s > 0$,

(1) $\mathrm{F}_{x,y}(\mathrm{t}) = \mathrm{H}(\mathrm{t})$ for all $t > 0$ if and only if $x = y$,

(2) $\mathrm{F}_{x,y}(\mathrm{t}) = \mathrm{F}_{y,x}(\mathrm{t})$,

(3) if $\mathrm{F}_{x,y}(\mathrm{t}) = 1$ & $\mathrm{F}_{y,z}(\mathrm{t}) = 1$, then $\mathrm{F}_{x,z}(\mathrm{t} + \mathrm{s}) = 1$.

The ordered triple $(X, \mathcal{F}, *)$ is called a **Menger space** if $(X, \mathcal{F})$ is a PM-space $*$ is a t-norm and the following inequality holds:

$$\mathrm{F}_{x,y}(\mathrm{t} + \mathrm{s}) \geq *(\mathrm{F}_{x,z}(\mathrm{t}), \mathrm{F}_{z,y}(\mathrm{t}))$$

for all $x, y, z \in X$ and $t, s > 0$.

Throughout this paper, CB(X) will denote the family of all non-empty closed and bounded subsets of a Menger space $(X, \mathcal{F}, *)$. For all $A, B \in CB(X)$ and for every $t > 0$, we define

$${}_D F_{A,B}(t) = \sup\{F_{a,b}(t);\ a \in A, b \in B\}$$

and

$${}_\delta F_{A,B}(t) = \inf\{F_{a,b}(t);\ a \in A, b \in B\}.$$

If set A consists of a single point a, we write

$${}_\delta F_{A,B}(t) = {}_\delta F_{a,B}(t).$$

If set B consists of a single point b, we write

$${}_\delta F_{A,B}(t) = {}_\delta F_{a,b}(t).$$

It follows immediately from the definition that ${}_\delta F_{A,B}(t) = 1$ for all $t > 0$ if and only if A=B= {a} for some $a \in X$.

Lemma 2.4[12] If a Menger space $(X,\mathcal{F},*)$ satisfies the condition $F_{x,y}(t) = C$ for all $t > 0$ with fixed $x, y \in X$. Then we have $C = 1$ and $x = y$.

Definition 2.5[1] Maps $f: X \longrightarrow X$ and $T: X \longrightarrow CB(X)$ are said to be weakly compatible if they commute at their coincidence points, that is $fx \in Tx$ for some $x \in X$ then $fTx = Tfx$.

Definition 2.6[1] Maps $f: X \longrightarrow X$ and $T: X \longrightarrow CB(X)$ are said to be occasionally weakly compatible (owc) if and only if there exist some point x in X such that $fx \in Tx$ and $fTx \subseteq Tfx$.

Example 2.7 Let $(X,\mathcal{F},*)$ be a Menger space, where $X = [0, \infty)$ and

$$F_{x,y}(t) = \begin{cases} \dfrac{t}{t+|x-y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0. \end{cases}$$

Let $A: X \to X$ & $B: X \to CB(X)$ be single valued and set-valued maps defined by

$$A(x) = \begin{cases} 0, & if\ x = 0; \\ x^2, & if\ x \in (0,\infty). \end{cases} \qquad B(X) = \begin{cases} \{0\}, & if\ x = 0; \\ \{3x\}, & if\ x \in (0,\infty). \end{cases}$$

Here, 0 and 3 are two coincidence points of A and B. That is $A0 = \{0\} \in B(0),\ A(3) = \{9\} \in B(3),\ but\ AB(0) = \{0\} = BA(0),\ AB(3) \neq BA(3)$. Thus A and B are owc but not weakly compatible.

Main Results

Theorem 2.1 Let $(X,\mathcal{F},*)$ be a menger space. Let $A, B : X \to X$ and $S, T: X \to CB(X)$ such that the pairs $\{A,S\}$ & $\{B,T\}$ are owc. If

$$_\delta F_{Sx,Ty}(t) \geq \min\{F_{Ax,By}(t),\ \ _\delta F_{Ax,Sx}(t),\ \ _\delta F_{By,Ty}(t),\ \ _\delta F_{Ax,Ty}(t),\ \ _\delta F_{By,Sx}(t)\} \qquad (2.1)$$

for all $x, y \in X$ & $t > 0$. Then A, B, S & T have a unique common fixed point.

Proof. Since the pairs $\{A,S\}$ & $\{B,T\}$ are owc, therefore, there exist two elements $u, v \in X$ such that $Au \in Su, ASu \subseteq SAu$ and $Bv \in Tv, BTv \subseteq TBv$.

First we prove that $Au = Bv$.

As $Au \in Su$ so $AAu \subset ASu \subset SAu, Bv \in Tv\ so\ BBv \subset BTv \subset TBv$ and hence $F_{A^2u,B^2v}(t) \geq\ _\delta F_{SAu,TBv}(t)$ and if $Au \neq Bv$ then $_\delta F_{SAu,TBv}(t) < 1$. Using (2.1) for $x = Au$ & $y = Bv$.

$$_\delta F_{SAu,TBv}(t) \geq \min\{F_{A^2u,B^2v}(t),\ \ _\delta F_{A^2u,SAu}(t),\ \ _\delta F_{B^2v,TBv}(t),\ \ _\delta F_{A^2u,TBv}(t),\ \ _\delta F_{B^2v,SAu}(t)\}$$

$$\geq \min\{\ _\delta F_{SAu,TBv}(t), 1, 1,\ \ _\delta F_{SAu,TBv}(t),\ \ _\delta F_{TBv,SAu}(t)\}$$

$$=\ _\delta F_{SAu,TBv}(t), \text{ a contradiction.}$$

Hence $Au = Bv$.

Also,

$F_{A^2u,Bu}(t) \geq\ _\delta F_{SAu,Tu}(t)$

$F_{A^2u,Tu}(t) \geq\ _\delta F_{SAu,Tu}(t)$.

Now we claim that $Au = u$. If not, then $_\delta F_{SAu,Tu}(t) < 1$.

Considering (2.1) for $Au = x, y = u$

$$_\delta F_{SAu,Tu}(t) \geq \min\{F_{A^2u,Bu}(t),\ \ _\delta F_{A^2u,SAu}(t),\ \ _\delta F_{Bu,Tu}(t),\ \ _\delta F_{A^2u,Tu}(t),\ \ _\delta F_{Bu,SAu}(t)\}$$

$$\geq \min\{\ _\delta F_{SAu,Tu}(t), 1, 1,\ \ _\delta F_{SAu,Tu}(t),\ \ _\delta F_{Tu,SAu}(t)\}$$

$$=\ _\delta F_{SAu,Tu}(t), \text{ which is again a contradiction and hence } Au = u.$$

Similarly, we can get $Bv = v$.
Thus A, B, S & T have a common fixed point.
For uniqueness let $u \neq u'$ be another fixed point of A, B, S & T, then (2.1) gives

$$_{\delta}F_{Su,Tu'}(t) \geq \min\{F_{Au,Bu'}(t),\ \ _{\delta}F_{Au,Su}(t),\ \ _{\delta}F_{Bu',Tu'}(t),\ \ _{\delta}F_{Au,Tu'}(t),\ \ _{\delta}F_{Bu',Su}(t)\}$$
$$\geq \min\{F_{Su,Tu'}(t), 1,1,\ \ _{\delta}F_{Su,Tu'}(t),\ \ _{\delta}F_{Tu',Su}(t)\}$$
$$= \ _{\delta}F_{Su,Tu'}(t), \text{ a contradiction.}$$

Hence $Su = Tu'. i.e., u = u'$.
Thus, A, B, S & T have a unique common fixed point.

Theorem 2.2 Let $(X, \mathcal{F}, *)$ be a menger space. Let $A, B : X \to X$ and $S, T : X \to CB(X)$ such that the pairs $\{A, S\}$ & $\{B, T\}$ are owc. If

$$_{\delta}F_{Sx,Ty}(t) \geq \min\{F_{Ax,By}(t),\ \ _{\delta}F_{Ax,Sx}(t),\ \ _{\delta}F_{By,Ty}(t), \frac{_{\delta}F_{Ax,Ty}(t) + \ _{\delta}F_{By,Sx}(t)}{2}\} \qquad (2.2)$$

for all $x, y \in X$ & $t > 0$. Then A, B, S & T have a unique common fixed point.

Proof. Since the pairs $\{A, S\}$ & $\{B, T\}$ are owc, therefore, there exist two elements $u, v \in X$ such that $Au \in Su, ASu \subseteq SAu$ and $Bv \in Tv, BTv \subseteq TBv$.
First we prove that $Au = Bv$.
As $Au \in Su$ so $AAu \subset ASu \subset SAu, Bv \in Tv$ so $BBv \subset BTv \subset TBv$ and hence $F_{A^2u,B^2v}(t) \geq \ _{\delta}F_{SAu,TBv}(t)$ and if $Au \neq Bv$ then $_{\delta}F_{SAu,TBv}(t) < 1$. Using (2.2) for $x = Au$ & $y = Bv$.

$$_{\delta}F_{SAu,TBv}(t)$$
$$\geq \min\{F_{A^2u,B^2v}(t),\ \ _{\delta}F_{A^2u,SAu}(t),\ \ _{\delta}F_{B^2v,TBv}(t), \frac{_{\delta}F_{A^2u,TBv}(t) + \ _{\delta}F_{B^2v,SAu}(t)}{2}\}$$
$$\geq \min\{\ _{\delta}F_{SAu,TBv}(t), 1,1, \frac{_{\delta}F_{SAu,TBv}(t) + \ _{\delta}F_{TBv,SAu}(t)}{2}\}$$
$$= \ _{\delta}F_{SAu,TBv}(t), \text{ a contradiction.}$$

Hence $Au = Bv$.
Also, $F_{A^2u,Bu}(t) \geq \ _{\delta}F_{SAu,Tu}(t)$
$F_{A^2u,Tu}(t) \geq \ _{\delta}F_{SAu,Tu}(t)$.
Now we claim that $Au = u$. If not, then $_{\delta}F_{SAu,Tu}(t) < 1$.
Considering (2.2) for $Au = x, y = u$

$$_{\delta}F_{SAu,Tu}(t) \geq \min\{F_{A^2u,Bu}(t),\ \ _{\delta}F_{A^2u,SAu}(t),\ \ _{\delta}F_{Bu,Tu}(t), \frac{_{\delta}F_{A^2u,Tu}(t) + \ _{\delta}F_{Bu,SAu}(t)}{2}\}$$
$$\geq \min\{\ _{\delta}F_{SAu,Tu}(t), 1,1, \frac{_{\delta}F_{SAu,Tu}(t) + \ _{\delta}F_{Tu,SAu}(t)}{2}\}$$
$$= \ _{\delta}F_{SAu,Tu}(t),$$ which is again a contradiction and hence $Au = u$.

Similarly, we can get $Bv = v$.
Thus A, B, S & T have a common fixed point.
For uniqueness let $u \neq u'$ be another fixed point of A, B, S & T, then (2.2) gives

$$_{\delta}F_{Su,Tu'}(t) \geq \min\{F_{Au,Bu'}(t),\ \ _{\delta}F_{Au,Su}(t),\ \ _{\delta}F_{Bu',Tu'}(t), \frac{_{\delta}F_{Au,Tu'}(t) + \ _{\delta}F_{Bu',Su}(t)}{2}\}$$
$$\geq \min\{F_{Su,Tu'}(t), 1,1, \frac{_{\delta}F_{Su,Tu'}(t) + \ _{\delta}F_{Tu',Su}(t)}{2}\}$$
$$= \ _{\delta}F_{Su,Tu'}(t), \text{ a contradiction.}$$

Hence $Su = Tu'. i.e., u = u'$.
Thus, A, B, S & T have a unique common fixed point.

Corollary 2.3 Let $(X,\mathcal{F},*)$ be a menger space. Let $A,B : X \to X$ and $S,T:X \to CB(X)$ such that the pairs $\{A,S\}$ & $\{B,T\}$ are owc. If

$$_\delta F_{Sx,Ty}(t) \geq \min\{F_{Ax,By}(t), \frac{_\delta F_{Ax,Sx}(t) + {_\delta F_{By,Ty}(t)}}{2}, \frac{_\delta F_{Ax,Ty}(t) + {_\delta F_{Sx,By}(t)}}{2}\} \quad (2.3)$$

for all $x,y \in X$ & $t > 0$. Then A,B,S & T have a unique common fixed point.

Proof. Condition (2.3) is special case of (2.2), therefore result follows from Theorem 2.2.

Corollary 2.4 Let $(X,\mathcal{F},*)$ be a menger space. Let $A,B : X \to X$ and $S,T:X \to CB(X)$ such that the pairs $\{A,S\}$ & $\{B,T\}$ are owc. If

$$_\delta F_{Sx,Ty}(t) \geq h\min\{F_{Ax,By}(t), \ {_\delta F_{Ax,Sx}(t)}, \ {_\delta F_{By,Ty}(t)}, \frac{_\delta F_{Ax,Ty}(t) + {_\delta F_{By,Sx}(t)}}{2}\} \quad (2.4)$$

for all $x,y \in X, h \in [0,1)$ & $t > 0$. Then A,B,S & T have a unique common fixed point.

Proof. Since (2.4) is a special case of (2.2), the result follows immediately from Theorem 2.2.

Theorem 2.5 Let $(X,\mathcal{F},*)$ be a menger space. Let $A,B : X \to X$ and $S,T:X \to CB(X)$ such that the pairs $\{A,S\}$ & $\{B,T\}$ are owc. If

$$_\delta F_{Sx,Ty}(t) \geq \alpha F_{Ax,By}(t) + \beta\min\{F_{Ax,By}(t), \ {_\delta F_{Ax,Sx}(t)}, \ {_\delta F_{By,Ty}(t)}\} + \gamma\min\{F_{Ax,By}(t) \ \ {_\delta F_{Ax,Ty}(t)}, \ {_\delta F_{By,Sx}(t)}\} \quad (2.5)$$

for all $x,y \in X$ & $t > 0, where\ \alpha,\beta,\gamma > 0\ and\ (\alpha+\beta+\gamma) = 1$. Then A,B,S & T have a unique common fixed point.

Proof. Since the pairs $\{A,S\}$ & $\{B,T\}$ are owc, therefore, there exist two elements $u,v \in X$ such that $Au \in Su, ASu \subseteq SAu$ and $Bv \in Tv, BTv \subseteq TBv.$

First we prove that $Au = Bv.$

As $Au \in Su$ so $AAu \subset ASu \subset SAu, Bv \in Tv\ so\ BBv \subset BTv \subset TBv$ and hence

$F_{A^2u,B^2v}(t) \geq \ {_\delta F_{SAu,TBv}(t)}$ and if $Au \neq Bv$ then ${_\delta F_{SAu,TBv}(t)} < 1.$ Using (2.5) for $x = Au$ & $y = Bv.$

$$\begin{aligned} {_\delta F_{SAu,TBv}(t)} &\geq \alpha F_{A^2u,B^2v}(t) + \beta\min\{F_{A^2u,B^2v}(t), \ {_\delta F_{A^2u,SAu}(t)}, \ {_\delta F_{B^2v,TBv}(t)}\} \\ &\quad + \gamma\min\{F_{A^2u,B^2v}(t), \ {_\delta F_{A^2u,TBv}(t)}, \ {_\delta F_{B^2v,SAu}(t)}\} \\ &= (\alpha+\beta+\gamma) \ \ {_\delta F_{SAu,TBv}(t)}, \end{aligned}$$

a contradiction as $(\alpha+\beta+\gamma) = 1.$

Hence $Au = Bv.$

Also, $F_{A^2u,Bu}(t) \geq \ {_\delta F_{SAu,Tu}(t)}$

$F_{A^2u,Tu}(t) \geq \ {_\delta F_{SAu,Tu}(t)}.$

Now we claim that $Au = u$. If not, then ${_\delta F_{SAu,Tu}(t)} < 1.$

Considering (2.5) for $Au = x, y = u$

$$\begin{aligned} {_\delta F_{SAu,Tu}(t)} &\geq \alpha F_{A^2u,Bu}(t) + \beta\min\{F_{A^2u,Bu}(t), \ {_\delta F_{A^2u,SAu}(t)}, \ {_\delta F_{Bu,Tu}(t)}\} \\ &\quad + \gamma\min\{F_{A^2u,Bu}(t), \ {_\delta F_{A^2u,Tu}(t)}, \ {_\delta F_{Bu,SAu}(t)}\} \\ &= (\alpha+\beta+\gamma) \ \ {_\delta F_{SAu,Tu}(t)}, \end{aligned}$$

which is again a contradiction as $(\alpha+\beta+\gamma) = 1$ and hence $Au = u.$

Similarly, we can get $Bv = v.$

Thus A,B,S & T have a common fixed point.

Uniqueness follows from (2.5).

Theorem 2.6 Let $(X,\mathcal{F},*)$ be a menger space with $t*t=t$ for all $t\in[0,1]$. Let $A,B:X\to X$ and $S,T:X\to CB(X)$ such that the pairs $\{A,S\}$ & $\{B,T\}$ are owc. If

$$_{\delta}F_{Sx,Ty}(t)\geq\min\{F_{Ax,By}(t),\ _{\delta}F_{Ax,Sx}(t),\ _{\delta}F_{By,Ty}(t),\ _{\delta}F_{Ax,Ty}(\alpha t)*\ _{\delta}F_{By,Sx}((2-\alpha)t)\}$$
(2.6)

for all $x,y\in X$ & $t>0,\alpha\in(0,2)$. Then A,B,S & T have a unique common fixed point.

Proof. Since the pairs $\{A,S\}$ & $\{B,T\}$ are owc, therefore, there exist two elements $u,v\in X$ such that $Au\in Su, ASu\subseteq SAu$ and $Bv\in Tv, BTv\subseteq TBv$.

First we prove that $Au=Bv$.

As $Au\in Su$ so $AAu\subset ASu\subset SAu, Bv\in Tv$ so $BBv\subset BTv\subset TBv$ and hence $F_{A^2u,B^2v}(t)\geq\ _{\delta}F_{SAu,TBv}(t)$ and if $Au\neq Bv$ then $_{\delta}F_{SAu,TBv}(t)<1$. Using (2.6) for $x=Au$ & $y=Bv$.

$$_{\delta}F_{SAu,TBv}(t)\geq\min\{F_{A^2u,B^2v}(t),\ _{\delta}F_{A^2u,SAu}(t),\ _{\delta}F_{B^2v,TBv}(t),\ _{\delta}F_{A^2u,TBv}(\alpha t)$$
$$*\ _{\delta}F_{B^2v,SAu}((2-\alpha)t)\}$$

Since $*$ is continuous, letting $\alpha\to1$, we get

$$\geq\min\{\ _{\delta}F_{SAu,TBv}(t),1,1,\ _{\delta}F_{SAu,TBv}(t)*\ _{\delta}F_{TBv,SAu}(t)\}$$
$$=\ _{\delta}F_{SAu,TBv}(t),$$ a contradiction.

Hence $Au=Bv$.

Also, $$F_{A^2u,Bu}(t)\geq\ _{\delta}F_{SAu,Tu}(t)$$

$F_{A^2u,Tu}(t)\geq\ _{\delta}F_{SAu,Tu}(t)$.

Now we claim that $Au=u$. If not, then $_{\delta}F_{SAu,Tu}(t)<1$.

Considering (2.6) for $Au=x, y=u, \alpha=1$

$$_{\delta}F_{SAu,Tu}(t)\geq\min\{F_{A^2u,Bu}(t),\ _{\delta}F_{A^2u,SAu}(t),\ _{\delta}F_{Bu,Tu}(t),\ _{\delta}F_{A^2u,Tu}(t)*\ _{\delta}F_{Bu,SAu}(t)\}$$
$$\geq\min\{\ _{\delta}F_{SAu,Tu}(t),1,1,\ _{\delta}F_{SAu,Tu}(t)*\ _{\delta}F_{Tu,SAu}(t)\}$$
$$=\ _{\delta}F_{SAu,Tu}(t),$$ which is again a contradiction and hence $Au=u$.

Similarly, we can get $Bv=v$.

Thus A,B,S & T have a common fixed point.

For uniqueness let $u\neq u'$ be another fixed point of A,B,S & T, then (2.6) gives

$$_{\delta}F_{Su,Tu'}(t)\geq\min\{F_{Au,Bu'}(t),\ _{\delta}F_{Au,Su}(t),\ _{\delta}F_{Bu',Tu'}(t),\ _{\delta}F_{Au,Tu'}(\alpha t)$$
$$*\ _{\delta}F_{Bu',Su}((2-\alpha)t)\}$$

Letting $\alpha\to1$,

$$\geq\min\{F_{Su,Tu'}(t),1,1,\ _{\delta}F_{Su,Tu'}(t)*\ _{\delta}F_{Tu',Su}(t)\}$$
$$=\ _{\delta}F_{Su,Tu'}(t),$$ a contradiction.

Hence $Su=Tu'.i.e.,u=u'$.

Thus, A,B,S & T have a unique common fixed point.

Corollary 2.7 Let $(X,\mathcal{F},*)$ be a menger space with $t*t=t$ for all $t\in[0,1]$. Let $A,B:X\to X$ and $S,T:X\to CB(X)$ such that the pairs $\{A,S\}$ & $\{B,T\}$ are owc. If

$$_{\delta}F_{Sx,Ty}(t)\geq\min\{F_{Ax,By}(t),\ _{\delta}F_{Ax,Sx}(\alpha t)*\ _{\delta}F_{By,Ty}((2-\alpha)t),\ _{\delta}F_{Ax,Ty}(\beta t)*\ _{\delta}F_{By,Sx}((2-\beta)t)\}\quad(2.7)$$

for all $x,y\in X$ & $t>0,\alpha,\beta\in(0,2)$. Then A,B,S & T have a unique common fixed point.

Corollary 2.8 Let $(X,\mathcal{F},*)$ be a menger space with $t*t=t$ for all $t\in[0,1]$. Let $A:X\rightarrow X$ and $S:X\rightarrow CB(X)$ such that the pairs $\{A,S\}$ is owc. If

$$_{\delta}F_{Ax,Sy}(t)\geq\min\{F_{Ax,Ay}(t),\ \ _{\delta}F_{Ax,Sx}(\beta t)*\ \ _{\delta}F_{Ay,Sy}((2-\beta)t),\ \ _{\delta}F_{Ax,Sy}(\alpha t)*\ \ _{\delta}F_{Ay,Sx}((2-\alpha)t)\}$$ (2.8)

for all $x,y\in X\ \&\ t>0,\alpha,\beta\in(0,2)$. Then $A\ \&\ S$ have a unique common fixed point.

Corollary 2.9 Let $(X,\mathcal{F},*)$ be a menger space with $t*t=t$ for all $t\in[0,1]$. Let $A:X\rightarrow X$ and $S,T:X\rightarrow CB(X)$ such that the pairs $\{A,S\}$ is owc. If

$$_{\delta}F_{Sx,Ty}(t)\geq\min\{F_{Ax,Ay}(t),\ \ _{\delta}F_{Ax,Sx}(t),\ \ _{\delta}F_{Ay,Ty}(t),\ \ _{\delta}F_{Ax,Ty}(\alpha t)*\ \ _{\delta}F_{Ay,Sx}((2-\alpha)t)\}$$ (2.9)

for all $x,y\in X\ \&\ t>0,\alpha\in(0,2)$. Then $A,S\ \&\ T$ have a unique common fixed point.

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