Vol.2.Issue.4.2014

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com

ISSN:2348-0580

RESEARCH ARTICLE

TYPE-2 FUZZY SHORTEST PATH ON SIMILARITY MEASURE

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Article Info:
Article received :08/11/2014
Revised on:19/11/2014
Accepted on:21/11/2014

ABSTRACT

In this paper, we have developed an algorithm for finding the shortest path in a fuzzy weighted network, using similarity measure. Here we are assigning discrete type-2 fuzzy number for each edge in the network. Also we are finding the shortest path length in fuzzy sense where illustrative example is included to demonstrate our proposed approach.

Key words : Type-2 fuzzy number Discrete type-2 fuzzy number Similarity Measure, Extension Principle

INTRODUCTION

The shortest path problem concentrates on finding the path with minimum distance to find the shortest path from source node to destination node is a fundamental matter in graph theory. A directed acyclic network is a network that consists of a finite set of nodes and a set of direct acyclic arcs.

Zadeh[13] proposed type-2 fuzzy sets as an extension of (type-1) fuzzy sets whose membership values are fuzzy sets on the interval [0,1]. There were further studied and applications of type-2 fuzzy sets such as Mizumoto and Tanaka [7]. Yager[12], Mendel et al.[4], Jammeh et al.[3], Mendoza et al.[5] and Wagner and Hagras[11]. The membership function of a type-2 fuzzy set provides additional degree of freedom for modeling uncertainties so that type-2 fuzzy sets can better improve certain kinds of inference than do fuzzy sets with increasing imprecision, uncertainty and fuzziness in information.

The concept of fuzzy measures was introduced by sugeno[10]. As an important tool for determining the similarity between two objects, Zadeh[14] initiated fuzzy similarity measure, and later on, various similarity measures for fuzzy set have been sequentially proposed. Pappis and Karacapilidis[9] proposed three similarity measures based on union and intersection operations, the maximum difference, and the difference and sum of membership grades. Based on this Yang and Lin[6] proposed a similarity measure between type-2 fuzzy sets.

The structure of paper is following: In Section 2, we have some basic concepts required for analysis. In section 3, an algorithm is proposed to find shortest path and shortest path length based on similarity measure with discrete type-2 fuzzy number. Section 4 gives the network terminology. To illustrate the proposed algorithm the numerical example is solved in section 5.

CONCEPTS

2.1 Type-2 Fuzzy Set:

A Type-2 fuzzy set denoted $\tilde{A}$, is characterized by a Type-2 membership function $\mu_{\tilde{A}}(x,u)$ where x $\in$ X and u $\in J_x \subseteq [0,1]$.

ie., $\tilde{A}$ = { ((x,u), $\mu_{\tilde{A}}(x,u)$) / $\forall$ x $\in$ X, $\forall$ u $\in J_x \subseteq [0,1]$ } in which $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$. $\tilde{A}$ can be expressed as $\tilde{A} = \int_{x\in X}\int_{u\in J_x} \mu_{\tilde{A}}(x,u)/(x,u)$ $J_x \subseteq [0,1]$, where $\int\int$ denotes union over all admissible x and u. For discrete universe of discourse $\int$ is replaced by $\sum$.

2.2 Type-2 Fuzzy Number:

Let $\tilde{A}$ be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied:

1. $\tilde{A}$ is normal,
2. $\tilde{A}$ is a convex set,
3. The support of $\tilde{A}$ is closed and bounded, then $\tilde{A}$ is called a type-2 fuzzy number.

2.3 Discrete Type-2 Fuzzy Number:

The discrete type-2 fuzzy number $\tilde{A}$ can be defined as follows:

$\tilde{A} = \sum_{x\in X} \mu_{\tilde{A}}(\mathrm{x})/\mathrm{x}$ where $\mu_{\tilde{A}}(\mathrm{x}) = \sum_{u\in J_x} f_x(\mathrm{u})/\mathrm{u}$ where J_x is the primary membership.

2.4 Extension Principle:

Let $A_1, A_2, \ldots, A_r$ be type-1 fuzzy sets in $X_1, X_2, \ldots, X_r$, respectively. Then, Zadeh's Extension Principle allows us to induce from the type-1 fuzzy sets $A_1, A_2, \ldots, A_r$ a type-1 fuzzy set B on Y, through f, i.e,B = f($A_1, \ldots, A_r$), such that

$$\mu_B(y) = \begin{cases} \sup\limits_{x_1,x_2,\ldots x_n \in f^{-1}(y)} \min\{\mu_{A_1}(x_1),\ldots,\mu_{A_n}(x_n) & iff^{-1}(y) \neq \phi \\ 0, & f^{-1}(y) = \phi \end{cases}$$

2.5 Addition On Type-2 Fuzzy Numbers:

2.6 Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number be $\tilde{A}=\sum \mu_{\tilde{A}}(x)/\mathrm{x}$ and $\tilde{B}=\sum \mu_{\tilde{B}}(\mathrm{y})/\mathrm{y}$ where $\mu_{\tilde{A}}(x)=\sum f_x(\mathrm{u})/\mathrm{u}$ and $\mu_{\tilde{B}}(x)=\sum g_y(\mathrm{w})/\mathrm{w}$. The addition of these two types-2 fuzzy numbers $\tilde{A}\oplus\tilde{B}$ is defined as

$$\mu_{\tilde{A}\oplus\tilde{B}}(\mathrm{z})=\bigcup_{z=x+y}(\mu_{\tilde{A}}(\mathrm{x})\bigcap\mu_{\tilde{B}}(\mathrm{y}))$$

$$=\bigcup_{z=x+y}((\sum_i \mathrm{f}_x(\mathrm{u}_i)/\mathrm{u}_i)\bigcap(\sum_j g_y(\mathrm{w}_j)/\mathrm{w}_j))$$

$$\mu_{\tilde{A}\oplus\tilde{B}}(\mathrm{z})==\bigcup_{z=x+y}((\sum_{i,j}(\mathrm{f}_x(\mathrm{u}_i)\wedge g_y(\mathrm{w}_j))/(\mathrm{u}_i\wedge\mathrm{w}_j))$$

2.7 Minimum of two discrete type-2 fuzzy number:

Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number then minimum of two type-2 fuzzy sets is denoted as Min($\tilde{A}$, $\tilde{B}$) is given by

$$Min(\tilde{A},\tilde{B})(z)=\underset{z=Min(x,y)}{Sup}\left[(f_x(u_i)\wedge g_y(w_j))/(\mathrm{u}_i\wedge w_j)\right]$$

Where $\tilde{A}=\sum f_x(\mathrm{u})/\mathrm{u}/\mathrm{x}$ and $\tilde{B}=\sum g_y(\mathrm{w})/\mathrm{w}/\mathrm{y}$.

2.8 Similarity Measure:

Let $\tilde{A}$ and $\tilde{B}$ be two discrete type-2 fuzzy number then the similarity measure is given by

$$s(\tilde{A},\tilde{B})=\frac{1}{n}\left[\sum_{x\in X}\frac{\sum_{u\in J_x}\min\{\mathrm{u}.\mathrm{f}_x(u),u.g_x(u)\}}{\sum_{u\in J_x}\max\{\mathrm{u}.\mathrm{f}_x(u),u.g_x(u)\}}\right]$$

Where $\tilde{A}=\sum f_x(\mathrm{u})/\mathrm{u}/\mathrm{x}$ and $\tilde{B}=\sum g_x(\mathrm{u})/\mathrm{u}/\mathrm{x}$.

ALGORITHM

Step 1 : Computation of Possible Paths

Form the possible paths from starting node to destination node and compute the corresponding path lengths, $\tilde{L}_i$ i = 1,2,. . . .n for possible n paths.

Step 2 : Initialization

i) Set $\tilde{L}$ = $\tilde{L}_1$

ii) Let i = 2

Step 3 : Computation of Shortest path length

i) Compute $\tilde{L}$ = Min ($\tilde{L}$, $\tilde{L}_i$) using def 2.6.

ii) i = i + 1

iii) If i $\leq$ n goto step 3

iv) The shortest path length is $\tilde{L}$

Step 4 : Computation of Similarity Measure

i) Let j =1

ii) Compute $s_j(\tilde{L},\tilde{L}_j)$ using def 2.7

iii) If j= n assign s $(\tilde{L},\tilde{L}_j)$ = $s_j(\tilde{L},\tilde{L}_j)$

iv) Compute s = Max (s, s_j)

v) Put j = j + 1

vi) If j $\leq$ n then go to Step 4(ii)

Step 5 : Shortest Path and Shortest Path Length

The highest similarity degree is s and the corresponding path is shortest path and the shortest path length is $\tilde{L}$

NETWORK TERMINOLOGY

Consider a directed network G(V,E) consisting of a finite set of nodes V = {1,2, . . .n} and a set of m directed edges $E \subseteq VXV$. Each edge is denoted by an ordered pair (i,j), where i,j $\in$ V and i $\neq$ j. In this network, we specify two nodes, denoted by s and t, which are the source node and the destination node, respectively. We define a path P_{ij} as a sequence P_{ij} = {i = i_1, (i_1,i_2),i_2,. . . ., i_{l-1}, (i_{l-1},i_l), i_l = j} of alternating nodes and edges. The existence of at least one path P_{si} in G(V,E) is assumed for every node i $\in$ V – {s}.

$\tilde{d}_{ij}$ denotes a Type-2 Fuzzy Number associated with the edge (i,j), corresponding to the length necessary to transverse (i,j) from i to j. The fuzzy distance along the path P is denoted as $\tilde{d}(P)$ is defined as $\tilde{d}(P) = \sum_{(i,j \in P)} \tilde{d}_{ij}$

NUMERICAL EXAMPLE

The problem is to find the shortest path and shortest path length between source node and destination node in the network having 6 vertices and 8 edges with type-2 fuzzy number.

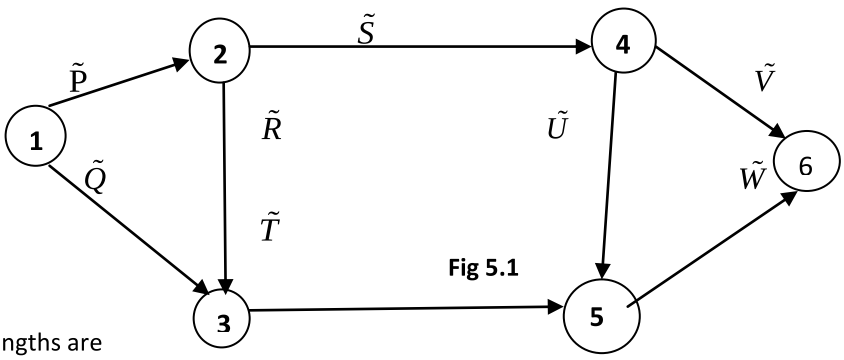

Fig 5.1

Solution:

The edge Lengths are

$\tilde{P}$ = (0.3/0.2+0.2/0.3)/2 + (0.3/0.1)/4

$\tilde{Q}$ = (0.5/0.2 + 0.3/0.4)/3

$\tilde{R}$ = (0.7/0.4)/1 + (0.5/0.3)/2

$\tilde{S}$ = (0.4/0.6 + 0.5/0.5)/2 + (0.2/0.1)/3

$\tilde{T}$ = (0.5/0.3)/2 + (0.7/0.6)/3

$\tilde{U}$ = (0.2/0.4)/1 + (0.3/0.5 + 0.4/0.6)/3

$\tilde{V}$ = (0.6/0.4)/2 +(0.7/0.5 + 0.4/0.6)/3

$\tilde{W}$ = (0.6/0.2)/1 +(0.4/0.5)/3

Step 1 : Computation of Possible Paths

Form the possible paths from starting node to destination node and compute the corresponding path lengths, $\tilde{L}_i$ i = 1,2,. . . .n for possible n paths.

$\tilde{P}_1$: 1 – 2 – 4 – 6

$\tilde{P}_2$: 1 – 2 – 4 – 5 – 6

$\tilde{P}_3$: 1 – 2 – 3 – 5 – 6

$\tilde{P}_4$: 1 – 3 – 5 – 6

$\tilde{L}_1$ = (0.3/0.2 + 0.2/0.3)/6+ (0.2/0.2+0.2/0.3)/7 + (0.2/0.1)/8 + (0.2/0.1)/9 (0.2/0.1)/10

$\tilde{L}_2$ = (0.2/0.2)/6 + (0.2/0.1)/7 + (0.2/0.2+ 0.2/0.3)/8 + (0.2/0.1)/9 + (0.2/0.2 + 0.2/0.3)/10 + (0.2/0.1) /11 + (0.3/0.1)/12 + (0.2/0.1)/13

$\tilde{L}_3$ = (0.3/0.2)/6 + (0.3/0.2)/7 + (0.3/0.2 +0.2 /0.3)/8 + (0.3/0.2 + 0.2/0.3)/9 + (0.3/0.2 + 0.2/0.3)/10 + (0.3/0.1)/11 + (0.3/0.1)/12

$\tilde{L}_4$ = (0.5/0.2 + 0.3/0.2)/6 + (0.5/0.2)/7 + 0.4/0.2 +0.3/0.3)/8 + (0.4/0.2 + 0.3/0.4)/9

Step 2 : Initialization

i) Set $\tilde{L}$ = $\tilde{L}_1$

ii) Let i = 2

Step 3 : Computation of Shortest path length

i) Compute $\tilde{L}$ = Min ($\tilde{L}$, $\tilde{L}_i$) using def 2.6.

ii) i = i + 1

iii) If i $\leq$ n goto step 3

iv) The shortest path length is $\tilde{L}$

$\tilde{L}$ = (0.2/0.1)/6 + (0.2/0.1)/7 + (0.2/0.1)/8 + (0.2/0.1)/9

Step 4 : Computation of Similarity Measure

i) Let j =1

ii) Compute $s_j(\tilde{L},\tilde{L}_j)$ using def 2.7

$s_1(\tilde{L},\tilde{L}_1)$ = 0.683325

iii) If j= n assign $s\ (\tilde{L},\tilde{L}_j)$ = $s_j(\tilde{L},\tilde{L}_j)$

$s\ (\tilde{L},\tilde{L}_1)$ = 0.683325

iv) Compute s = Max (s, s_j)

S = 0.683325

v) Put j = j + 1

vi) If j $\leq$ n then go to Step 4(ii)

ii)Compute $s_j(\tilde{L},\tilde{L}_j)$ using def 2.7

$s_2(\tilde{L},\tilde{L}_2)$ = 0.725

iii) Compute s = Max (s, s_j)

S = 0.725

iv) Put j = 2 + 1

v) If 3 $\leq$ 4 then go to Step 4(ii)

ii)Compute $s_j(\tilde{L},\tilde{L}_j)$ using def 2.7

$s_3(\tilde{L},\tilde{L}_3)$ = 0.3333

vi) Compute s = Max (s, s_j)

S = 0.725

vii) Put j = 3 + 1

viii) If 4 $\leq$ 4 then go to Step 4(ii)

ii)Compute $s_j(\tilde{L}, \tilde{L}_j)$ using def 2.7

$s_4(\tilde{L}, \tilde{L}_4)$ = 0.22213

ix) Compute s = Max (s, s_j)

S = 0.725

x) Put j = 4 + 1

xi) If 5 $\leq$ 4 then stop the procedure

s = 0.725

Step 5 : Shortest Path and Shortest Path Length

The highest similarity degree is s and the corresponding path is shortest path and the shortest path length is $\tilde{L}$

S = 0.725

Shortest path is 1 – 2 - 4 - 5 – 6 and

shortest path length is $\tilde{L}$ = (0.2/0.1)/6 + (0.2/0.1)/7 + (0.2/0.1)/8 + (0.2/0.1)/9

CONCLUSION

Type-2 fuzzy number has more flexible in capturing uncertainties in the real world due to the fact that it is described by primary and secondary membership. In this paper, new approach is proposed based on similarity degree, to find the shortest path shortest path length, using discrete type-2 fuzzy number. This approach is not suitable for complement type-2 fuzzy number.

REFERENCES

[1]. V. Anusuya and R, Sathya, " Type-2 fuzzy shortest path", International Journal of fuzzy mathematical Archive, Vol.2, 2013, 36-42.

[2]. S. Elizabeth and L. Sujatha, " Fuzzy shortest path problem based on index ranking", Joural of Mathematics Research, Vol.3, No.4, Nov. 2011, 80 – 88.

[3]. E. A. Jammeh, M. Fleury, C.Wagner, H.Hagras and M.Ghanbari, " Interval type-2 fuzzy logic congestion control for video streaming across IP network", IEEE Trans. Fuzzy Systems, Vol.17, 2009, pp.1123 – 1142.

[4]. J. M. Mendel, F. Liu and D.Zhai, " α – plane representation for type-2 fuzzy sets: theory and applications", IEEE Tras. Fuzzy Systems, Vol. 17, 2009, pp. 1189 – 1207.

[5]. O, Mendoza, P. Melin and G. Licea, " A hybrid approach for image recognition combining type-2 fuzzy logic, modular neural networks and the sugeno integral", Information Sciences, Vol. 179, 2009, pp. 2078 – 2101.

[6]. Miin – Shen Yang, Der – Chen Lin, " On similarity ad Inclusion Measures between type-2 fuzzy sets with ad application to Clustering ", An International Journal of Computers ad Mathematics with Applications, 57, 2009, 896 – 907.

[7]. M. Mizumoto and K. Tanaka, " Some properties of fuzzy sets of type-2", Information and Congtrol, Vol. 31, 1976, pp. 312 – 340.

[8]. Nagoorgani, A. and Anusuya, V. Fuzzy Shortest Path by Linear Multiple Objective Programming, Proceedings of the International Conference On Mathematics and Computer Science.

[9]. C.P. Pappis, N.I. Karacapilidis, " A comparative assessment of measures of similarity of fuzzy values", Fuzzy sets and systems, 56, (1993), 171 – 174.

[10]. Sugeno,M. " Fuzzy measures and Fuzzy integrals- a survey", In Gupta, saridis, and Gaines, 1977, pp.89-102.

[11]. C. Wagner and H. Hagras, " Towards general type-2 fuzzy logic systems based on zSlices", IEEE Trans. Fuzzy systems, Vol. 18, 2010, pp. 637 – 660.

[12]. R.R. Yager, " Fuzzy Subssets of type-II in decisions", J.Cybern., Vol.10, 1980, pp. 137 – 159.

[13]. L.A. Zadeh, " The Concept of a Linguistic Variable and its Application to Approximate Reasoning – 1", Inform. Sci. 8 (1975) 199 – 249.

[14]. L.A. Zadeh, " Similarity relations and Fuzzy ordering", Information sciences, 3 (1971), 177 – 200.