Vol.2.Issue.4.2014

http://www.bomsr.com

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

ISSN:2348-0580

RESEARCH ARTICLE

OBSERVATIONS ON THE HYPERBOLA $y^2 = 34x^2 + 1$

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Article Info:
Article received :11/11/2014
Revised on:20/11/2014
Accepted on:22/11/2014

ABSTRACT

The binary quadratic equation $y^2 = 34x^2 + 1$ is considered and a few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration, a special pythogorean triangle is obtained.

Keywords : binary quadratic, hyperbola, integral solutions, pell equation

2010 Mathematics Subject Classification : 11D09

INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$, where D is a non-square positive integer, has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values [1,2,3,4]. In [5] infinitely many pythogorean triangles in each of which hypotenuse is four times the product of the generators added with unity are obtained by employing the non-integral solutions of binary quadratic equation $y^2 = 3x^2 + 1$. In [6], a special pythogorean triangle is obtained by employing the integral solutions of $y^2 = 10x^2 + 1$. In [7], different patterns of infinitely many pythogorean triangles are obtained by employing the non-trivial solutions of $y^2 = 12x^2 + 1$. In this context one may also refer [8-16]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation $y^2 = 34x^2 + 1$ representing a hyperbola. A few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration, a special pythogorean triangle is obtained.

Notations Used:

$t_{m,n}$ – Polygonal number of rank n with size m

P_n^m -Pyramidal number of rank n with size m

2. Method of Analysis

The binary quadratic equation representing hyperbola under consideration is

$$y^2 = 34x^2 + 1 \quad (1)$$

whose general solution (x_n, y_n) is given by $x_n = \frac{g}{2\sqrt{34}}, y_n = \frac{f}{2}$ where

$f = (35 + 6\sqrt{34})^{n+1} + (35 - 6\sqrt{34})^{n+1}$ and

$g = (35 + 6\sqrt{34})^{n+1} - (35 - 6\sqrt{34})^{n+1}$, n = 0, 1, 2, ….

The recurrence relations satisfied by x and y are given by

$$y_{n+2} - 70y_{n+1} + y_n = 0, y_0 = 35, y_1 = 2449$$

$$x_{n+2} - 70x_{n+1} + x_n = 0, x_0 = 6, x_1 = 420$$

Some numerical examples of x and y satisfying (1) are given in the following table:

N	x_n	y_n
0	6	35
1	420	2449
2	29394	171395
3	2057160	11995201
4	143971806	839492675
5	10075969260	58752492049

From the above table we observe some interesting properties:

1. X_n is always even.
2. Y_n is always odd.
3. $y_{2n} \equiv 0 mod(5)$.
4. $x_n \equiv 0 mod(6)$.

A few interesting properties between the solutions are given below:

1. $70y_{2n+2} - 408x_{2n+2} + 2$ is a perfect square
2. $6(70y_{2n+2} - 408x_{2n+2} + 2)$ is a nasty number
3. $70y_{3n+3} - 408x_{3n+3} + 3(70y_{n+1} - 408x_{n+1})$ is a cubical integer
4. $x_{n+1} + 2y_{3n+2} - 35x_n$ is a cubical integer
5. $2(2y_{3n+2} + 6y_n)y_n$ is a quartic integer
6. $2y_{3n+2} + 6y_n$ is a cubical integer
7. $2y_{3n+2} + 210y_{n+1} - 1224x_{n+1}$ is a cubical integer
8. $(2y_{3n+2} + 210y_{n+1} - 1224x_{n+1})(70y_{n+1} - 408x_{n+1})$ is a quartic integer

9. $2y_{2n+1} + 2 = (70y_{n+1} - 408x_{n+1})^2$
10. $y_{n+1} = 35y_n + 204x_n$
11. $y_{n+2} = 2449y_n + 14280x_n$
12. $x_{n+1} = 35x_n + 6y_n$
13. $x_{n+2} = 2449x_n + 420y_n$
14. $y_{3n+2} + 3y_n = 2y_n(y_{2n+1} + 1)$
15. $(y_{3n+2} + 3y_n)^2 = 16y_n^6$
16. Let $Y = 70y_{n+1} - 408x_{n+1}$ and $X = 35x_{n+1} - 6y_{n+1}$. Then the pair (X, Y) satisfies the hyperbola $Y^2 = 136X^2 + 4$.

3. Remarkable Observations

1. Let α be any non-zero positive integer such that $\alpha_s = \frac{y_s - 1}{2}, s = 0,1,2, \ldots$, it is seen that $408t_{3,\alpha_s}$ is a Nasty Number.

2. Let p, q be the generators of the pythogorean triangle $T(\alpha, \beta, \gamma)$ with $\alpha = 2pq, \beta = p^2 - q^2, \gamma = p^2 + q^2, p > q > 0$. Let $q_s = x_s, p_s = x_s + y_s$. Then T satisfies the following relations.

(i) $\alpha - 17\beta + 16\gamma + 1 = 0.$

(ii) $68\frac{A}{P} - 1 = 18\alpha - \gamma$ where A and P represent the area and perimeter of the pythogorean triangle.

(iii) $\gamma - 4\frac{A}{P} = 18(\gamma - \beta) + 1.$

(iv) $P^2 + 2P(17(\beta - \gamma) - \alpha - 1) = 4A.$

(v) Let $x_n = p - q, p > q > 0$ and let N be a positive integer defined by $N = \frac{y_n - 1}{2}$. Then $17(\gamma - \alpha)$ is four times a triangular number.

n	x_n	p	q	y_n	N	α	γ	17(γ-α) $=4t_{3,N}$
0	6	8 9	2 3	35	17	32 54	68 90	612 $=4t_{3,17}$
1	420	620 450	200 30	2449	1224	248000 27000	424400 203400	2998800 $=4t_{3,1224}$

3. Employing the solutions of (1), the following relations among the special polygonal and pyramidal numbers are observed:

(i) $\left(\frac{P_{y_s}^5}{t_{3,y_s}}\right)^2 = 34\left(\frac{3P_{x_s}^3}{t_{3,x_s+1}}\right)^2 + 1$

(ii) $\left(\frac{6P_{y_s-1}^4}{t_{3,2y_s-2}}\right)^2 = 34\left(\frac{P_{x_s}^5}{t_{3,x_s}}\right)^2 + 1$

(iii) $\left(\frac{P_{y_s}^5}{t_{3,y_s}}\right)^2 = 34\left(\frac{3P_{x_s-2}^3}{t_{3,x_s-2}}\right)^2 + 1$

CONCLUSION

To conclude, one may search for other choices of hyperbolas for patterns of solutions and their corresponding properties.

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