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SOME CHARACTERIZATIONS OF WEAKLY FUZZY g'''-CLOSED SETS

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ABSTRACT

In this paper, we offer a new class of sets called weakly fuzzy g'''-continuous functions in fuzzy topological spaces. It turns out that this class lies between the class of fuzzy continuous function and the class of fuzzy generalized continuous functions and some of their characterizations are investigated.

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Key words and Phrases: Fuzzy Topological space, fuzzy g'''-closed set, fαg-closed set, fgsp-closed set and weakly fuzzy g'''-closed set , fuzzy R-map, fuzzy perfectly continuous, fuzzy gsp-continuous, fuzzy gs-continuous, fuzzy g'''-irresolute , weakly fuzzy g'''-continuous, Wfg'''-open, and Wfg'''-closed.

INTRODUCTION

Quite Recently, Jeyaraman et al. [6] have introduced the concept of fuzzy g'''-closed sets and studied its basic fundamental properties in fuzzy topological spaces. In this paper, we introduce a new class of fuzzy generalized continuous function called weakly fuzzy g'''-continuous function which contains the above mentioned class. Also, we investigate the relationships among related fuzzy generalized continuous functions.

2. PRELIMINARIES

Throughout this paper (X, τ) and (Y, σ) (or X and Y) represent fuzzy topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a fuzzy subset A of a space (X, τ), cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively.

We recall the following definitions which are useful in the sequel.

Definition 2.1:

A fuzzy subset A of a space (X, τ) is called:

(i) fuzzy semi-open set [1] if A ≤ cl(int(A));
(ii) fuzzy α -open set [4] if A ≤ int(cl(int(A)));
(iii) fuzzy semi-preopen set [14] if A ≤ cl(int(cl(A)));
(iv) fuzzy regular open set [1] if A = int(cl(A)).

The complements of the above mentioned open sets are called their respective closed sets.

The fuzzy semi-closure [16] (resp. fuzzy α -closure [8], fuzzy semi-preclosure [14]) of a fuzzy subset A of X, denoted by scl(A) (resp. α cl(A), spcl(A)) is defined to be the intersection of all fuzzy semi-closed (resp. fuzzy α -closed, fuzzy semi-preclosed) sets of (X, τ) containing A. It is known that scl(A) (resp. α cl(A), spcl(A)) is a fuzzy semi-closed (resp. fuzzy α -closed, fuzzy semi-preclosed) set.

Definition 2.2 : A fuzzy subset A of a space (X, τ) is called:

I. a fuzzy generalized closed (briefly fg-closed) set [2] if cl(A) ≤ U whenever A ≤ U and U is fuzzy open in (X, τ). The complement of fuzzy g-closed set is called fuzzy g-open set;
II. a fuzzy semi-generalized closed (briefly fsg-closed) set [3] if scl(A) ≤ U whenever A ≤ U and U is fuzzy semi-open in (X, τ). The complement of fsg-closed set is called fsg-open set;
III. a fuzzy generalized semi-closed (briefly fgs-closed) set [11] if scl(A)≤U whenever A ≤ U and U is fuzzy open in (X, τ). The complement of fgs-closed set is called fgs-open set;
IV. a fuzzy α -generalized closed (briefly f α g-closed) set [12] if α cl(A) ≤ U whenever A ≤ U and U is fuzzy open in (X, τ). The complement of f α g-closed set is called f α g-open set;
V. a fuzzy generalized semi-preclosed (briefly fgsp-closed) set [10] if spcl(A) ≤ U whenever A ≤ U and U is fuzzy open in (X, τ). The complement of fgsp-closed set is called fgsp-open set;
VI. a fuzzy g''' -closed set [6] if cl(A) ≤ U whenever A ≤ U and U is fgs-open in (X, τ). The complement of fuzzy g''' -closed set is called fuzzy g''' -open set.

Remark 2.4: Every fuzzy open set is fgs-open set but not conversely.

Example 2.5: Let X = {a, b} with τ = {0_x, A, 1_x} where A is fuzzy set in X defined by A(a)=1, A(b)=0. Then (X, τ) is a fuzzy topological space. Clearly B defined by B(a)=0.5, B(b)=0 is fgs-open set but not fuzzy open.

Definition 2.6[7] :

A fuzzy subset A of a fuzzy topological space (X, τ) is called a weakly fuzzy g''' -closed (briefly w f g''' -closed) set if cl(int(A)) ≤ U whenever A ≤ U and U is fgs-open in (X, τ).

The complement of weakly fuzzy g''' -closed set is called weakly fuzzy g''' -open set

Corollary 2.7[7]: If a subset A of a fuzzy topological space (X, τ) is both fuzzy open and weakly fuzzy g''' -closed set, then it is both fuzzy regular open and fuzzy regular closed in (X, τ).

Theorem 2.8[7]: Every fuzzy open set is weakly fuzzy g''' -open set in (X, τ).

Theorem 2.9[7]: Every fuzzy g''' -open set is weakly fuzzy g''' -open set but not conversely.

(i) Every fuzzy regular open set is weakly fuzzy g''' -open set but not conversely.

3. WEAKLY FUZZY g''' -CONTINUOUS FUNCTIONS

Definition 3.1:

Let X and Y be fuzzy topological spaces. A function f : X → Y is called

I. fuzzy completely continuous (resp. fuzzy R-map) if f^{-1}(V) is fuzzy regular open in X for each fuzzy open (resp.fuzzy regular open) set V in Y.
II. fuzzy perfectly continuous if f^{-1}(V) is both fuzzy open and fuzzy closed in X for each fuzzy open set V in Y.
III. fuzzy gsp-continuous if f^{-1}(V) is fuzzy gsp-closed in X for each fuzzy closed set V in Y.
IV. fuzzy g''' -continuous if f^{1}(V) is fuzzy g''' -closed in X for every fuzzy closed set V in Y.
V. fuzzy g''' -irresolute if f^{-1}(V) is fuzzy g''' -closed in X for every fuzzy g''' -closed set V in Y.
VI. fuzzy gs-irresolute if f^{-1}(V) is fuzzy gs-open in X for every fuzzy gs-open set V in Y.

Definition 3.2: Let X and Y be fuzzy topological spaces. A function f : X → Y is called weakly fuzzy g'''-continuous (briefly wf g'''-continuous) if $f^{-1}(U)$ is a wf g'''-open set in X, for each fuzzy open set U in Y.

Example 3.3: Let X=Y={a, b} with τ ={0_x, A, 1_x} where A is fuzzy set in X defined by A(a)=1, A(b)=0 and σ ={0_x, B, 1_x} where B is fuzzy set in Y defined by B(a)=0.5, B(b)=0. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f: (X, τ)→ (Y, σ) be the fuzzy identity function. Clearly f is weakly fuzzy g'''- continuous.

Proposition 3.4: Every fuzzy continuous function is wf g'''-continuous.

Proof: It follows from Theorem 2.8.

The converse of Proposition 3.4 need not be true as seen in the following example.

Example 3.5: Let X = Y = {a, b} with τ = {0_x, λ, 1_x} where λ is fuzzy set in X defined by λ(a)=0.6, λ(b)=0.5 and σ ={0_x, β, 1_x} where β is fuzzy set in Y defined by β(a)=0.5, β(b)=0.5. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f: (X, τ)→ (Y, σ) be the fuzzy identity function.. Clearly f is weakly fuzzy g'''-continuous but it is not fuzzy continuous in (X, τ).

Theorem 3.6: Every fuzzy g'''-continuous function is wf g'''-continuous.

Proof: It follows from Proposition 2.9 (i).

The converse of Theorem 3.6 need not be true as seen in the following example.

Example 3.7: Let X = Y = {a, b} with τ = {0_x, λ, 1_x} where λ is fuzzy set in X defined by λ(a)=0.6, λ(b)=0.5 and σ ={0_x, β, 1_x} where β is fuzzy set in Y defined by β(a)=0.5, β(b)=0.5. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f: (X, τ)→ (Y, σ) be the fuzzy identity function.. Clearly f is weakly fuzzy g'''-continuous but it is not fuzzy g'''-continuous in (X, τ).

Theorem 3.8: Every fuzzy completely continuous function is wf g'''-continuous.

Proof: It follows from Proposition 2.9 (ii).

Remark 3.9: The converse of Theorem 3.8 need not be true in general. The function f in the Example 3.7 is wf g'''-continuous but not fuzzy completely continuous.

Theorem 3.10:A function f : X → Y is wf g'''-continuous if and only if $f^{-1}(U)$ is a wf g'''-closed set in X, for each fuzzy closed set U in Y.

Proof: Let U be any fuzzy closed set in Y. According to the assumption

$f^{-1}(U^c) = X \setminus f^{-1}(U)$ is wf g'''-open in X, so $f^{-1}(U)$ is wf g'''-closed in X.

The converse can be proved in a similar manner.

Proposition 3.11: If f : X → Y is fuzzy perfectly continuous and wf g'''-continuous, then it is fuzzy R-map.

Proof: Let V be any fuzzy regular open subset of Y. According to the assumption, $f^{-1}(V)$ is both fuzzy open and fuzzy closed in X. Since $f^{-1}(V)$ is fuzzy closed, it is wf g'''-closed. We have $f^{-1}(V)$ is both fuzzyopen and wf g'''-closed. Hence, by Corollary 2.7, it is fuzzy regular open in X, so f is fuzzy R-map.

4. WEAKLY FUZZY g''' -OPEN FUNCTIONS AND WEAKLY FUZZY g''' -CLOSED FUNCTIONS

Definition 4.1: Let X and Y be fuzzy topological spaces. A function f : X → Y is called weakly fuzzy g''' -open (briefly wf g'''-open) if f(V) is a wf g'''-open set in Y, for each fuzzy open set V in X.

Definition 4.2: Let X and Y be fuzzy topological spaces. A function f : X → Y is called weakly fuzzy g'''-closed (briefly wf g'''-closed) if f(V) is a wf g'''-closed set in Y, for each fuzzy closed set V in X.

It is clear that an fuzzy open function is wf g'''-open and a fuzzy closed function is wf g'''-closed.

Theorem 4.3: Let X and Y be fuzzy topological spaces. A mapping f : X → Y is wf g'''-closed if and only if for each fuzzy subset B of Y and for each fuzzy open set G containing

$f^{-1}(B)$ there exists a wf g'''-open set F of Y such that $B \subseteq F$ and $f^{-1}(F) \subseteq G$.

Proof: Let B be any fuzzy subset of Y and let G be an fuzzy open subset of X such that $f^{-1}(B) \subseteq G$. Then F = Y \ f(X \ G) is wf g''' -open set containing B and $f^{-1}(F) \subseteq G$.

Conversely, let U be any fuzzy closed subset of X. Then $f^{-1}(Y \setminus f(U)) \subseteq X \setminus U$ and X \ U is fuzzy open. According to the assumption, there exists a wf g''' -open set F of Y such that $Y \setminus f(U) \subseteq F$ and $f^{-1}(F) \subseteq X \setminus U$. Then $U \subseteq X \setminus f^{-1}(F)$.

From $Y \setminus F \subseteq f(U) \subseteq f(X \setminus f^{-1}(F)) \subseteq Y \setminus F$ follows that f(U) = Y \ F, so f(U) is wf g''' -closed in Y. Therefore f is a wf g''' -closed function.

Theorem 4.4: If f : X $\to$ Y is fuzzy gs-irresolute and wf g''' -closed function and A is a wf g''' -closed set in X, then f(A) is wf g''' -closed in Y.

Proof: Let $f(A) \subseteq U$, where U is a fuzzy gs-open set of Y. According to the assumption $f^{-1}(U)$ is fuzzy gs-open in X, so $cl(int(A)) \subseteq f^{-1}(U)$. Since f is wf g''' -closed we can conclude that f(cl(int(A))) is wf g''' -closed set contained in the fuzzy gs-open set U, which implies $cl(int(f(cl(int(A))))) \subseteq U$. Therefore $cl(int(f(A))) \subseteq U$. Hence f(A) is wf g''' -closed in Y.

Corollary 4.5: If f : X $\to$ Y is fuzzy gs-irresolute and fuzzy closed function and A is a wf g''' -closed set in X, then f(A) is wf g''' -closed in Y.

Remark 4.6: The composition of two wf g''' -closed functions need not be wf g''' -closed as we can see from the following example.

Example 4.7: Let X = Y = Z = {a, b} with $\tau = \{0_x, \alpha, 1_x\}$ where $\alpha(a)= 0.4$, $\alpha(b)= 0$,
$\sigma = \{0_x, \beta, 1_x\}$ where $\beta(a) =1$, $\beta(b)= 0$ and $\eta = \{0_x, \gamma, \beta, 1_x\}$ where $\gamma(a) = 0.5$,
$\gamma(b) = 0$. Then (X, τ), (Y, σ) and (Z, η) are fuzzy topological spaces.
Let f : (X, τ) $\to$ (Y, σ) be the identity map and g : (Y, σ) $\to$ (Z, η) be the identity fuzzy map. Clearly both f and g are weakly fuzzy g''' -closed maps but their composition
g $\circ$ f : (X, τ) $\to$ (Z, η) is not an weakly fuzzy g''' -closed map.

Theorem 4.8: Let X, Y and Z be fuzzy topological spaces. If f : X $\to$ Y be a fuzzy closed function and g : Y $\to$ Z be a wf g''' -closed function, then g$\circ$f : X $\to$ Z is a wf g''' -closed function.

Definition 4.9:A function f : X $\to$ Y is called a weakly fuzzy g''' -irresolute (briefly wf g''' -irresolute) function if $f^{-1}(U)$ is a wf g''' -open set in X, for each wf g''' -open set U in Y.

Example 4.10: Let X = Y = {a, b} with $\tau = \{0_x, \alpha, 1_x\}$ where $\alpha(a)= 1$, $\alpha(b)= 0$ and
$\sigma = \{0_x, \beta, 1_x\}$ where $\beta(a) =0.5$, $\beta(b)= 0$. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f : (X, τ) $\to$ (Y, σ) be the identity map. Clearly f is a wf g''' -irresolute.

Remark 4.11: The following examples show that fuzzy gs-irresoluteness and wf g''' -irresoluteness are independent.

Example 4.12: Let X = Y = {a, b} with $\tau = \{0_x, \alpha, 1_x\}$ where $\alpha(a) =1$, $\alpha(b)= 0$ and
$\sigma = \{0_x, \beta, 1_x\}$ where $\beta(a) =0.5$, $\beta(b)= 0$. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f : (X, τ) $\to$ (Y, σ) be the identity map. Clearly f is wf g''' -irresolute but not fuzzy gs-irresolute.

Example 4.13: Let X = Y = {a, b} with $\tau = \{0_x, \alpha, 1_x\}$ where $\alpha(a)= 0.5$, $\alpha(b)= 0$ and
$\sigma = \{0_x, \beta, 1_x\}$ where $\beta(a) =1$, $\beta(b)= 0$. Then (X, τ) and (Y, σ) are fuzzy topological spaces. Let f : (X, τ) $\to$ (Y, σ) be the identity map. Clearly f is fuzzy gs-irresolute but not wf g''' -irresolute.

Theorem 4.14: The composition of two wf g''' -irresolute functions is also wf g''' -irresolute.

Theorem 4.15: Let f : X $\to$ Y and g : Y $\to$ Z be fuzzy function such that g $\circ$ f : X $\to$ Z is wf g''' -closed function. Then the following statements hold:

I. if f is fuzzy continuous and injective, then g is wf g''' -closed.
II. if g is wf g''' -irresolute and surjective, then f is wf g''' -closed.

III. Proof:

(i) Let F be a fuzzy closed set of Y. Since $f^{-1}(F)$ is fuzzy closed in X, we can conclude that (g $\circ$ f)($f^{-1}(F)$) is wf g'''-closed in Z. Hence g(F) is wf g'''-closed in Z. Thus g is a wf g'''-closed function.

(ii) It can be proved in a similar manner as (i).

Theorem 4.16: If f : X $\to$ Y is an wf g'''-irresolute function, then it is wf g'''-continuous.

Remark 4.17: The converse of the above need not be true in general. The function f : X $\to$ Y in the Example 5.13 is wf g'''-continuous but not wf g'''-irresolute.

Definition 4.18:A function f : X $\to$ Y is pre-fgs-open if f(U) is fgs-open in Y, for each fgs-open set U in X.

Theorem 4.19: If f : X $\to$ Y is bijective pre-fgs-open and wf g'''-continuous function, then f is wf g'''-irresolute.

Proof: Let F be any wf g'''-closed set in Y and $f^{-1}(F) \subseteq U$, where U is a fgs-open set in X. Then F $\subseteq$ f(U) and cl(int(F)) $\subseteq$ f(U). It follows that f^{-1}(cl(int(F))) $\subseteq$ U. Since f is wf g'''-continuous and cl(int(F)) is fuzzy closed in Y, f^{-1}(cl(int(F))) is wf g'''-closed in X. Since f^{-1}(cl(int(F))) $\subseteq$ U and f^{-1}(cl(int(F))) is wf g'''-closed, we have cl(int(f^{-1}(cl(int(F)))) $\subseteq$ U, so cl(int(f^{-1}(F))) $\subseteq$ U. Therefore f^{-1}(F) is wf g'''-closed and hence f is wf g'''-irresolute.

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