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GRACEFUL LABELING OF EXTENDED KOMODO DRAGON GRAPHS

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ABSTRACT

A labeled graph G which can be gracefully numbered is said to be graceful. Labeling the nodes of G with distinct nonnegative integers and then labeling the e edges of G with the absolute differences between node values, if the graph edge numbers run from 1 to e, the graph G is gracefully numbered. In this paper, we have discussed the gracefulness of some the graphs formed from dragon graphs.

Keywords: Labeling; Graceful graph; Dragon graph

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INTRODUCTION

Labeled graphs form useful models for a wide range of applications. An graceful labeling f of a graph G with q edges is an injective function from the vertices of G to the set {0,1,2,…,q} such that when each edge xy is assigned the label |f(x)-f(y)| , the resulting edge labels are distinct and nonzero. The concept above was put forward by Rosa in 1967. In this paper, some new classes of graphs have been constructed by combining some subdivisions of dragon graphs with the star graphs St(n), (n ≥ 1). Only finite simple undirected graphs are considered here. Our notations and terminology are as in [1]. We refer to [2] for some basic concepts.

2. SOME RESULTS ON GRACEFUL EXTENDED KOMODO DRAGON GRAPHS

Definition 2.1: A dragon graph is formed by attaching a path to the vertex of a cycle.

Definition 2.2: A komodo dragon graph is formed by attaching a path to a three degree vertex of a cycle with a chord and attaching star graphs to the end points of the path.

Definition 2.3: A komodo dragon graph with many tails D(t, b, h, m, n) is formed by attaching many paths of length two to an endpoint of the path in the komodo dragon graph. In this paper we form D(t, b, h, m, n) as follows: Let $a, a_1, a_2, \dots, a_h, a$, where h is odd, be a cycle. Form the chord a, a $_{(k+1)/2}$. Let P:$v_1, v_2, v_3, .., v_b$ be a path. Join the vertex v_1 to the vertex a and rename it as v_1. Let St(m) be a star graph with center u and pendant vertices $u_1, u_2, \dots, u_m$. Let St(n) be a star graph with center w and pendant vertices $w_1, w_2, \dots, w_n$. Attach the vertex u to the vertex v_1 and rename it as v_1 and merge the vertex v_b with the vertex w and rename it as v_b, to form a komodo dragon graph. Let P_{i1}:s_{i1}, s_{i2}, s_{i3} , i = 1,2,…,t be paths of length two. Attach the vertices $s_{11},s_{21},s_{31},…,s_{t1}$ to the vertex v_b and rename the vertex as v_b, to form a komodo dragon graph with t tails D(t,b,h,m,n). It has (m + n + h + b + 2t) vertices and (m + n + h + b + 2t +1) edges.

We investigate the gracefulness of the extended komodo dragon graph , which is a komodo dragon graph with many tails, with b≥ 4. In particular we choose b = 4.

Remark: Let G = D(t,b,h,m,n). The vertex set V(G) = {s_{i2}, s_{i3}, v_j, a_k, u_x,w_y / i = 1,2,…,t; j = 1,2,…,b; k = 1,2,…,h; x = 1,2,…,m and y = 1,2,…,n}. The edge set E(G) ={a_1v_1, a_hv_1, $a_{(h+1)/2}v_1$}∪{a_ka_{k+1} / k = 1,2,…,(h–1)}∪{v_j v_{j+1} / j=1,2,…,(b–1)}∪{$s_{i2}v_b$ / i = 1, 2 ,…,t} ∪{$s_{i2}s_{i3}$ / i = 1,2,…,t}∪{u_xv_1 / x = 1, 2, …,m}∪{w_yv_b / y = 1,2,…, n}.

Let f be the labeling on the set of vertices of G and g be the induced labeling on the set of edges of G. The vertex label set of G can be written as A∪B∪T_1∪T_2∪…∪T_i ∪U∪W, i = 1,2,…, t, where A = {f(a_k) / k = 1,2,…,h}, B = {f(v_j) / j = 1,2,…, b}, T_i = {f(s_{iz}) / z = 1,2}, U = {f(u_x) / x = 1,2,…,m} and U = {f(w_y) / y = 1,2,…, n}.

The edge label set of G can be written as J_1∪J_2∪K∪L_1∪L_2∪M∪N where J_1 = {g(a_1v_1), g(a_hv_1), g($a_{(h+1)/2}v_1$)}, J_2 = {g(a_ka_{k+1}) / k = 1,2,…, (h – 1)}, K = {g(v_j v_{j+1}) / j = 1,2,…,(b – 1)}, L_1 = {g($s_{i2}v_b$) / i = 1, 2,…,t}, L_2 = {g($s_{i2}s_{i3}$) / i = 1,2,…,t}, M = {g(u_xv_1) / x = 1,2,…,m} and N = {g(w_yv_b) / y = 1,2,…,n}. Let m ≥ 1, n ≥ 1.

Definition 2.4 Let A be any graph and B be any tree graph. A ○ B denotes the new graph formed by attaching a center vertex of B to a vertex of A. [3],[4],[5],[6].

Theorem 2.5: D(t, b, h, m, n) is graceful for t = 1, b = 4, h = 3, m, n ≥ 1.

Proof: Consider a komodo dragon graph with 1 tail G = D(t, b, h, m, n), where t = 1, b = 4, h = 3, m, n ≥ 1.G has (m + n + 9) vertices and (m + n + 10) edges.

Let the labeling f on the vertices of G be defined by f(a_k) = (m+n+5+k) for k = 1, f(a_k) = (m + n + 8 + k) for k = 2, f(a_k) = (k – 3) for k = 3, f(v_j) = (m + n + 8 + j) for j = 1, f(v_j) = (m–1+ j) for j=2, f(v_j) = (m+n+5+j) for j=3, f(v_j) = (m–2+j) for j=4, f(s_{i2}) = (m+n +6 + i) for i = 1, f(s_{i3}) = (m + 2 + i) for i = 1, f(u_x) = (x) for 1 ≤ x ≤ m, f(w_y) = (m + 3 + y) for y = 1, f(w_y) = (m + 5 + y) for 2 ≤ y ≤ n.

The induced labeling g on the edges of G is defined by g(a_kv_1) = (k + 2) for k = 1, g(a_kv_1) = (k – 1) for k = 2, g(a_kv_1) = (m+n+6+k) for k = 3, g(a_ka_{k+1}) = (k + 3) for k = 1, g(a_ka_{k+1}) = (m+n+8+k) for k = 2, g(v_jv_{j+1}) = (n+7+j) for j=1, g(v_jv_{j+1}) = (n+5+j) for j = 2, g(v_jv_{j+1}) = (n+3+ j) for j = 3, g($s_{i2}v_4$) = (n+ 4+ i) for i = 1, g($s_{i2}s_{i3}$) = (n + 3 + i) for i = 1, g(u_xv_1) = (m+n+9–x) for 1≤ x ≤ m, g(w_yv_4) = (1+y) for y=1, g(w_yv_4) = (3+y) for 2≤ y ≤ n.

The vertex labels of G can be arranged in the following order. A = {0,(m + n + 6), (m+n+10)}, B ={(m+1), (m+2), (m+n+8), (m+n+9)}, T_1 ={(m+3), (m+n+7)}, U ={1,2, …, m}, W = {(m+4), (m+7),...,(m+n+5)}. The set of vertex labels of G is A∪B∪T_1∪U∪ W = { 0,1,2,…,(m + 4), (m + 7), …, (m + n + 10)}.

The edge labels of G can be arranged in the following order. J_1 = {1,3,(m +n+ 9)}, J_2 = {4,(m + n+ 10)}, K = {(n + 6), (n + 7), (n + 8)}, L_1 = {(n + 5)}, L_2 = {(n + 4)}, M = {(n + 9),…,(m + n + 8) }, N = {2,5, …,(n + 3)}. The set of edge labels of G is J_1 ∪ J_2 ∪ K ∪ L_1 ∪ L_2 ∪ M ∪ N = { 1, 2, 3, … , (m + n + 10)}.

Therefore the set of vertex labels and edge labels are distinct . So f is a graceful labeling. Hence G = D(t, b, h, m, n), is graceful for t = 1, b = 4, h = 3, m, n ≥ 1.

Corollary 2.6: D(t, b, h, m, n) ○ St(r) is graceful for t = 1, b = 4 ,h = 3, m, n, r ≥ 1.

Theorem 2.7: D(t, b, h, m, n) is graceful for t = 2, b = 4, h = 3, m, n ≥ 1.

Proof: Consider a komodo dragon graph with 2 tails G = D(t, b, h, m, n), where t = 2, b = 4, h = 3, m, n ≥ 1. G has (m + n + 11) vertices and (m + n + 12) edges. .

Let the labeling f on the vertices of G be defined by f(a_k) = (m+n+7+k) for k = 1, f(a_k) = (m+n+10+k) for k = 2, f(a_k) = (k – 3) for k = 3, f(v_j) = (m + n+10+ j) for j = 1, f(v_j) = (m – 1 + j) for j = 2, f(v_j) = (m + n + 7 + j) for j = 3, f(v_j)= (m – 2 + j) for j = 4, f(s_{i2}) = (m + 6 + i) for i = 1, f(s_{i3}) = (m + 4 + i) for i = 1, f(s_{i2}) = (m + n+7+i) for i = 2, f(s_{i3}) = (m + 1 + i) for i = 2, f(u_x) = (x) for 1 ≤ x ≤ m, f(w_y) = (m + 7 + y) for 1 ≤ y ≤ n.

The induced labeling g on the edges of G is defined by g(a_kv_1) = (k + 2) for k = 1, g(a_kv_1) = (k –1) for k = 2, g(a_kv_1) = (m + n + 8 + k) for k = 3,g(a_ka_{k+1}) = (k + 3) for k= 1, g(a_ka_{k+1}) = (m+n+10+k) for k= 2, g(v_jv_{j+1}) = (n+9+j) for j=1, g(v_jv_{j+1}) = (n+7+j) for j =2, g(v_jv_{j+1}) = (n + 5 + j) for j = 3, g($s_{i2}v_4$) = (4 + i) for i = 1, g($s_{i2}s_{i3}$) = (1+ i) for i = 1,

$g(s_{i2}v_4) = (n + 5 + i)$ for $i = 2$, $g(s_{i2}s_{i3}) = (n + 4 + i)$ for $i = 2$, $g(u_xv_1) = (m + n + 11 – x)$ for $1 \le x \le m$, $g(w_yv_4) = (5 + y)$ for $1 \le y \le n$.

The vertex labels of G can be arranged in the following order. A ={0, (m + n + 8), (m + n + 12)}, B = {(m + 1), (m + 2), (m + n + 10), (m + n + 11)}, T_1= {(m+5), (m+7)}, T_2 = {(m+3), (m+n+9)}, U = {1,2,…,m}, W = {(m+8),...,(m+n+7)}. The set of vertex labels of G is $A \cup B \cup T_1 \cup T_2 \cup U \cup W$ = {0,1,2,…,(m + 3), (m + 5), (m+7), …, (m+n+12)}.

The edge labels of G can be arranged in the following order. J_1 ={1,3,(m+n+ 11)}, J_2 = {4, (m+n+12)}, K = {(n+8), (n+9), (n+10)}, L_1 = {5, (n+7)}, L_2 = {2, (n+6)}, M = {(n + 11),…,(m + n + 10)}, N = {6,7, …,(n + 5)}. The set of edge labels of G is $J_1 \cup J_2 \cup K \cup L_1 \cup L_2 \cup M \cup N$ = {1, 2, 3, … , (m + n + 12)}.

Therefore the set of vertex labels and edge labels are distinct. So f is a graceful labeling. Hence G = D(t, b, h, m, n), is graceful for t = 2, b = 4, h = 3, m, n ≥ 1.

Corollary 2.8: D(t, b, h, m, n) ○ St(r) is graceful for t = 2, b = 4, h = 3, m, n, r ≥ 1

Theorem 2.9: D(t, b, h, m, n) is graceful for t = 3, b = 4, h = 3, m, n ≥ 1.

Proof: Consider a komodo dragon graph with 3 tails G = D(t, b, h, m, n), where t = 3, b = 4, h = 3, m, n ≥ 1.G has (m + n + 13) vertices and (m + n + 14) edges. .

Let the labeling f on the vertices of G be defined by $f(a_k) = (m+n+9+k)$ for k =1, $f(a_k) = (m + n +12+k)$ for k = 2, $f(a_k) = (k – 3)$ for k = 3, $f(v_j) = (m+n+12+j)$ for j = 1, $f(v_j) = (m – 1+ j)$ for j = 2, $f(v_j) = (m+n+9+j)$ for j = 3, $f(v_j) = (m–2+j)$ for j = 4, $f(s_{i2}) = (m+n+10+i)$ for i = 1, $f(s_{i3}) = (m+2+i)$ for i = 1, $f(s_{i2}) = (m+8+i)$ for i = 2, $f(s_{i3}) = (m + 3 + i)$ for i = 2, $f(s_{i2}) = (m + 6 + i)$ for i = 3, $f(s_{i3}) = (m + 4 + i)$ for i = 3, $f(u_x) = (x)$ for $1 \le x \le m$, $f(w_y) = (m + 7 + y)$ for y = 1, $f(w_y) = (m + 9 + y)$ for $2 \le y \le n$.

The induced labeling g on the edges of G is defined by $g(a_kv_1) = (k + 2)$ for k = 1, $g(a_kv_1) = (k – 1)$ for k = 2, $g(a_kv_1) = (m+n+10+k)$ for k = 3, $g(a_ka_{k+1}) = (k + 3)$ for k = 1, $g(a_ka_{k+1}) = (m+n+12+k)$ for k = 2, $g(v_jv_{j+1})= (n+11+j)$ for j=1, $g(v_jv_{j+1})= (n+9+j)$ for j=2, $g(v_jv_{j+1}) = (n + 7 + j)$ for j = 3, $g(s_{i2}v_4) = (n + 8 + i)$ for i = 1, $g(s_{i2}s_{i3}) = (n +7+ i)$ for i = 1, $g(s_{i2}v_4) = (6+i)$ for i = 2, $g(s_{i2}s_{i3}) = (3 + i)$ for i = 2, $g(s_{i2}v_4) = (4 + i)$ for i = 3, $g(s_{i2}s_{i3}) = (i – 1)$ for i = 3, $g(u_xv_1) = (m+n+13–x)$ for $1\le x \le m$, $g(w_yv_4) = (5+ y)$ for y = 1, $g(w_yv_4) = (7 + y)$ for $2 \le y \le n$.

The vertex labels of G can be arranged in the following order. A = {0,(m+ n+ 10), (m+n+14)}, B = {(m + 1), (m + 2), (m + n + 12), (m+n+13)}, T_1 = {(m + 3), (m+n +11)}, T_2 = {(m+5), (m+10)}, T_3 = {(m+7), (m+9)}, U = {1,2,…,m }, W = {(m+8), (m+11),..., (m+n+9)}. The set of vertex labels of G is $A \cup B \cup T_1 \cup T_2 \cup T_3 \cup U \cup W$ = {0,1,2,…, (m+3), (m + 5), (m + 7), …, (m + n + 14)}.

The edge labels of G can be arranged in the following order. J_1 ={1,3,(m +n+13)}, J_2 = {4, (m+n+14)}, K = {(n+10), (n+11), (n+12)}, L_1 = {7, 8, (n+9)}, L_2 = {2,5,(n+8)}, M = {(n + 13),…, (m+ n + 12)}, N = {6,9, …, (n + 7)}. The set of edge labels of Gis $J_1 \cup J_2 \cup K \cup L_1 \cup L_2 \cup M \cup N$ = {1,2,3,… ,(m+n+14)}.

Therefore the set of vertex labels and edge labels are distinct. So f is a graceful labeling. Hence G = D(t, b, h, m, n), is graceful for t = 3, b = 4, h = 3, m, n ≥ 1.

Corollary 2.10: D(t, b, h, m, n) ○ St(r) is graceful for t = 3, b = 4, h = 3, m, n, r ≥ 1.

Theorem 2.11: D(t, b, h, m, n) is graceful for t = 4, b = 4, h = 3, m, n ≥ 1.

Proof: Consider a komodo dragon graph with 4 tails G = D(t, b, h, m, n), where t = 4, b = 4, h = 3, m, n ≥ 1. G has (m + n + 15) vertices and (m + n + 16) edges.

Let the labeling f on the vertices of G be defined by $f(a_k) = (m+n+11+k)$ for k = 1, $f(a_k) = (m + n + 14+k)$ for k = 2, $f(a_k) = (k – 3)$ for k = 3, $f(v_j) = (m + n + 14+ j)$ for j = 1, $f(v_j) = (m – 1 + j)$ for j = 2, $f(v_j) = (m + n + 11 + j)$ for j = 3, $f(v_j) = (m–2+j)$ for j = 4, $f(s_{i2}) = (m +n+12+ i)$ for i = 1, $f(s_{i3}) = (m + 2 + i)$ for i = 1, $f(s_{i2}) = (m + n+9+ i)$ for i = 2, $f(s_{i3}) = (m + 3 + i)$ for i = 2, $f(s_{i2}) = (m +n+7 + i)$ for i = 3, $f(s_{i3}) = (m +n+ 5+ i)$ for i = 3, $f(s_{i2}) = (m + n + 5 + i)$ for i = 4, $f(s_{i3}) = (m + i)$ for i = 4, $f(u_x) = (x)$ for $1 \le x \le m$, $f(w_y) = (m + 6 + y)$ for $1 \le y \le n$.

The induced labeling g on the edges of G is defined by $g(a_kv_1) = (k + 2)$ for k = 1, $g(a_kv_1) = (k – 1)$ for k = 2, $g(a_kv_1) = (m + n + 12+k)$ for k = 3, $g(a_ka_{k+1}) = (k +3)$ for k =1, $g(a_ka_{k+1}) =(m+n+14+k)$ for k=2, $g(v_jv_{j+1}) = (n+13+j)$ for j=1, $g(v_jv_{j+1}) =(n+11+j)$ for j=2, $g(v_jv_{j+1}) = (n+9+j)$ for j=3, $g(s_{i2}v_4) = (n+10+i)$ for i=1, $g(s_{i2}s_{i3}) = (n+9+i)$ for i =1, $g(s_{i2}v_4) = (n + 7 + i)$ for i = 2, $g(s_{i2}s_{i3}) = (n + 4 + i)$ for i = 2, $g(s_{i2}v_4) = (n + 5 + i)$ for i = 3, $g(s_{i2}s_{i3}) = (i – 1)$ for i = 3,

$g(s_{i2}v_4) = (n + 3 + i)$ for $i = 4$, $g(s_{i2}s_{i3}) = (n + 1 + i)$ for $i = 4$, $g(u_xv_1) = (m + n + 15 – x)$ for $1 \le x \le m$, $g(w_yv_4) = (4 + y)$ for $1 \le y \le n$.

The vertex labels of G can be arranged in the following order. A = {0, (m+n+12), (m + n + 16)}, B = {(m + 1), (m + 2), (m + n + 14), (m + n + 15)}, T_1 = {(m + 3), (m + n + 13)}, T_2 = {(m + 5), (m + n + 11)}, T_3 = {(m + n + 8), (m + n + 10)}, T_4 = {(m + 4), (m + n + 9)}, U = {1,2,…,m }, W = {(m + 7), ...,(m + n + 6)}. The set of vertex labels of G is $A \cup B \cup T_1 \cup T_2 \cup T_3 \cup T_4 \cup U \cup W$ = {0, 1, 2,…,(m + 5), (m + 7),…, (m + n + 6), (m + n + 8), …, (m + n + 16)}.

The edge labels of G can be arranged in the following order. J_1 ={1,3, (m+n+15)}, J_2 = {4, (m + n + 16)}, K = {(n + 12), (n + 13), (n + 14)}, L_1 = {(n + 7), (n + 8), (n + 9), (n + 11)}, L_2 = {2, (n + 5), (n + 6), (n + 10)}, M = {(n + 15),…,(m + n + 14)}, N = {5,6, …, (n + 4)}. The set of edge labels of G is $J_1 \cup J_2 \cup K \cup L_1 \cup L_2 \cup M \cup N$ = {1,2,3, … , (m + n + 16)}.

Therefore the set of vertex labels and edge labels are distinct. So f is a graceful labeling. Hence G = D(t, b, h, m, n), is graceful for t = 4, b = 4, h = 3, m, n ≥ 1.

Corollary 2.12: D(t, b, h, m, n) ○ St(r) is graceful for t = 4, b = 4, h = 3, m, n, r ≥ 1.

Theorem 2.13: D(t, b, h, m, n) is graceful for t = 5, b = 4, h = 3, m, n ≥ 1.

Proof: Consider a komodo dragon graph with five tails G = D(t, b, h, m, n), where t = 5, b = 4, h = 3, m, n ≥ 1. G has (m + n + 17) vertices and (m + n + 18) edges.

Let the labeling f on the vertices of G be defined by $f(a_k) = (m+n+13+k)$ for k = 1, $f(a_k) = (m + n+16 + k)$ for k = 2, $f(a_k) = (k – 3)$ for k = 3, $f(v_j) = (m + n + 16 + j)$ for j = 1, $f(v_j) = (m – 1 + j)$ for j = 2, $f(v_j) = (m + n + 13 + j)$ for j = 3, $f(v_j) = (m – 2 + j)$ for j = 4, $f(s_{i2}) = (m +n+14+ i)$ for i = 1, $f(s_{i3}) = (m+2+ i)$ for i = 1, $f(s_{i2}) = (m +n +11+ i)$ for i = 2, $f(s_{i3}) = (m + 4 + i)$ for i = 2, $f(s_{i2}) = (m + n + 9 + i)$ for i = 3, $f(s_{i3}) = (m + 1 + i)$ for i = 3, $f(s_{i2}) = (m+n+7 + i)$ for i = 4, $f(s_{i3}) = (m+1+ i)$ for i = 4, $f(s_{i2}) = (m + n + 2 + i)$ for i = 5, $f(s_{i3}) = (m +n+4+ i)$ for i = 5, $f(u_x) = (x)$ for $1 \le x \le m$, $f(w_y) = (m+6 + y)$ for $1\le y \le n$.

The induced labeling g on the edges of G is defined by $g(a_kv_1) = (k + 2)$ for k = 1, $g(a_kv_1)= (k – 1)$ for k = 2, $g(a_kv_1) = (m +n+14+ k)$ for k = 3, $g(a_ka_{k+1}) = (k + 3)$ for k = 1, $g(a_ka_{k+1}) = (m+n+16+k)$ for k = 2, $g(v_jv_{j+1}) = (n+15+j)$ for j = 1, $g(v_jv_{j+1}) = (n+13+j)$ for j = 2, $g(v_jv_{j+1}) = (n+11+j)$ for j=3, $g(s_{i2}v_4) = (n+12+i)$ for i=1, $g(s_{i2}s_{i3}) =(n+11+i)$ for i =1, $g(s_{i2}v_4) = (n + 9 + i)$ for i = 2, $g(s_{i2}s_{i3}) = (n +5 + i)$ for i = 2, $g(s_{i2}v_4) = (n + 7+ i)$ for i = 3, $g(s_{i2}s_{i3}) = (n + 5 + i)$ for i = 3, $g(s_{i2}v_4) = (n + 5 + i)$ for i = 4, $g(s_{i2}s_{i3}) = (n + 2+ i)$ for i = 4, $g(s_{i2}v_4) = (n + i)$ for i = 5, $g(s_{i2}s_{i3}) = (i – 3)$ for i = 5, $g(u_xv_1) = (m+n+17– x)$ for $1 \le x \le m$, $g(w_yv_4) = (4 + y)$ for $1 \le y \le n$.

The vertex labels of G can be arranged in the following order. A = {0,(m+n + 14), (m + n + 18) }, B = {(m + 1), (m + 2), (m + n + 16), (m + n + 17)}, T_1 = {(m + 3), (m + n + 15)}, T_2 = {(m + 6), (m + n +13)}, T_3 = {(m + 4), (m + n + 12)}, T_4 = {(m + 5), (m + n + 11}, T_5 = {(m + n + 7), (m +n+ 9)}, U = {1,2,…,m}, W = {(m+7),...,(m+n +6)}. The set of vertex labels of G is $A \cup B \cup T_1 \cup T_2 \cup T_3 \cup T_4 \cup T_5 \cup U \cup W$ = {0,1,2,…,(m+n+7), (m + n + 9), (m + n + 11), …, (m + n + 18)}.

The edge labels of G can be arranged in the following order. J_1 = {1,3,(m+n+17)}, J_2 = {4, (m + n + 18)}, K = {(n + 14), (n + 15), (n + 16)}, L_1 = {(n + 5), (n + 9), (n + 10), (n + 11), (n + 13)}, L_2 = {2, (n + 6), (n + 7), (n + 8), (n + 12)}, M = {(n + 17),…, (m + n + 16)}, N = {5,6,…,(n + 4)}. The set of edge labels of G is $J_1 \cup J_2 \cup K \cup L_1 \cup L_2 \cup M \cup N$ = { 1, 2, 3, … , (m + n + 18)}.

Therefore the set of vertex labels and edge labels are distinct. So f is a graceful labeling. Hence G = D(t, b, h, m, n), is graceful for t = 5, b = 4, h = 3, m, n ≥ 1.

Corollary 2.14: D(t, b, h, m, n) ○ St(r) is graceful for t = 5, b = 4, h = 3, m, n, r ≥ 1.

Theorem 2.15: D(t,b,h,m,n) is graceful for t = 6, b = 4, h = 3, m, n ≥ 1.

Proof: Consider a komodo dragon graph with six tails G = D(t, b, h, m, n), where t = 6, b = 4, h = 3, m, n ≥ 1. G has (m + n + 19) vertices and (m + n + 20) edges. .

Let the labeling f on the vertices of G be defined by $f(a_k) = (m+n+15+k)$ for k = 1, $f(a_k) = (m + n+18 + k)$ for k = 2, $f(a_k) = (k – 3)$ for k = 3, $f(v_j) = (m + n + 18 + j)$ for j = 1, $f(v_j) = (m – 1 + j)$ for j = 2, $f(v_j) = (m + n + 15 + j)$ for j = 3, $f(v_j) = (m – 2 + j)$ for j = 4, $f(s_{i2}) = (m +n+16+ i)$ for i = 1, $f(s_{i3}) = (m +2+ i)$ for i = 1, $f(s_{i2}) = (m + n +13+ i)$ for i = 2, $f(s_{i3}) = (m+5+ i)$ for i = 2, $f(s_{i2}) = (m +n+11+ i)$ for i = 3, $f(s_{i3}) = (m +n+6+ i)$ for i = 3, $f(s_{i2}) = (m + n + 9 +$

i) for i = 4, $f(s_{i3})$ = (m + i) for i = 4, $f(s_{i2})$ = (m + n + 7 + i) for i = 5, $f(s_{i3})$ = (m + i) for i = 5, $f(s_{i2})$ = (m + n + 2 + i) for i = 6, $f(s_{i3})$ = (m + n + 4 + i) for i = 6, $f(u_x)$ = (x) for 1 ≤ x ≤ m, $f(w_y)$ = (m + 7 + y) for 1 ≤ y ≤ n.

The induced labeling g on the edges of G is defined by $g(a_kv_1)$ = (k + 2) for k = 1, $g(a_kv_1)$ = (k – 1) for k = 2, $g(a_kv_1)$ = (m+n+16 + k) for k = 3, $g(a_ka_{k+1})$ = (k + 3) for k = 1, $g(a_ka_{k+1})$ =(m+n+18+k) for k=2, $g(v_jv_{j+1})$ =(n+17+j) for j=1, $g(v_jv_{j+1})$ =(n+15+j) for j=2, $g(v_jv_{j+1})$ = (n+13+ j) for j = 3, $g(s_{i2}v_4)$ = (n+14+ i) for i = 1, $g(s_{i2}s_{i3})$ = (n+13+i) for i = 1, $g(s_{i2}v_4)$ = (n +11+ i) for i = 2, $g(s_{i2}s_{i3})$ = (n +6+ i) for i = 2, $g(s_{i2}v_4)$ = (n + 9 + i) for i = 3, $g(s_{i2}s_{i3})$ = (2+ i) for i = 3, $g(s_{i2}v_4)$ = (n + 7 + i) for i = 4, $g(s_{i2}s_{i3})$ = (n + 5 + i) for i = 4, $g(s_{i2}v_4)$ = (n + 5 + i) for i = 5, $g(s_{i2}s_{i3})$ = (n + 2 + i) for i = 5, $g(s_{i2}v_4)$ = (n + i) for i = 6, $g(s_{i2}s_{i3})$ = (i–4) for i=6, $g(u_xv_1)$ = (m+n+19–x) for 1≤ x ≤ m, $g(w_yv_4)$ = (5+y) for 1≤ y ≤ n.

The vertex labels of G can be arranged in the following order. A = {0,(m+n +16), (m+n+20)}, B = {(m + 1), (m + 2), (m + n + 18), (m +n +19)}, T_1 = {(m+3), (m+n+17)}, T_2 = {(m + 7), (m + n + 15)}, T_3 = {(m + n + 9), (m + n + 14)}, T_4 = {(m+4), (m+n+13)}, T_5 = {(m+5), (m+n+12)}, T_6 = {(m+n+8), (m+n+10)}, U = {1,2,…,m}, W = {(m+8), ..., (m + n + 7)}. The set of vertex labels of G is $A \cup B \cup T_1 \cup T_2 \cup T_3 \cup T_4 \cup T_5 \cup T_6 \cup U \cup W$ = 0,1,2,…,(m + 5), (m +7), …, (m + n + 10), (m + n + 12), …, (m + n + 20)}.

The edge labels of G can be arranged in the following order. J_1 ={1,3, (m+n+19)}, J_2 = {4, (m + n + 20)}, K = {(n + 16), (n + 17), (n + 18)}, L_1 = {(n + 5), (n + 6), (n + 10), (n + 11), (n + 12), (n + 13)}, L_2 = {2,5,(n + 7), (n + 8), (n + 9), (n + 14)}, M = {(n + 19),…,(m + n + 18) }, N = {6,7, …,(n + 5)}. The set of edge labels of G is $J_1 \cup J_2 \cup K \cup L_1 \cup L_2 \cup M \cup N$ = {1, 2, 3, … , (m + n + 20)}.

Therefore the set of vertex labels and edge labels are distinct. So f is a graceful labeling. Hence G = D(t, b, h, m, n), is graceful for t = 6, b = 4, h = 3, m, n ≥ 1.

Corollary 2.16: D(t, b, h, m, n) ○ St(r) is graceful for t = 6, b = 4, h = 3, m, n, r ≥ 1

Theorem 2.17: D(t, b, h, m, n) is graceful for t = 7, b = 4,h = 3, m, n ≥ 1

Proof: Consider a komodo dragon graph with seven tails G = D(t, b, h, m, n), where t = 7, b = 4, h = 3, m, n ≥ 1. G has (m + n + 21) vertices and (m + n + 22) edges.

Let the labeling f on the vertices of G be defined by $f(a_k)$ = (m+n+17+k) for k = 1, $f(a_k)$ = (m +n+ 20 + k) for k = 2, $f(a_k)$ = (k – 3) for k = 3, $f(v_j)$ = (m + n + 20 + j) for j = 1, $f(v_j)$ = (m – 1 + j) for j = 2, $f(v_j)$ = (m + n + 17 + j) for j = 3, $f(v_j)$ = (m – 2 + j) for j = 4, $f(s_{i2})$ = (m+n+18+ i) for i = 1, $f(s_{i3})$ = (m+2+ i) for i = 1, $f(s_{i2})$ = (m + n + 15 + i) for i = 2, $f(s_{i3})$ = (m+n+10+ i) for i = 2, $f(s_{i2})$ = (m+n+13+ i) for i = 3, $f(s_{i3})$ = (m+n+7+ i) for i = 3, $f(s_{i2})$ = (m+n+11+ i) for i = 4, $f(s_{i3})$ = (m+1 + i) for i = 4, $f(s_{i2})$ = (m + n + 9 + i) for i = 5, $f(s_{i3})$ = (m + 1 + i) for i = 5, $f(s_{i2})$ = (m + n + 7 + i) for i = 6, $f(s_{i3})$ = (m – 2 + i) for i = 6, $f(s_{i2})$ = (m+n+2+ i) for i = 7, $f(s_{i3})$ = (m + n + 4 + i) for i = 7, $f(u_x)$ = (x) for 1 ≤ x ≤ m, $f(w_y)$ = (m + 8 + y) for 1 ≤ y ≤ n.

The induced labeling g on the edges of G is defined by $g(a_kv_1)$ = (k + 2) for k = 1, $g(a_kv_1)$ = (k – 1) for k = 2, $g(a_kv_1)$ = (m+n+18+k) for k = 3, $g(a_ka_{k+1})$ = (k + 3) for k = 1, $g(a_ka_{k+1})$ =(m+n+20+k) for k=2, $g(v_jv_{j+1})$ =(n+19+j) for j=1, $g(v_jv_{j+1})$ = (n+17+j) for j=2, $g(v_jv_{j+1})$ = (n+15+ j) for j = 3, $g(s_{i2}v_4)$ = (n+16+ i) for i = 1, $g(s_{i2}s_{i3})$ = (n+15+i) for i = 1, $g(s_{i2}v_4)$ = (n + 13 + i) for i = 2, $g(s_{i2}s_{i3})$ = (3 + i) for i = 2, $g(s_{i2}v_4)$ = (n + 11+ i) for i = 3, $g(s_{i2}s_{i3})$ = (3 + i) for i = 3, $g(s_{i2}v_4)$ = (n + 9 + i) for i = 4, $g(s_{i2}s_{i3})$ = (n + 6 + i) for i = 4, $g(s_{i2}v_4)$ = (n +7 + i) for i = 5, $g(s_{i2}s_{i3})$ = (n + 3 + i) for i = 5, $g(s_{i2}v_4)$ = (n + 5 + i) for i = 6, $g(s_{i2}s_{i3})$ = (n + 3 + i) for i = 6, $g(s_{i2}v_4)$ = (n + i) for i = 7, $g(s_{i2}s_{i3})$ = (i – 5) for i = 7, $g(u_xv_1)$ = (m + n + 21 – x) for 1 ≤ x ≤ m, $g(w_yv_4)$ = (6 + y) for 1 ≤ y ≤ n.

The vertex labels of G can be arranged in the following order. A = {0, (m+n+18), (m + n + 22)}, B = {(m + 1), (m + 2), (m + n + 20), (m + n + 21)}, T_1 = {(m + 3), (m + n + 19)}, T_2 = {(m + n + 12), (m + n + 17)}, T_3 = {(m + n +10), (m + n + 16)}, T_4 = {(m + 5), (m + n + 15)}, T_5 = {(m + 6), (m + n + 14)}, T_6 = {(m + 4), (m + n + 13)}, T_7 = {(m + n + 9), (m + n + 11)}, U = {1,2,…,m}, W = {(m + 9), ...,(m + n + 8)}. The set of vertex labels of G is $A \cup B \cup T_1 \cup T_2 \cup T_3 \cup T_4 \cup T_5 \cup T_6 \cup T_7 \cup U \cup W$ = {0,1,2,…, (m + 6), (m + 9), …, (m + n + 22)}.

The edge labels of G can be arranged in the following order. J_1 ={1,3, (m+n+21)}, J_2 = {4, (m + n+ 22)}, K = {(n + 18), (n + 19), (n + 20)}, L_1 = {(n + 7), (n + 11), (n + 12), (n + 13), (n + 14), (n + 15), (n + 17)}, L_2 = {2, 5, 6, (n + 8), (n + 9) (n + 10) (n + 16)}, M = {(n + 21),…,(m + n + 20)}, N = {7, …, (n + 6)}. The set of edge labels of G is $J_1 \cup J_2 \cup K \cup L_1 \cup L_2 \cup M \cup N$ = {1, 2, 3, … , (m + n + 22)}.

Therefore the set of vertex labels and edge labels are distinct. So f is a graceful labeling. Hence G = D(t, b, h, m, n), is graceful for t = 7, b = 4, h = 3, m, n ≥ 1.

Corollary 2.18: D(t, b, h, m, n) ○ St(r) is graceful for t = 7, b = 4, h = 3, m, n, r ≥ 1.

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