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RESEARCH ARTICLE

GRACEFUL LABELING OF EXTENDED SQUID GRAPHS

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ABSTRACT

A labeled graph G which can be gracefully numbered is said to be graceful. Labeling the nodes of G with distinct nonnegative integers and then labeling the e edges of G with the absolute differences between node values, if the graph edge numbers run from 1 to e, the graph G is gracefully numbered. In this paper, we have discussed the gracefulness of some the graphs formed from dragon graphs.

Keywords: Labeling; Graceful graph; Dragon graph

AMS Mathematics Subject Classification (2010): 05C78

INTRODUCTION

Labeled graphs form useful models for a wide range of applications such as coding theory, X-ray crystallography, radar, astronomy, circuit design, communication network addressing and database management. An graceful labeling f of a graph G with q edges is an injective function from the vertices of G to the set {0,1,2,…,q} such that when each edge xy is assigned the label $|f(x)-f(y)|$, the resulting edge labels are distinct and nonzero. The concept above was put forward by Rosa in 1967. Graphs consisting of any number of pairwise disjoint paths with common end vertices are called generalized theta graphs. Various labelings have been found for these graphs.

In this paper, some new classes of graphs have been constructed by combining some subdivisions of theta graphs with the star graphs St(n), (n ≥ 1). Only finite simple undirected graphs are considered here. Our notations and terminology are as in [1]. We refer to [2] for some basic concepts.

SOME RESULTS ON THE GRACEFULNESS OF EXTENDED SQUID GRAPHS

Definition 2.1 The theta graph θ (2, 2, … ,2) consists of n edge disjoint paths of lengths two having the same end points. Let the theta graph θ (2, 2,...,2) have the paths P_1: a,v_1,b; P_2: a, v_2, b; P_3: a, v_3, b;…; P_n: a, v_n, b;.

Definition 2.2 Attach the center of a star graph St(l) to the end point ‘a’ of the theta graph θ (2, 2,...,2) to form the squid graph Sq(l, n).This graph has (2 +n + l) vertices and (2n + l) edges.

Definition 2.3 Attach the center of a star graph St(m) to the end point 'b' of the theta graph θ (2, 2,...,2) in the squid graph Sq(l, n) to form the extended squid graph Sqt(l,n,m). This graph has (2+n+l+m) vertices and (2n+l+m) edges.

Definition 2.4 Let A be any graph and B be any tree graph. $A_i \circ B$ denotes the new graph formed by attaching a center vertex of B to a vertex v_i of A.

Theorem 2.5 Sqt(l, n, m) is graceful, for l = (n – 1), odd m, odd n and l, n ≥ 1.

Proof. Consider the theta graph θ (2,2,…,2) with end vertices a and b and inner vertices $v_1,v_2,…,v_n$. Let St(l) be a star graph with (l + 1) vertices $h,h_1,h_2,…,h_l$ for $l \geq 1$, where h is the center vertex and $h_1,h_2,…,h_l$ are pendant vertices. Let St(m) be a star graph with (m +1) vertices $t,t_1,t_2,…,t_s$ for s ≥ 1,where t is the center vertex and $t_1,t_2,…,t_s$ are pendant vertices. To form the graph G = Sqt (l,n,m) attach the center vertex h of St(l) with the end vertex 'a' of θ (2,2,…,2) and join the center vertex t of St(m) with the end vertex 'b' of θ (2,2,…,2). G has (n + 2 + l + m) vertices and (2n + l + m) edges.

The vertex set V(G) = {a, v_i, b, h_j, t_s / i = 1,2,…,n, j = 1,2,…, l and t = 1,2,…,m}. The edge set E(G) ={av_i / i = 1,2,…,n} ∪ {v_i b / i = 1,2,…,n}∪{h_ja / j = 1,2,…, l} ∪{t_sb / s = 1,2,…,m}.

Let f be the labeling on the set of vertices of G and g be the induced labeling on the set of edges of G.

The vertex label set of G can be written as A∪B∪H∪T whereA = {f(a), f(b)}, B = {f(v_i) / i = 1,2,…, n}, H = {f(h_j) / j = 1,2,…, l}, T = {f(t_s) / s = 1,2,…, m}. The edge label set of G can be written as N_1∪N_2∪L∪O where N_1 = {g(av_i) / i = 1,2,…,n}, N_2 = {g(v_ib) / i = 1,2,…, n}, L={g(h_ia) / j = 1,2,…, l}, O = {g(t_sb) / s = 1,2,…, m}.

Let l = (n – 1). Let m and n be odd. Let n = (2p + 1), m = (2q + 1) where p and q are positive integers and p > 0, q > 0. Therefore l = (2p).

Let the labeling f on the vertices of G be defined by f(v_i) = (i–1) for 1≤ i ≤ (2p+1), f(h_j) = (4p + 2q + j) for 1 ≤ j ≤ (2p), f(t_s) = (2p + s) for 1≤ s ≤ (2q + 1), f(a) = (6p+2q+3), f(b) = (4p + 2q + 2).

The induced labeling g on the edges of G is defined by g(av_i) = (6p + 2q + 4 – i) for 1 ≤ i ≤ (2p + 1), g(v_i b) = (4p + 2q + 3 – i) for 1≤ i ≤ (2p + 1), g(h_j a) = (2p + 1 – j) for 1 ≤ j ≤ 2p, g(t_sb) = (2p + 2q + 2 – s) for 1 ≤ s ≤ (2q + 1).

The vertex labels of G can be arranged in the following order. A = {(4p + 2q + 2), (6p + 2q + 3)}, B = {0,1,…,(2p)}, H = {(4p + 2q + 3),…, (6p + 2q + 2)}, T = {(2p + 1), (2p + 2),…, (2p + 2q+1)}. The set of vertex labels of G is A∪B∪H∪T = {0,1,2, …, (2p), (2p+ l),…, (2p + 2q + 1), (4p + 2q + 2), (4p + 2q + 3),..., (6p + 2q + 2), (6p + 2q + 3)}.

The edge labels of G can be arranged in the following order. N_1 = {(4p+2q+ 2),..., (6p + 2q + 3)}, N_2 = { (2p+ 2q + 2) ,...,(4p + 2q + 2))}, L = {1,2,...,2p}, O = {(2p + 1), (2p + 2),..., (2p + 2q + 1)}. The set of edge labels of G is N_1 ∪ N_2 ∪ L ∪ O = {1, 2, 3, … , (6p + 2q + 3)}.

Therefore the set of vertex labels and edge labels are distinct. So f is a graceful labeling. Hence G = Sqt(l, n, m) is graceful, for l = (n – 1), odd m, odd n and l, n ≥ 1.

Theorem 2.6 Sqt(l, n, m) is graceful, for l = (n – 1), even m, even n and l, n ≥ 2.

Proof. Consider the theta graph θ (2,2,…,2) with end vertices a and b and inner vertices v_1, v_2,…,v_n . Let St(l) be a star graph with (l +1) vertices $h,h_1,h_2,…,h$ (m +1) vertices t, t_1, t_2,…,t_s for s ≥ 1, where t is the center vertex and $t_1,t_2,…,t_s$ are pendant vertices. To form the graph G = Sqt (l, n, m) attach the center vertex h of St(l) with the end vertex 'a' of θ (2,2,…,2) and join the center vertex t of St(m) with the end vertex 'b' of θ (2,2,…,2). G has (n + 2 + l + m) vertices and (2n + l + m) edges.

The vertex set V(G) = {a, v_i, b, h_j, t_s / i = 1,2,…,n, j = 1,2,…, l and t = 1,2,…,m}. The edge set E(G) ={av_i / i = 1,2,…,n} ∪ {v_i b / i = 1,2,…,n}∪{h_ja / j = 1,2,…, l } ∪{t_sb / s = 1,2,…,m}.

Let f be the labeling on the set of vertices of G and g be the induced labeling on the set of edges of G.

The vertex label set of G can be written as A∪B∪H∪T where A = {f(a), f(b)}, B = {f(v_i) / i = 1,2,…, n}, H = {f(h_j) / j = 1,2,…, l}, T = {f(t_s) / s = 1,2,…, m}. The edge label set of G can be written as N_1∪N_2∪L∪O where N_1 = {g(av_i) / i = 1, 2,…, n}, N_2 = {g(v_ib) / i = 1,2,…, n}, L = {g(h_ia) / j = 1,2,…, l }, O = {g(t_sb) / s = 1,2,…,m}.

Let l = (n – 1). Let m and n be even and l, n ≥ 2. Let n = (2p), m = (2q) where p and q are positive integers and p > 0, q > 0. Therefore l = (2p – 1).

Let the labeling f on the vertices of G be defined by $f(v_i)$ = (i – 1) for 1 ≤ i ≤ (2p), (h_j) = (4p+2q–1+ j) for 1≤ j ≤ (2p– 1), $f(t_s)$ = (2p – 1+s) for 1 ≤ s ≤ (2q), f(a) = (6p+2q–1), f(b) = (4p + 2q – 1).

The induced labeling g on the edges of G is defined by $g(av_i)$ = (6p+2q– i) for 1≤ i ≤ (2p), $g(v_i b)$ = (4p+2q– i) for 1≤ i ≤ (2p), $g(h_j a)$ =(2p – j) for 1 ≤ j ≤ (2p – 1), $g(t_s b)$ = (2p + 2q – s) for 1 ≤ s ≤ (2q).

The vertex labels of G can be arranged in the following order. A = {(4p + 2q – 1), (6p + 2q – 1)}, B = {0,1,…,(2p – 1)}, H = {(4p + 2q),…, (6p + 2q – 2)}, T = {(2p), (2p + 1),…,(2p + 2q – 1)}. The set of vertex labels of G is A∪B∪H∪T = {0,1,2, …, (2p – 1), (2p),…, (2p + 2q – 1), (4p + 2q – 1), (4p + 2q),…,(6p + 2q – 2), (6p + 2q - 1)}.

The edge labels of G can be arranged in the following order. N_1={(4p+2q),..., (6p+2q–1)}, N_2 = {(2p+2q) ,...,(4p+2q–1))}, L = {1,2,...,(2p–1)}, O = {(2p), (2p+1),..., (2p+2q–1)}. The set of edge labels of G is N_1 ∪ N_2 ∪ L ∪ O = {1, 2, 3, … , (6p+2q – 1)}.

Therefore the set of vertex labels and edge labels are distinct. So f is a graceful labeling. Hence G = Sqt(l, n, m) is graceful, for l = (n – 1), even m, even n and l, n ≥ 2.

Theorem 2.7 Sqt(l, n, m) is graceful, for l = (n–1), even m, odd n and m≥ 2, n ≥ 1.

Proof. Consider the theta graph θ (2,2,…,2) with end vertices a and b and inner vertices $v_1,v_2,…,v_n$. Let St(l) be a star graph with (l + 1) vertices h, h_1, h_2, …, h_l for l ≥ 1, where h is the center vertex and h_1, h_2, …, h_l are pendant vertices. Let St(m) be a star graph with (m + 1) vertices t,t_1,t_2,…,t_s for s ≥ 1, where t is the center vertex and t_1, t_2,…, t_s are pendant vertices. To form the graph G = Sqt (l, n, m) attach the center vertex h of St(l) with the end vertex 'a' of θ (2,2,…,2) and join the center vertex t of St(m) with the end vertex 'b' of θ (2,2,…,2). G has (n + 2 + l + m) vertices and (2n + l + m) edges.

The vertex set V(G) = {a, v_i, b, h_j, t_s / i = 1, 2,…, n, j = 1,2,…, l and t = 1,2,…,m}. The edge set E(G) = {av_i / i = 1,2,…, n} ∪ {v_i b / i = 1, 2,…, n}∪{h_ja / j = 1, 2,…, l} ∪{t_sb / s = 1, 2,…,m}.

Let f be the labeling on the set of vertices of G and g be the induced labeling on the set of edges of G.

The vertex label set of G can be written as A∪B∪H∪T whereA = {f(a), f(b)}, B = {$f(v_i)$ / i = 1, 2, …, n}, H = {$f(h_j)$ / j = 1,2,…, l}, T = {$f(t_s)$ / s = 1,2,…, m}. The edge label set of G can be written as N_1 ∪ N_2 ∪ L ∪ O where N_1 = {$g(av_i)$ / i = 1, 2,…, n}, N_2 = {$g(v_ib)$ / i = 1, 2, …, n}, L = {$g(h_ia)$ / j = 1, 2,…, l },O = {$g(t_sb)$ / s = 1, 2, …, m}.

Let l = (n – 1). Let m be even and n be odd and m ≥ 2, n ≥ 1.Let n = (2p + 1), m = (2q) where p and q are positive integers and p > 0, q > 0. Therefore l = (2p).

Let the labeling f on the vertices of G be defined by $f(v_i)$ = (i–1) for1≤ i ≤ (2p+1), $f(h_j)$ = (4p+2q+1+ j)for 1 ≤ j ≤ (2p), $f(t_s)$= (2p + s) for 1 ≤ s ≤ (2q), f(a) = (6p + 2q + 2), f(b) = (4p + 2q + 1).

The induced labeling g on the edges of G is defined by $g(av_i)$ = (6p+2q+3 – i) for 1 ≤ i ≤ (2p + 1), $g(v_i b)$ = (4p+2q+2– i) for 1 ≤ i ≤ (2p + 1), $g(h_j a)$ = (2p + 1– j) for 1 ≤ j ≤ 2p, $g(t_sb)$ = (2p + 2q + 1 – s) r 1 ≤ s ≤ (2q).

The vertex labels of G can be arranged in the following order. A = {(4p + 2q + 1), (6p + 2q + 2)}, B = {0,1,…,(2p) }, H = {(4p + 2q + 2),…, (6p + 2q + 1)}, T = {(2p + 1), (2p + 2),…, (2p + 2q)}. The set of vertex labels of G is A∪B∪H∪T = {0, 1,2, …,(2p), (2p + l),…, (2p + 2q), (4p + 2q + 1), (4p + 2q +2),..., (6p + 2q + 1), (6p + 2q + 2)}.

The edge labels of G can be arranged in the following order. N_1 = {(4p+2q+ 2),..., (6p+2q+2)}, N_2 = {(2p+2q+1,..., (4p+2q+1))}, L = {1,2,...,2p}, O = {(2p +1), (2p+2),..., (2p + 2q)}. The set of edge labels of G is N_1∪ N_2 ∪ L ∪ O = {1, 2, 3, … , (6p + 2q + 2)}.

Therefore the set of vertex labels and edge labels are distinct. So f is a graceful labeling. Hence G = Sqt(l, n, m) is graceful, for l = (n – 1), even m, odd n and m ≥ 2, n ≥ 1.

Theorem 2.8 Sqt(l, n, m) is graceful, for l = (n–1), odd m, even n and m≥ 1, n ≥ 2.

Proof. Consider the theta graph θ (2,2,…,2) with end vertices a and b and inner vertices $v_1,v_2,…,v_n$. Let St(l) be a star graph with (l + 1) vertices h, h_1, h_2, …, h_l for l ≥ 1, where h is the center vertex and h_1, h_2,…, h_l are pendant vertices. Let St(m) be a star graph with (m +1) vertices t, t_1, t_2,…,t_s for s ≥ 1, where t is the center vertex and $t_1, t_2,\ldots,t_s$ are pendant vertices. To form the graph G = Sqt (*l*, n, m) attach the center vertex h of St(*l*) with the end vertex 'a' of θ (2,2,…,2) and join the center vertex t of St(m) with the end vertex 'b' of θ (2,2,…,2). G has (n + 2 + *l* + m) vertices and (2n + *l* + m) edges.

The vertex set V(G) ={a, v_i, b, h_j, t_s / i = 1, 2,…, n, j = 1,2,…, *l* and t = 1,2,…,m}. The edge set E(G) = {av_i / i = 1,2,…,n} ∪ {$v_i b$ / i = 1,2,…,n}∪{$h_j a$ / j = 1,2,…, *l*} ∪{$t_s b$/s = 1,2,…,m}.

Let f be the labeling on the set of vertices of G and g be the induced labeling on the set of edges of G.

The vertex label set of G can be written as A∪B∪H∪T whereA ={f(a), f(b)}, B = { f(v_i) / i = 1,2,…, n}, H = { f(h_j) / j = 1,2,…, *l*}, T = { f(t_s) /s = 1,2,…, m}. The edge label set of G can be written as N_1∪N_2∪L∪O where N_1 = {g(av_i) / i = 1,2,…,n}, N_2 = {g($v_i b$) / i = 1, 2,…, n}, L = {g($h_i a$) / j = 1, 2,…, *l*}, O = {g($t_s b$) / s = 1,2,…,m}.

Let *l* = (n – 1). Let m be odd, n be even and m ≥ 1, n ≥ 2. Let n = (2p), m = (2q + 1) where p and q are positive integers and p > 0, q > 0. Therefore *l* = (2p – 1).

Let the labeling f on the vertices of G be defined by f(v_i) = (i – 1) for 1≤ i ≤ (2p), f(h_j) = (4p+q+j) for 1≤ j ≤ (2p–1), f(t_s) =(2p – 1 + s) for 1 ≤ s ≤ (2q + 1), f(a) = (6p + 2q), f(b) =(4p + 2q).

The induced labeling g on the edges of G is defined by g(av_i) = (6p+2q+1– i) for 1 ≤ i ≤ (2p), g(v_i b) = (4p+2q+1– i) for 1 ≤ i ≤ (2p), g(h_j a) = (2p – j) for 1 ≤ j ≤ (2p – 1), g($t_s b$) = (2p + 2q + 1 – s) for 1 ≤ s ≤ (2q + 1).

The vertex labels of G can be arranged in the following order. A = {(4p + 2q), (6p + 2q)}, B = {0,1,…,(2p – 1)}, H = {(4p + 2q + 1),…, (6p + 2q – 1) }, T = {(2p), (2p + 1),…, (2p + 2q)}. The set of vertex labels of G is A∪B∪H∪T = {0, 1, 2, …, (2p–1), (2p),…, (2p + 2q), (4p + 2q), (4p + 2q + 1),..., (6p + 2q – 1), (6p + 2q)}.

The edge labels of G can be arranged in the following order. N_1 = {(4p+2q+1),..., (6p+2q)}, N_2 = {(2p+2q+1),..., (4p+2q))}, L = {1,2,...,(2p–1)}, O = {(2p), (2p+1),..., (2p+ 2q)}. The set of edge labels of G is N_1∪N_2∪L∪O = {1, 2, 3, … , (6p + 2q)}.

Therefore the set of vertex labels and edge labels are distinct. So f is a graceful labeling. Hence G = Sqt(*l*, n, m) is graceful, for *l* = (n – 1), odd m, even n and m ≥1, n ≥2.

Theorem 2.9 Sqt_i (*l*, n, m)○ St(r) is graceful, for *l* = (n – 1), *l*, n, m, r ≥ 1.

Proof. Consider the theta graph θ (2,2,…,2) with end vertices a and b and inner vertices $v_1,v_2,\ldots,v_n$. Form the graph Sqt(*l*, n, m). Let St(r) be a star graph with (r + 1) vertices $w,w_1,w_2,\ldots, w_r$ for r ≥ 1, where w is the center vertex and $w_1,w_2,\ldots,w_r$ are pendant vertices. To form the graph G = Sqt (*l*, n, m) ○ St(r) attach the vertex w of St(r) with a vertex v_i (1 ≤ i ≤ n) of Sqt(*l*, n, m), say v_1. G has (n + 2 + *l* + m + r) vertices and (2n + *l* + m + r) edges.

The vertex set V(G) = {a, v_i, b, h_j, t_s, w_k / i = 1, 2,…, n, j = 1,2,…*l*, s = 1, 2,…, m and k = 1, 2,…, r}. The edge set E(G) ={av_i / i = 1, 2,…,n} ∪ {v_i b / i = 1, 2,…, n} ∪{$h_j a$ / j = 1,2,…, *l*}∪{$t_s b$ / s = 1,2,…,m} ∪{$w_k v_1$ / k = 1, 2,…, r}.

Let f be the labeling on the set of vertices of G and g be the induced labeling on the set of edges of G.

The vertex label set of G can be written as A∪B∪H∪T ∪C where A ={f(a), f(b)}, B = { f(v_i) / i = 1,2,…, n}, H = { f(h_j) / j = 1,2,…, *l*}, T = { f(t_s) /s = 1,2,…, m}, C = { f(w_k) / k = 1,2,…, r}. The edge label set of G can be written as N_1 ∪ N_2 ∪ L ∪ O ∪M where N_1 = {g(av_i) / i = 1,2,…,n}, N_2 = {g($v_i b$) / i = 1, 2,…, n}, L = {g($h_i a$) / j = 1, 2,…, *l*}, O={g($t_s b$) / s = 1, 2,…, m}, M = {g($w_k v_1$) / k = 1, 2,…, r}.

Let the labeling f on the vertices of G be defined by f(v_i) = (i – 1) for 1 ≤ i ≤ n, f(h_j) = (2n + m – 1 + j) for 1 ≤ j ≤ *l*, f(t_s) = (n–1+s) for 1 ≤ s ≤ m, f(w_k) = (2n+*l*+ m + k) for 1 ≤ k ≤ r, f(a) = (3n + m – 1), f(b)= (2n + m – 1).

The induced labeling g on the edges of G is defined by g(av_i) =(2n+*l* +m+1–i) for 1≤i ≤ n, g(v_i b) =(n+*l*+m+1– i) for 1≤ i ≤ n, g(h_j a) =(*l*+1– j) for 1 ≤ j ≤ *l*, g($t_s b$) =(n+m– s) for 1 ≤ s ≤ m, g($w_k v_1$) = (2n + *l* + m + k) for 1 ≤ k ≤ r.

The vertex labels of G can be arranged in the following order. A = {(2n+m–1), (3n+m–1)}, B ={0,1,…,(n–1)}, H ={(2n+m), (2n+m+1),…,(3n+m–2)},T = {n,...,(n+m–1)}, C = {(2n+ *l* +m+1), (2n+ *l* +m+2),…, (2n+*l* +m+r)}. The set of vertex labels of G is A∪B∪H∪T∪C = {0,1,2, …, (n–1), n, (n+l),..., (n+m–1), (2n+m–1), (2n+m), (2n+m+1) ,…, (3n + m – 2), (3n + m – 1), (2n + *l* + m + 1), (2n + *l* + m + 2),…, (2n + *l* +m + r)}.

The edge labels of G can be arranged in the following order. N_1 = { (2n + m), (2n+m+1),..., (3n+m–1)}, N_2 = {(n+m), (n+m+1),..., (2n+m–l)}, L = {1,2,..., (n–1)}, O = {n,...,(n+m–1)}, M = {(2n+ *l* +m+1), (2n+ *l* +m+2),..., (2n+ *l* +m+r)}. The set of edge labels of G is $N_1 \cup N_2 \cup L \cup O \cup M$ = {1, 2, 3, … , (2n + *l* +m + r)}.

Therefore the set of vertex labels and edge labels are distinct. So f is a graceful labeling. Hence G = Sqt_i (*l*, n, m)○ St(r) is graceful, for *l* = (n – 1), *l*, n, m, r ≥ 1.

Theorem 2.10 Sqt(*l*, n, m) is graceful, for *l* = m, even n, even m and m ≥ 2, n ≥ 2.

Proof. Consider the theta graph θ (2,2,…,2) with end vertices a and b and inner vertices $v_1, v_2,…,v_n$. Let St(*l*) be a star graph with (*l* + 1) vertices h, $h_1, h_2,…, h_l$ for *l* ≥ 1, where h is the center vertex and $h_1, h_2, …, h_l$ are pendant vertices. Let St(m) be a star graph with (m + 1) vertices t, $t_1, t_2, …, t_s$ for s ≥ 1, where t is the center vertex and $t_1, t_2, …, t_s$ are pendant vertices. To form the graph G = Sqt (*l*, n, m) attach the center vertex h of St(*l*) with the end vertex 'a' of θ (2,2,…,2) and join the center vertex t of St(m) with the end vertex 'b' of θ (2,2,…,2). G has (n + 2 + *l* + m) vertices and (2n + *l* + m) edges.

The vertex set V(G) = {a, v_i, b, h_j, t_s / i = 1,2,…,n, j = 1,2,…,l and t = 1, 2,…, m}.

The edge set E(G) = {av_i / i = 1, 2,…, n} ∪ {v_i b / i = 1, 2,…, n}∪{h_ja / j = 1,2,…,*l*} ∪{t_sb / s = 1,2,…,m}.

Let f be the labeling on the set of vertices of G and g be the induced labeling on the set of edges of G.

The vertex label set of G can be written as A∪B∪H∪T where A = { f(a), f(b)}, B = {f(v_i) / i = 1,2,…, n}, H = {f(h_j) / j = 1, 2,…, *l*}, T = {f(t_s) / s = 1, 2, …, m}. The edge label set of G can be written as N_1∪ N_2 ∪ L ∪ O where N_1 = { g (av_i) / i = 1,2,…, n }, N_2 = {g (v_ib) / i = 1,2,…, n}, L = {g (h_ia) / j = 1, 2, …, *l* }, O = {g(t_sb) / s = 1, 2, …, m}.

Let *l* = (m). Let m and n be even and m, n ≥ 2. Let n = (2p), m = (2q) where p and q are positive integers and p > 0, q > 0. Therefore *l* = (2q).

Let the labeling f on the vertices of G be defined by f(v_i) =(i – 1) for 1≤ i ≤ (2p), f(h_j) = (4p – 1 + 2 j) for 1 ≤ j ≤ (2q), f(t_s) = (2p + 4q – 2s) for1 ≤ s ≤ (2q), f(a) = (4p+4q), f(b) = (2p + 4q).

The induced labeling g on the edges of G is defined by g(av_i) = (4p + 4q + 1– i) for 1 ≤ i ≤ (2p), g(v_i b) = (2p+4q+1– i) for 1 ≤ i ≤ (2p), g(h_j a) = (4q+1– 2j) for 1≤ j≤ (2q), g(t_sb) = (2s) for 1 ≤ s ≤ (2q).

The vertex labels of G can be arranged in the following order. A ={(2p+4q), (4p+4q)},B = {0,1,…,(2p–1)}, H = {(4p+1), (4p+3),…,(4p+4q–1)},T = {(2p), (2p+2),…, (2p + 4q – 2)}. The set of vertex labels of G is A∪B∪H∪T = {0,1,2, …, (2p–1), (2p), (2p+2),…,(2p+4q – 2), (2p+4q), (4p+1), (4p+3),..., (4p+4q–1), (4p+4q)}.

The edge labels of G can be arranged in the following order. N_1 = { (2p + 4q + 1), (2p + 4q + 2),...,(4p + 4q)}, N_2 = {(4q + 1), (4q + 2),.., (4q + 2p)}, L ={1,3,5,...,(4q–1)}, O = {2,4,6,…,(4q)}. The set of edge labels of G is N_1∪N_2∪L∪O ={1,2,3,…,(4p + 4q)}.

Therefore the set of vertex labels and edge labels are distinct. So f is a graceful labeling. Hence G = Sqt(*l*, n, m) is graceful, for *l* = m, even n, even m and m ≥ 2, n ≥ 2.

Theorem 2.11 Sqt(*l* , n, m) is graceful, for *l* = m, even n, odd m and m ≥ 1, n ≥ 2.

Proof. Consider the theta graph θ (2,2,…,2) with end vertices a and b and inner vertices $v_1, v_2,…, v_n$. Let St(*l*) be a star graph with (*l* +1) vertices h, $h_1, h_2, …, h_l$ for *l* ≥ 1, where h is the center vertex and $h_1, h_2,…, h_l$ are pendant vertices. Let St(m) be a star graph with (m +1) vertices t, $t_1, t_2,…, t_s$ for s ≥ 1, where t is the center vertex and $t_1, t_2, …, t_s$ are pendant vertices. To form the graph G = Sqt (*l*, n, m) attach the center vertex h of St(*l*) with the end vertex 'a' of θ (2,2,…,2) and join the center vertex t of St(m) with the end vertex 'b' of θ (2,2,…,2). G has (n + 2 + *l* + m) vertices and (2n + *l* + m) edges.

. The vertex set V(G) = {a, v_i, b, h_j, t_s / i = 1,2,…,n, j = 1,2,…,*l* and t = 1, 2,…, m}. The edge set E(G) = {av_i / i = 1, 2,…,n} ∪ {v_i b / i = 1, 2,…,n}∪{h_ja / j = 1, 2,…, *l*} ∪{t_sb/s = 1,2,…,m}.

Let f be the labeling on the set of vertices of G and g be the induced labeling on the set of edges of G.

The vertex label set of G can be written as A∪B∪H∪T whereA = { f(a), f(b) }, B = { f(v_i) / i = 1, 2,…, n }, H = { f(h_j) / j = 1,2,…, *l* }, T = {f(t_s) / s = 1,2, …, m }. The edge label set of G can be written as N_1∪N_2∪L∪O where N_1 = { g(av_i) / i = 1, 2,…,n }, N_2 = { g(v_ib) / i = 1, 2,…,n }, L = { g(h_ia) / j = 1,2,…, *l* }, O = {g(t_sb) / s = 1, 2,…, m}.

Let l = (n–1). Let m be odd, n be even and m ≥ 1, n ≥ 2. Let n = (2p), m = (2q + 1) where p and q are positive integers and p > 0, q > 0. Therefore l = (2q + 1).

Let the labeling f on the vertices of G be defined by $f(v_i)$ = (i – 1) for 1≤ i ≤ (2p), $f(h_j)$ = (4p+2j–1)for 1 ≤ j ≤ (2q + 1), $f(t_s)$ = (2p+4q+2 – 2s) for 1 ≤ s ≤ (2q + 1), f(a) = (4p + 4q + 2), f(b)= (2p + 4q + 2).

The induced labeling g on the edges of G is defined by $g(av_i)$ = (4p+4q+3– i) for 1≤ i ≤ (2p), $g(v_i b)$ = (2p+4q+3– i) for 1≤ i ≤ (2p), $g(h_j a)$ = (4q+3–2j) for 1≤ j ≤ (2q+1), $g(t_s b)$ = (2s) for 1 ≤ s ≤ (2q + 1).

The vertex labels of G can be arranged in the following order. A = {(2p + 4q + 2), (4p+4q+2)}, B = {0,1,…,(2p–1)}, H = {(4p+1), (4p+3),…, (4p+4q+1)}, T = {(2p), (2p+2),…, (2p+4q)}. The set of vertex labels of G is A∪B∪H∪T = {0,1,2,…,(2p–1), (2p), (2p+ 2),…, (2p+4q), (2p+4q+2), (4p+1), (4p+3), ..., (4p+4q+1),(4p+4q+2)}.

The edge labels of G can be arranged in the following order. N_1 = { (2p + 4q + 3), (2p + 4q + 5),...,(4p + 4q + 2)}, N_2 = {(4q + 3), (4q + 4) ,.., (4q + 2p + 2)}, L = { 1, 3,5,..., (4q + 1) }, O = { 2, 4, 6,…, (4q + 2)}. The set of edge labels of G is N_1 ∪ N_2 ∪ L ∪ O = {1, 2, 3, … , (4p + 4q + 2)}.

Therefore the set of vertex labels and edge labels are distinct . So f is a graceful labeling. Hence G = Sqt(l, n, m) is graceful, for l = m, even n, odd m and m ≥ 1, n ≥ 2.

Theorem 2.12 Sqt_i (l, n, m)○ St(r) is graceful, for l = m, even n and l, n, m, r ≥ 1.

Proof. Consider the theta graph θ (2,2,…,2) with end vertices a and b and inner vertices $v_1,v_2,…,v_n$. Form the graph Sqt(l, n, m). Let St(r) be a star graph with (r + 1) vertices $w,w_1,w_2,…,w_r$ for r ≥ 1, where w is the center vertex and $w_1,w_2,…,w_r$ are pendant vertices. To form the graph G = Sqt (l, n, m)○ St(r) attach the vertex w of St(r) with a vertex v_i (1 ≤ i ≤ n) of Sqt (l, n, m), say v_1. G has (n + 2 + l +m + r) vertices and (2n +l + m + r) edges.

The vertex set V(G) = {a, v_i, b, h_j, t_s, w_k / i = 1,2,…, n, j = 1,2,…, l, s = 1, 2,…, m and k = 1, 2,…, r }. The edge set E(G) = {av_i / i = 1, 2,…,n} ∪ {v_i b / i = 1, 2, …,n} ∪ { h_ja / j = 1,2,…, l}∪{t_sb / s = 1, 2, …, m } ∪ { w_kv_1 / k = 1, 2,…, r }.

Let f be the labeling on the set of vertices of G and g be the induced labeling on the set of edges of G.

The vertex label set of G can be written as A∪B∪H∪T ∪C where A = {f(a), f(b)}, B = {$f(v_i)$ / i = 1,2,…, n }, H = {$f(h_j)$ / j = 1, 2,…, l }, T = {$f(t_s)$ / s = 1,2,…, m}, C = { $f(w_k)$ / k = 1, 2,…, r }. The edge label set of G can be written as N_1∪ N_2∪ L ∪ O ∪ M where N_1 = {$g(av_i)$ / i = 1, 2,…, n }, N_2 = {$g(v_ib)$ / i = 1,2,…, n }, L = {$g(h_ia)$ / j = 1, 2,…, l }, O = {$g(t_sb)$ / s = 1, 2,…, m }, M = {$g(w_kv_1)$ / k = 1, 2,…, r}.

Let the labeling f on the vertices of G be defined by $f(v_i)$ = (i – 1) for 1 ≤ i ≤ n, $f(h_j)$ = (2n+2j–1) for 1≤ j ≤ m, $f(t_s)$ = (n+m–2s) for 1≤ s ≤ m, $f(w_k)$ = (2n+2m+k) for 1≤ k ≤ r, f(a) = (2n + 2m), f(b) = (n + 2m).

The induced labeling g on the edges of G is defined by $g(av_i)$ = (2n+2m+1– i) for 1≤ i ≤n, $g(v_i b)$ = (n+2m+1– i) for 1 ≤ i ≤ n, $g(h_j a)$ = (2m+1–2j) for 1≤ j ≤ m, $g(t_sb)$ = (2s) for 1 ≤ s ≤ m, $g(w_kv_1)$ = (2n+2m+k) for 1≤ k ≤ r.

The vertex labels of G can be arranged in the following order. A = {(n+2m), (2n+2m)}, B = {0,1,…,(n – 1)}, H = {(2n+1), (2n+3),…, (2n+2m–1)}, T = { n, (n + 2),..., (n+2m–2)}, C ={(2n+2m+1), (2n+2m+2),…,(2n+2m+r)}. The set of vertex labels of G is A∪B∪H∪T∪C = {0,1,2,…,(n–1), n, (n+2), (n+4),...,(n+2m–2), (n+2m), (2n+1), (2n+m), (2n+3),…,(2n+2m–1), (2n+2m), (2n+2m+1), (2n+2m+2),…,(2n+2m+r)}.

The edge labels of G can be arranged in the following order. N_1 = { (n + 2m + 1), (n + 2m + 2),..., (2m + 2n) }, N_2 = { (2m + 1), (2m + 2),...,(2m + n) }, L ={ 1, 3, 5,..., (2m – 1)}, O = { 2, 4, 6,...,2m}, M = { (2n + 2m + 1), (2n + 2m + 2),..., (2n + 2m + r)}. The set of edge labels of G is N_1 ∪ N_2 ∪ L∪ O ∪ M = {1, 2, 3, … , (2n + 2m + r)}.

Therefore the set of vertex labels and edge labels are distinct. So f is a graceful labeling. Hence G = Sqt_i (l, n, m)○ St(r) is graceful, for l = m, even n and l, n, m, r ≥ 1.

Theorem 2.13 Sqt(l,n,m) is graceful, for l = m, odd n, even m, m<n and m ≥ 2, n ≥1.

Proof. Consider the theta graph θ (2,2,…,2) with end vertices a and b and inner vertices $v_1,v_2,…,v_n$. Let St(l) be a star graph with (l +1) vertices h, h_1, h_2, …, h_l for l ≥ 1,where h is the center vertex and h_1, h_2, …, h_l are pendant vertices. Let St(m) be a star graph with (m +1) vertices t, t_1, t_2,…, t_s for s ≥ 1, where t is the center vertex and $t_1, t_2, \ldots, t_s$ are pendant vertices. To form the graph G = Sqt (l, n, m) attach the center vertex h of St(l) with the end vertex 'a' of θ (2,2,…,2) and join the center vertex t of St(m) with the end vertex 'b' of θ (2,2,…,2). G has (n + 2 + l + m) vertices and (2n + l + m) edges.

. The vertex set V(G) = {a, v_i, b, h_j, t_s / i = 1,2,…, n, j = 1,2,…, l and t = 1,2,…,m}. The edge set E(G) = { av_i / i = 1,2,…,n } ∪ { v_i b / i = 1,2,…,n } ∪ { $h_j a$ / j = 1,2,…, l } ∪{$t_s b$ / s = 1,2,…,m}.

Let f be the labeling on the set of vertices of G and g be the induced labeling on the set of edges of G.

The vertex label set of G can be written as A∪B∪H∪T where A = { f(a), f(b) }, B = { f(v_i) / i = 1, 2,…, n }, H = { f(h_j) / j = 1, 2,…, l }, T = {f(t_s) / s = 1, 2,…, m }. The edge label set of G can be written as N_1 ∪ N_2 ∪ L ∪ O where N_1 = { g(av_i) / i = 1, 2,…, n }, N_2 = {g($v_i b$) / i = 1,2,…,n}, L = {g($h_i a$) / j = 1, 2,…, l }, O = {g($t_s b$) / s = 1, 2,…, m}.

Let l = (n – 1). Let m be even, n be odd, m < n and m ≥ 2, n ≥ 1. Let n = (2p + 1), m = (2q) where p and q are positive integers and p > 0, q > 0. Therefore l = (2q).

Let the labeling f on the vertices of G be defined by f(v_i) = (i –1) for 1≤ i ≤ (2p+1), f(h_j) = (4p + 2q + 1 + j) for 1 ≤ j ≤ (2q), f(t_s) = (2p +2q + 1 –s) for 1 ≤ s ≤ (2q), f(a) = (4p + 4q + 2), f(b) =(2p + 4q).

The induced labeling g on the edges of G is defined by g(av_i) = (4p + 4q + 3– i) for 1 ≤ i ≤ (2p + 1), g(v_i b) = (2p + 4q + 2 – i) for 1≤ i ≤ (2p+1), g(h_j a) = (2q + 1 – j) for 1 ≤ j ≤ 2q, g($t_s b$) = (2q + s) for 1 ≤ s ≤ (2q).

The vertex labels of G can be arranged in the following order. A = {(2p + 4q), (4p + 4q + 2)}, B = {0,1,…,(2p) }, H = { (4p + 2q + 2),…, (4p+ 4q + 1) }, T = {(2q + 1), (2q + 2),…,(4q) }. The set of vertex labels of G is A ∪ B ∪ H ∪ T = {0, 1, 2, …, (2p), (2p + 4q + 2), (4p + 2q + 2)…,(4p + 4q + 1), (2q + 1),…, 4q, (4p + 4q + 2) }.

The edge labels of G can be arranged in the following order. N_1 = {(2p+4q+2),..., (4p+4q+2)}, N_2 ={(4q+1),...,(2p+4q+1))}, L = {1,2,...,2q}, O = {(2q+1), (2q+2),..., (4q)}. The set of edge labels of G is N_1 ∪ N_2 ∪ L ∪ O = { 1, 2, 3, … , (4p + 4q + 2)}.

Therefore the set of vertex labels and edge labels are distinct. So f is a graceful labeling. Hence G =Sqt(l,n,m) is graceful, for l = m, odd n, even m, m< n and m≥ 2, n ≥1.

Theorem 2.14 Sqt(l, n, m) is graceful, for l = m, odd n, odd m, m<n and m≥ 3, n ≥1.

Proof. Consider the theta graph θ (2,2,…,2) with end vertices a and b and inner vertices $v_1, v_2, \ldots, v_n$. Let St(l) be a star graph with (l +1) vertices h, $h_1, h_2, \ldots, h_l$ for l ≥ 1, where h is the center vertex and $h_1, h_2, \ldots, h_l$ are pendant vertices. Let St(m) be a star graph with (m + 1) vertices t, $t_1, t_2, \ldots, t_s$ for s ≥ 1, where t is the center vertex and $t_1, t_2, \ldots, t_s$ are pendant vertices. To form the graph G = Sqt (l, n, m) attach the center vertex h of St(l) with the end vertex 'a' of θ (2,2,…,2) and join the center vertex t of St(m) with the end vertex 'b' of θ (2,2,…,2). G has (n + 2 + l + m) vertices and (2n + l + m) edges.

. The vertex set V(G) = {a, v_i, b, h_j, t_s / i = 1,2,…,n, j = 1,2,…,l and t = 1, 2,…, m}. The edge set E(G) = {av_i / i = 1,2,…, n} ∪ {v_i b / i = 1, 2,…, n} ∪ {$h_j a$ / j = 1, 2,…, l } ∪{$t_s b$ / s = 1,2,…, m}.

Let f be the labeling on the set of vertices of G and g be the induced labeling on the set of edges of G.

The vertex label set of G can be written as A∪B∪H∪T where A = {f(a), f(b)}, B = {f(v_i) / i = 1, 2,…, n}, H = {f(h_j) / j = 1, 2,…, l}, T = {f(t_s) /s = 1, 2,…, m}. The edge label set of G can be written as N_1∪N_2∪L∪O where N_1 = {g(av_i) / i = 1, 2,…, n}, N_2 = {g($v_i b$) / i = 1,2,…,n}, L = {g($h_i a$) / j = 1, 2,…, l}, O = {g($t_s b$) / s = 1, 2,…, m}.

Let l = (n – 1). Let m, n be odd, m < n and m ≥ 3, n ≥ 1. Let n = (2p + 1), m = (2q + 1) where p and q are positive integers and p > 0, q > 0. Therefore l = (2q + 1).

Let the labeling f on the vertices of G be defined by

f(v_i) = (i – 1) for 1 ≤ i ≤ (2p + 1), f(h_j) =(4p + 2q + 2 + j) for 1 ≤ j ≤ (2q + 1), f(t_s) =(2p + 2q + 2 – s) for 1 ≤ s ≤ (2q + 1), f(a) = (4p + 4q + 4), f(b) = (2p + 4q + 3).

The induced labeling g on the edges of G is defined by g(av_i) = (4p + 4q + 5– i) for 1 ≤ i ≤ (2p +1), g(v_i b) = (2p + 4q +4 – i) for 1≤ i ≤ (2p + 1), g(h_j a) = (2q + 2 + j) for 1 ≤ j ≤ (2q + 1), g($t_s b$) = (2q + 1 + s) for 1 ≤ s ≤ (2q + 1).

The vertex labels of G can be arranged in the following order. A = {(2p + 4q + 3), (4p + 4q + 4)}, B = {0,1,…,(2p)}, H = {(4p + 2q + 3),…, (4p+ 4q + 3)}, T = {(2q + 2), (2q + 3),…,(4q + 2)}. The set of vertex labels of G is A ∪ B ∪ H ∪ T = {0,1,2, …, (2p), (2p + 4q + 3), (4p + 2q + 3)…,(4p + 4q + 3), (2q + 2),…,(4q+2), (4p + 4q + 4)}.

The edge labels of G can be arranged in the following order. N_1 = {(2p+4q+4),..., (4p + 4q + 4)}, N_2 = {(4q + 3) ,...,(2p + 4q + 3))}, L = {1,2,...,(2q+ 1)}, O = { (2q +2), (2q + 3),..., (4q + 2)}. The set of edge labels of G is N_1 ∪ N_2 ∪ L ∪ O = {1, 2, 3, … , (4p + 4q + 4)}.

Therefore the set of vertex labels and edge labels are distinct. So f is a graceful labeling. Hence G = Sqt(*l*,n,m) is graceful, for *l* = m, odd n, odd m, m< n and m ≥3, n ≥1.

Theorem 2.15 Sqt_i (*l*, n, m)○St(r) is graceful, for *l* = m, odd n, m<n and *l*, n, m, r≥1.

Proof. Consider the theta graph θ (2,2,…,2) with end vertices a and b and inner vertices v_1, v_2, …,v_n. Form the graph Sqt(*l*, n, m). Let St(r) be a star graph with (r + 1) vertices w,w_1,w_2,…,w_r for r ≥ 1, where w is the center vertex and w_1,w_2,…,w_r are pendant vertices. To form the graph G = Sqt (*l*, n, m)○St(r) attach the vertex w of St(r) with a vertex v_i (1 ≤ i ≤ n) of Sqt(*l*, n, m), say v_1. G has (n + 2 + *l* +m + r) vertices and (2n + *l* + m + r) edges.

. The vertex set V(G) = {a, v_i, b, h_j, t_s, w_k / i = 1, 2,…, n, j = 1, 2,…, *l*, s = 1,2,…,m and k = 1,2,…r}. The edge set E(G) = {av_i / i = 1,2,…,n} ∪ {v_i b / i = 1,2,…,n}∪{h_ja /j = 1, 2,…, *l*}∪{t_sb / s = 1, 2,…, m} ∪{w_kv_1 / k = 1, 2,…, r}.

Let f be the labeling on the set of vertices of G and g be the induced labeling on the set of edges of G.

The vertex label set of G can be written as A ∪ B ∪ H ∪ T ∪C where A = {f(a), f(b)}, B = { f(v_i) / i = 1, 2,…, n}, H = {f(h_j) / j = 1, 2,…, *l*}, T = {f(t_s) / s = 1, 2,…, m}, C = {f(w_k) / k = 1,2,…, r}. The edge label set of G can be written as N_1 ∪ N_2 ∪ L ∪ O ∪ M where N_1 = {g(av_i) / i = 1,2,…, n}, N_2 = {g(v_ib) / i = 1,2,…,n}, L = { g(h_ia) / j = 1, 2,…, *l*}, O = {g(t_sb) / s = 1, 2,…,m }, M = {g(w_kv_1) / k = 1, 2,…, r}.

Let the labeling f on the vertices of G be defined by f(v_i) = (i – 1) for 1 ≤ i ≤ n, f(h_j) = (2n+m+j– 1) for 1 ≤ j ≤ m, f(t_s) = (n+m–s) for 1 ≤ s ≤ m, f(w_k) = (2n + 2m + k) for 1 ≤ k≤ r, f(a) = (2n + 2m), f(b)= (n + 2m).

The induced labeling g on the edges of G is defined by g(av_i) = (2n+2m+1– i) for 1≤ i ≤ n, g(v_i b) = (n+2m+1– i) for 1≤ i ≤ n, g(h_j a) = (m+1– j) for 1 ≤ j ≤ m, g(t_sb) = (m + s) for 1 ≤ s ≤ m, g(w_kv_1) = (2n+2m+k) for1≤ k≤ r.

The vertex labels of G can be arranged in the following order. A ={(n + 2m), (2n + 2m)}, B = {0,1,…,(n – 1)}, H = {(2n + m),…,(2n + 2m – 1)}, T = {(m + 1), (m + 2), …, (2m)}, C = {(2n + 2m + 1), (2n + 2m + 2),…, (2n + 2m + r)}. The set of vertex labels of G is A ∪ B ∪ H ∪ T ∪ C = {0, 1, 2, …,(n – 1), (n + 2m), (2n + m),..., (2n + 2m – 1), (m + 1), (m + 2),…,(2m),(2n + 2m), (2n + 2m + 1), (2n + 2m + 2),…, (2n + 2m + r)}.

The edge labels of G can be arranged in the following order. N_1 = {(n + 2m + 1), (n + 2m + 2),...,(2m +2n)}, N_2 = {(2m + 1), (2m + 2),..., (2m + n)}, L = {1,2,...,(m)}, O = {(m + 1), (m + 2),...,2m}, M = {(2n + 2m +1), (2n + 2m + 2),...,(2n + 2m + r)}. The set of edge labels of G is N_1 ∪ N_2 ∪ L∪ O ∪ M = {1, 2, 3, … , (2n + 2m + r)}.

Therefore the set of vertex labels and edge labels are distinct. So f is a graceful labeling. Hence G = Sqt_i (*l*, n, m) ○ St(r) is graceful, for *l* = m, odd n, m < n and *l*, n, m, r ≥1.

REFERENCES

[1] F.Harary, *Graph Theory*, Addision Wesley , New Delhi,1972.

[2] J.A. Gallian, *A dynamic survey of graph labeling*, The Electronic Journal of Combinatorics, 19(2012), # DS6.

[3] R. Raziya Begam, M. Palanivelrajan, K. Gunasekaran and A. R. Shahul Hameed, *Graceful labeling of some theta related graphs*, Intern. J. Fuzzy Mathematical Archive, 2(2013), 78-84.

[4] A. R. Shahul Hameed, M. Palanivelrajan, K. Gunasekaran and R. Raziya Begam, *On Graceful labeling of some Bicyclic graphs*, Intern. J. Fuzzy Mathematical Archive, 3(2013), 1-8.

[5] A. R. Shahul Hameed, M. Palanivelrajan, K. Gunasekaran and R. Raziya Begam, *Graceful labeling of some combinations of hut graphs*, *Archimedes J*. *Math*.**,** **4**(1)(2014)**, 27-35.**

[6] A. R. Shahul Hameed, M. Palanivelrajan, K. Gunasekaran and R. Raziya Begam, *On Graceful labeling of Certain Subdivisions of Some Tricyclic graphs*, *Bulletin of Mathematics and Statistics Research,* **2**(3)(2014)**, 331-336.**