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RESEARCH ARTICLE

ON TERNARY QUADRATIC DIOPHANTINE EQUATION $8(x^2+y^2)-15xy=80z^2$

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ABSTRACT

The ternary quadratic equation $8(x^2+y^2)-15xy=80z^2$ representing cone is analyzed by its non-zero distinct integer points on it. Employing the integer solutions, a few relations between the solutions and special polygonal numbers are presented. Also, knowing an integer solution formula for generating sequence of solutions are given.

Keywords: Ternary, Quadratic, Integer solutions, polygonal numbers.

2010 Mathematics subject classification number:11D09.

INTRODUCTION

The ternary quadratic Diophantine equation offers an unlimited field for research because of their variety [1-2].In particular, one may refer [3-22] for finding points on some specific three dimensional surfaces. This communication concerns with yet another ternary quadratic equation $8(x^2+y^2)-15xy=80z^2$ representing cone for determining its infinitely many integer solutions. Employing integral solutions on the cone,a few interesting relations among the special polygonal and pyramidal numbers are given.

NOTATIONS:

$$t_{m,n}=n\left(1+\frac{(n-1)(m-2)}{2}\right).$$

$$P_n^m=\left(\frac{n(n+1)}{6}\right)[(m-2)n+(5-m)]$$

$$SO_n=n(2n^2-1)$$

$$S_n=6n(n-1)+1$$

$$\Pr_n=n(n+1)$$

$$OH_n = \frac{1}{3}[n(2n^2+1)]$$

Method of analysis:

Consider the equation

$$8(x^2+y^2)-15xy=80z^2 \quad (1)$$

The substitutions of the linear transformations

$$x=u+v, y=u-v \ (u\neq v\neq 0) \quad (2)$$

in (1) leads to

$$u^2+31v^2=80z^2 \quad (3)$$

The above equation is solved through different methods and using (2),different patterns of integer solutions to (1) are obtained.

Pattern :I

Write 80 as

$$80=(7+i\sqrt{31})(7-i\sqrt{31}) \quad (4)$$

Assume

$$z=a^2+31b^2 \quad (5)$$

where a and b are non-zero integers.

Using (4) and (5) in (3) and employing the method of factorization, define,

$$(u+i\sqrt{31}v)=(7+i\sqrt{31})(a+i\sqrt{31}b)^2 \quad (6)$$

Equating real and imaginary parts, we have

$$u=u(a,b)=7a^2-217b^2-62ab$$

$$v=v(a,b)=a^2-31b^2-14ab$$

substituting the values of u and v in (2),the values of x and y are given by

$$x=x(a,b)=8a-248b^2-48ab$$

$$y=y(a,b)=6a^2-186b^2-76ab \quad (7)$$

Thus (5) and (7) represent non-zero distinct integral solutions of (1) in two parameters.

Properties:

- $6x(t_{3,a},t_{3,a+2})-8y(t_{3,a},t_{3,a+2})=1920Pt_a$
- $3[y(a,a)-x(a,a)]$ is a nasty number.
- $6x(a,2a^2-1)-8y(a,2a^2-1)=320So_a$
- $x(a,a)-y(a,a)+z(a,a)=0$
- $x(a,1)+y(a,1)-t_{30,a}+111\Pr_a-111t_{4,a}+434=0$
- $x(1,b)+z(1,b)+169t_{4,b}+96t_{3,b}=9$
- $y(2^n,1)-x(2^n,1)-32Ky_n\equiv 0(\bmod 2)$
- $6x(a(a+1),a)-8y(a(a+1),a)=640P_a^5$

Pattern: II

Consider (3) as

$$u^2-49z^2=31(z^2-v^2) \quad (8)$$

write (8) in the form of ratio as

$$\frac{u+7z}{z+v}=\frac{31(z-v)}{u-7z}=\frac{a}{b}(b\neq 0)$$

which is equivalent to the following two equations

$$bu - av + (7b - a)z = 0$$

$$-au - 31bv + (31b + 7a)z = 0$$

On employing the method of cross multiplication, we get

$$u = 7a^2 + 62ab - 217b^2$$

$$v = -a^2 + 14ab + 31b^2 \quad (9)$$

$$z = a^2 + 31b^2 \quad (10)$$

Substituting the values of u, v from (9) in (2),the non-zero distinct integral values of x y are given by

$$x = x(a,b) = 6a^2 - 186b^2 + 76ab$$

$$y = y(a,b) = 8a^2 - 248b^2 + 48ab \quad (11)$$

Thus (10) and (11) represents the non-zero distinct integer solutions of (1) in two parameters

Properties:

- $4x(a(a+1), a+2) - 3y(a(a+1), a+2) = 960P_a^3$
- $x[a, a(a+1)] + y[a, a(a+1)] - 14t_{4,a} - 248P_a^5 + 434(P_{ra})^2 = 0$
- $4x[a, a(a+1)] - 3y[a, a(a+1)] = 320P_a^5$
- $x(a,1) + y(a,1) + z(a,1) - 30t_{3,a} \equiv 33 \pmod{109}$
- $4x(a, a+1) - 3y(a, a+1) - 160P_{ra} = 0$

Note: (8) can also be expressed in the form of ratio in three different ways as follows:

1. $\dfrac{u+7z}{31(z+v)} = \dfrac{z-v}{u-7z} = \dfrac{a}{b} (b \neq 0)$
2. $\dfrac{u+7z}{z-v} = \dfrac{31(z+v)}{u-7z} = \dfrac{a}{b} (b \neq 0)$
3. $\dfrac{u+7z}{31(z-v)} = \dfrac{z+v}{u-7z} = \dfrac{a}{b} (b \neq 0)$

Repeating the analysis as above, we get different patterns of solutions to (1).

Pattern: III

Write (3) as

$$80z^2 - u^2 = 31v^2 \quad (12)$$

Assume

$$v = 80a^2 - b^2 \quad (13)$$

Write 31 as

$$31 = (\sqrt{80} + 7)(\sqrt{80} - 7) \quad (14)$$

Using (13) and (14) in (12) ,employing the method of factorization, define,

$$(\sqrt{80}z + u) = (\sqrt{80} + 7)(\sqrt{80}a + b)^2 \quad (15)$$

Equating rational and real parts, we get,

$$u = u(a,b) = 560a^2 + 7b^2 + 160ab$$

$$z = z(a,b) = 80a^2 + b^2 + 14ab$$

Substituting the values of u and v in (2),we get

$$x = x(a,b) = 640a^2 + 6b^2 + 160ab$$

$$y = y(a,b) = 480a^2 + 8b^2 + 160ab$$

$$z = z(a,b) = 80a^2 + b^2 + 14ab$$

Properties:

- $8x(a,1)-6y(a,1)-320S_a-640t_{3,a}\equiv 0(\bmod 2)$
- $x(a(a+1),a)+y(a(a+1),a)-248P_a^5=0$
- $3\{[y(a(a+1),a]-x[a(a+1),a]+160(P_{ra})^2\}$ is a nasty number.
- $x(a,2a^2+1)+y(a,2a^2+1)-372(OH_a)=0$
- $8[x[a(a+1),1]-6y[a(a+1),1]=2240(P_{ra})^2+640t_{3,a}$

Remarkable Observations: If (u_0,v_0,z_0) is the initial solution of the equation (3),then each of the following triples $(x_{n,}y_n,z_n)$ satisfies (1).

Triple:1

$$x_n=7^n u_0+\frac{1}{14}\{[7^n(1750)-(-7)^n(1736)]v_0+[7^n(-2800)+(-7)^n(2800)]z_0\}$$

$$y_n=7^n u_0-\frac{1}{14}\{[7^n(1750)-(-7)^n(1736)]v_0+[7^n(-2800)+(-7)^n(2800)]z_0\}$$

$$z_n=\frac{1}{14}\{[7^n(1085)-(-7)^n(1085)]v_0+[7^n(-1736)+(-7)^n(1750)]z_0\}$$

Triple:1I

$$x_n=\frac{1}{14}\{[7^n(70)-(-7)^n(560)]u_0+[7^n(-560)+(-7)^n(560)]z_0\}+7^n v_0$$

$$y_n=\frac{1}{70}\{[7^n(7)-(-7)^n(7)]u_0+[7^n(-56)+(-7)^n(70)]z_0\}-7^n v_0$$

$$z_n=\frac{1}{14}\{[7^n(7)-(-7)^n(7)]u_0+[7^n(-56)+(-7)^n(70)]z_0\}$$

CONCLUSION

In this paper, we have presented different patterns of integer solutions to the ternary quadratic equation $8(x^2+y^2)-15xy=80z^2$ representing the cone .As this Diophantine equations are rich in variety,one may attempt to find integer solutions to other choices of equations along with suitable properties.

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