



 ON SOME OF OPEN SETS IN FUZZY TOPOLOGY

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Article Info:

Article received :05/12/2014

Revised on:12/12/2014

Accepted on:16/12/2014

ABSTRACT

In this paper, we offer a new class of sets called fuzzy \ddot{g} -open sets in fuzzy topological spaces and we study some of its basic properties. It turns out that this class lies between the class of fuzzy open sets and the class of fuzzy g -open sets.

2010 Mathematics Subject Classification: 54C10, 54C08, 54C05

Key words and Phrases: Fuzzy Topological space, $f\ddot{g}$ -open set, $f\ddot{g}_\alpha$ -open set, $f\alpha g_s$ -open, $f\omega$ -open set.

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INTRODUCTION

The study of fuzzy sets was initiated with the famous paper of Zadeh [22] and thereafter Chang [7] paved the way for subsequent tremendous growth of the numerous fuzzy topological concepts. The theory of fuzzy topological spaces was developed by several authors by considering the basic concepts of general topology. The extensions of functions in fuzzy settings can be carried out using the concepts of quasi coincidences and q -neighborhoods by Pu and Liu [14]. Generalized fuzzy open sets and regular generalized fuzzy open sets were defined in [16, 14].

In this paper, we introduce a new class of sets namely fuzzy \ddot{g} -open (briefly $f\ddot{g}$ -open) sets in fuzzy topological spaces. This class lies between the class of fuzzy open sets and the class of fg -open sets. This class also lies between the class of fuzzy open sets and the class of $f\omega$ -open sets.

2. PRELIMINARIES

Throughout this paper (X, τ) and (Y, σ) (or X and Y) represent fuzzy topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a fuzzy space (X, τ) , $cl(A)$, $int(A)$ and A^c denote the closure of A , the interior of A and the complement of A respectively.

We recall the following definitions which are useful in the sequel

Definition 2.1: A subset A of a space (X, τ) is called:

- (i) Fuzzy semi-open set [3] (briefly fs-open) if $A \leq \text{cl}(\text{int}(A))$.
- (ii) Fuzzy preopen set [6] (briefly fp-open) if $A \leq \text{int}(\text{cl}(A))$.
- (iii) Fuzzy α -open set [6] (briefly $f\alpha$ -open) if $A \leq \text{int}(\text{cl}(\text{int}(A)))$.
- (iv) Fuzzy β -open set [21] (briefly $f\beta$ -open) if $A \leq \text{cl}(\text{int}(\text{cl}(A)))$.
- (v) Fuzzy regular open set [3] if $A = \text{int}(\text{cl}(A))$.

The complements of the above mentioned fuzzy open sets are called their respective closed sets. The semi closure (resp. α -closure, semi-pre-closure) of a subset A of X , denoted by $\text{scl}(A)$ (resp. $\alpha \text{cl}(A)$, $\text{spcl}(A)$) is defined to be the intersection of all fuzzy semi-closed (resp. fuzzy α -closed, fuzzy semi-preclosed) sets of (X, τ) containing A . It is known that $\text{scl}(A)$ (resp. $\alpha \text{cl}(A)$, $\text{spcl}(A)$) is a fuzzy semi-closed (resp. fuzzy α -closed, fuzzy semi-pre closed) set.

Definition 2.2: A subset A of a space (X, τ) is called:

- (i) a fuzzy generalized closed (briefly fg-closed) set [4] if $\text{cl}(A) \leq U$ whenever $A \leq U$ and U is fuzzy open in (X, τ) . The complement of fg-closed set is called fg-open set.
- (ii) a fuzzy semi-generalized closed (briefly fsg-closed) set [5] if $\text{scl}(A) \leq U$ whenever $A \leq U$ and U is fuzzy semi-open in (X, τ) . The complement of fsg-closed set is called fsg-open set.
- (iii) a fuzzy generalized semi-closed (briefly fgs-closed) set [18] if $\text{scl}(A) \leq U$ whenever $A \leq U$ and U is fuzzy open in (X, τ) . The complement of fgs-closed set is called fgs-open set.
- (iv) a fuzzy α -generalized closed (briefly $f\alpha$ g-closed) set [16] if $\alpha \text{cl}(A) \leq U$ whenever $A \leq U$ and U is fuzzy open in (X, τ) . The complement of $f\alpha$ g-closed set is called $f\alpha$ g-open set.
- (v) a fuzzy generalized semi-preclosed (briefly fgsp-closed) set [18] if $\text{spcl}(A) \leq U$ whenever $A \leq U$ and U is fuzzy open in (X, τ) . The complement of fgsp-closed set is called fgsp-open set.
- (vi) a $f\omega$ -closed set [20] if $\text{cl}(A) \leq U$ whenever $A \leq U$ and U is fuzzy semi-open in (X, τ) . The complement of $f\omega$ -closed set is called $f\omega$ -open set.
- (vii) a fuzzy \ddot{g} -closed set (briefly $f\ddot{g}$ -closed set) [10] if $\text{cl}(A) \leq U$ whenever $A \leq U$ and U is fsg-open in (X, τ) .
- (viii) a fuzzy \ddot{g}_α -closed set (briefly $f\ddot{g}_\alpha$ -closed set) [10] if $\alpha \text{cl}(A) \leq U$ whenever $A \leq U$ and U is fsg-open in (X, τ) . The complement of $f\ddot{g}_\alpha$ -closed set is called $f\ddot{g}_\alpha$ -open set.
- (ix) a fuzzy ψ -closed set (briefly $f\psi$ -closed set) [10] if $\text{scl}(A) \leq U$ whenever $A \leq U$ and U is fsg-open in (X, τ) . The complement of $f\psi$ -closed set is called $f\psi$ -open set.
- (x) a $f\alpha g_s$ -closed set (briefly $f\alpha g_s$ -closed set) [10] if $\alpha \text{cl}(A) \leq U$ whenever $A \leq U$ and U is fuzzy semi-open in (X, τ) . The complement of $f\alpha g_s$ -closed set is called $f\alpha g_s$ -open set.

Remark 2.3: The collection of all $f\ddot{g}$ -closed (resp. $f\omega$ -closed, fg-closed, fgs-closed, fgsp-closed, $f\alpha$ g-closed, fsg-closed, $f\alpha$ -closed, fuzzy semi-closed) sets is denoted by $F\ddot{G}C(X)$ (resp. $F\omega C(X)$, $FGC(X)$, $FGS C(X)$, $FGSP C(X)$, $F\alpha g C(X)$, $FSG C(X)$, $F\alpha C(X)$, $FS C(X)$).

The collection of all $f\ddot{g}$ -open (resp. $f\omega$ -open, fg-open, fgs-open, fgsp-open, $f\alpha$ g-open, fsg-open, $f\alpha$ -open, fuzzy semi-open) sets is denoted by $F\ddot{G}O(X)$ (resp. $F\omega O(X)$, $FGO(X)$, $FGSO(X)$, $FGSPO(X)$, $F\alpha GO(X)$, $FSGO(X)$, $F\alpha O(X)$, $FO(X)$).

We denote the power set of X by $P(X)$.

Theorem 2.4[10]: If A and B are $f\ddot{g}$ -closed sets in (X, τ) , then $A \vee B$ is $f\ddot{g}$ -closed in (X, τ) .

Theorem 2.5[10]: If A is $f\ddot{g}$ -closed in (X, τ) and $A \leq B \leq \text{cl}(A)$, then B is $f\ddot{g}$ -closed in (X, τ) .

3. FUZZY \ddot{g} -OPEN SETS

We introduce the following definition.

Definition 3.1: A fuzzy set A in a fuzzy topological space is called fuzzy \ddot{g} -open set (briefly $f\ddot{g}$ -open set) if A^c is fuzzy \ddot{g} -closed.

Proposition 3.2: Every fuzzy open set is $f\ddot{g}$ -open.

Proof: If A is any fuzzy open set in (X, τ) . Then A^c is fuzzy closed set and G is any fsg-open set such that $G \geq A^c = \text{cl}(A^c)$. Thus A^c is $f\ddot{g}$ -closed. Hence A is $f\ddot{g}$ -open set.

The converse of Proposition 3.2 need not be true as seen from the following example.

Example 3.3: Let $X = \{a, b\}$ and $\alpha : X \rightarrow [0, 1]$ be defined by $\alpha(a) = \alpha(b) = 0.5$ and $\beta(a) = \beta(b) = 0.6$. Then (X, τ) is a fuzzy topological space with $\tau = \{0_x, \alpha, \beta, 1_x\}$. In (X, τ) , the fuzzy subset $\lambda = (0.7, 0.7)$ is $f\ddot{g}$ -open set in (X, τ) but not fuzzy open in (X, τ) .

Proposition 3.4: Every $f\ddot{g}$ -open set is fgsp-open.

Proof: If A is any fuzzy open set in (X, τ) . Then A^c is a $f\ddot{g}$ -closed subset of (X, τ) and G is any fuzzy open set such that $G \geq A^c$, every fuzzy open set is fsg-open, we have

$G \geq \text{cl}(A^c) \geq \text{spcl}(A^c)$. Thus A^c is fgsp-closed. Hence A is fgsp-open in (X, τ) .

The converse of Proposition 3.4 need not be true as seen from the following example.

Example 3.5: Let $X = \{a, b\}$ and $\alpha : X \rightarrow [0, 1]$ be defined by $\alpha(a) = \alpha(b) = 0.5$. Then (X, τ) is a fuzzy topological space with $\tau = \{0_x, \alpha, 1_x\}$. In (X, τ) , the fuzzy subset $\lambda = (0.6, 0.6)$ is fgsp-open set but not $f\ddot{g}$ -open set in (X, τ) .

Proposition 3.6: Every $f\ddot{g}$ -open set is $f\omega$ -open.

Proof: Suppose that $A^c \leq G$ and G is fuzzy semi-open in (X, τ) . Since every fuzzy semi-open set is fsg-open and A^c is $f\ddot{g}$ -closed, therefore $\text{cl}(A^c) \leq G$. Thus A^c is $f\omega$ -closed in (X, τ) . Hence A is $f\omega$ -open.

The converse of Proposition 3.6 need not be true as seen from the following example.

Example 3.7: Let $X = \{a, b\}$ and $\alpha : X \rightarrow [0, 1]$ be defined by $\alpha(a) = \alpha(b) = 0.5$. Then (X, τ) is a fuzzy topological space with $\tau = \{0_x, \alpha, 1_x\}$. In (X, τ) , the fuzzy subset $\lambda = (0.6, 0.6)$ is $f\omega$ -open but not $f\ddot{g}$ -open set in (X, τ) .

Proposition 3.8: Every $f\ddot{g}$ -open set is fg-open.

Proof: If A is a $f\ddot{g}$ -open subset of (X, τ) . Then A^c is a $f\ddot{g}$ -closed subset of (X, τ) and G is any fuzzy open set such that $G \geq A^c$, since every fuzzy open set is fsg-open, we have

$G \geq \text{cl}(A^c)$. Thus A^c is fg-closed in (X, τ) . Hence A is fg-open.

The converse of Proposition 3.8 need not be true as seen from the following example.

Example 3.9: Let $X = \{a, b\}$ and $\alpha : X \rightarrow [0, 1]$ be defined by $\alpha(a) = \alpha(b) = 0.5$. Then (X, τ) is a fuzzy topological space with $\tau = \{0_x, \alpha, 1_x\}$. In (X, τ) , the fuzzy subset $\lambda = (0.6, 0.6)$ is fg-open but not $f\ddot{g}$ -open set in (X, τ) .

Proposition 3.10: Every $f\ddot{g}$ -open set is $f\alpha$ g-open.

Proof: If A is a $f\ddot{g}$ -open subset of (X, τ) , A^c is a $f\ddot{g}$ -closed subset of (X, τ) and G is any fuzzy open set such that $G \geq A^c$, since every fuzzy open set is fsg-open, we have $G \geq \text{cl}(A^c) \geq \alpha \text{cl}(A^c)$. Thus A^c is $f\alpha$ g-closed in (X, τ) . Hence A is $f\alpha$ g-open.

The converse of Proposition 3.10 need not be true as seen from the following example.

Example 3.11:

Let $X = \{a, b\}$ and $\alpha : X \rightarrow [0, 1]$ be defined by $\alpha(a) = 1$ $\alpha(b) = 0$. Then (X, τ) is a fuzzy topological space with $\tau = \{0_x, \alpha, 1_x\}$. In (X, τ) , the fuzzy subset $\lambda = (1, 0.5)$ is $f\alpha$ g-open set but not $f\ddot{g}$ -open set in (X, τ) .

Proposition 3.12: Every $f\ddot{g}$ -open set is fgs-open.

Proof: If A is a $f\ddot{g}$ -open subset of (X, τ) , A^c is a $f\ddot{g}$ -closed subset of (X, τ) and G is any fuzzy open set such that $G \geq A^c$, since every fuzzy open set is fsg-open,

we have $G \geq \text{cl}(A^c) \geq \text{scl}(A^c)$. Thus A^c is fgs-closed set in (X, τ) . Hence A is fgs-open.

The converse of Proposition 3.12 need not be true as seen from the following example.

Example 3.13: Let $X = \{a, b\}$ and $\alpha : X \rightarrow [0, 1]$ be defined by $\alpha(a) = 1, \alpha(b) = 0$. Then (X, τ) is a fuzzy topological space with $\tau = \{0_x, \alpha, 1_x\}$. In (X, τ) , the fuzzy subset $\lambda = (1, 0.5)$ is fgs-open set but not $f\ddot{g}$ -open set in (X, τ) .

Proposition 3.14: Every $f\ddot{g}$ -open set is $f\ddot{g}_\alpha$ -open.

Proof: If A is a $f\ddot{g}$ -open subset of (X, τ) . Then A^c is $f\ddot{g}$ -closed subset of (X, τ) and G is any fsg-open set such that $G \geq A^c$, then $G \geq \text{cl}(A^c) \geq \alpha \text{cl}(A^c)$. $\therefore A^c$ is $f\ddot{g}_\alpha$ -open in

(X, τ) . Hence A is $f\ddot{g}_\alpha$ -open. The converse of Proposition 3.15 need not be true as seen from the following example.

Example 3.15: Let $X = \{a, b\}$ and $\alpha : X \rightarrow [0, 1]$ be defined by $\alpha(a) = 1, \alpha(b) = 0$. Then (X, τ) is a fuzzy topological space with $\tau = \{0_x, \alpha, 1_x\}$. In (X, τ) , the fuzzy subset $\lambda = (1, 0.5)$ is $f\ddot{g}_\alpha$ -open set in (X, τ) but not $f\ddot{g}$ -open in (X, τ) .

Proposition 3.16: Every $f\alpha$ -open set is $f\ddot{g}_\alpha$ -open.

Proof: If A is a $f\alpha$ -open subset of (X, τ) . Then A^c is $f\alpha$ -closed subset of (X, τ) and G is any fsg-open set such that $G \geq A^c$, then $G \geq \text{cl}(A^c) \geq \alpha \text{cl}(A^c)$. Thus A^c is $f\ddot{g}_\alpha$ -closed. Hence A is $f\ddot{g}_\alpha$ -open.

The converse of Proposition 3.17 need not be true as seen from the following example

Example 3.17: Let $X = \{a, b\}$ and $\alpha : X \rightarrow [0, 1]$ be defined by $\alpha(a) = 1, \alpha(b) = 0$. Then (X, τ) is a fuzzy topological space with $\tau = \{0_x, \alpha, 1_x\}$. In (X, τ) , the fuzzy subset $\lambda = (0.5, 1)$ is $f\ddot{g}_\alpha$ -open but not $f\alpha$ -open set in (X, τ) .

Proposition 3.18: Every $f\ddot{g}$ -open set is $f\psi$ -open.

Proof: If A is a $f\ddot{g}$ -open subset of (X, τ) . Then A^c is a $f\ddot{g}$ -closed subset of (X, τ) and G is any fsg-open set such that $G \geq A^c$, then $G \geq \text{cl}(A^c) \geq \text{scl}(A^c)$. Thus A^c is $f\psi$ -closed in (X, τ) . Hence A is $f\psi$ -open. The converse of Proposition 3.20 need not be true as seen from the following example.

Example 3.19: Let $X = \{a, b\}$ and $\alpha : X \rightarrow [0, 1]$ be defined by $\alpha(a) = 1, \alpha(b) = 0$. Then (X, τ) is a fuzzy topological space with $\tau = \{0_x, \alpha, 1_x\}$. In (X, τ) , the fuzzy subset $\lambda = (1, 0.5)$ is $f\psi$ -open set but not $f\ddot{g}$ -open set in (X, τ) .

Proposition 3.20: Every $f\psi$ -open set is fsg-open.

Proof: If A is a $f\psi$ -open subset of (X, τ) . Then A^c is a $f\psi$ -closed. Suppose that $A^c \leq G$ and G is fuzzy semi-open in (X, τ) . Since every fuzzy semi-open set is fsg-open and A^c is $f\psi$ -closed, therefore $\text{scl}(A^c) \leq G$. Thus A^c is fsg-closed in (X, τ) . Hence A is fsg-open.

The converse of Proposition 3.22 need not be true as seen from the following example.

Example 3.21: Let $X = \{a, b\}$ and $\alpha : X \rightarrow [0, 1]$ be defined by $\alpha(a) = \alpha(b) = 0.5$. Then (X, τ) is a fuzzy topological space with $\tau = \{0_x, \alpha, 1_x\}$. In (X, τ) , the fuzzy subset $\beta = (0.6, 0.6)$ is fsg-open set but not $f\psi$ -open set in (X, τ) .

Proposition 3.22: Every fs-open set is $f\psi$ -open.

Proof: If A is fs-open set in (X, τ) . Then A^c is fs-closed set in (X, τ) and G is any fuzzy semi-open set such that $G \geq A^c$, since every fuzzy semi-open set is fsg-open, we have

$G \geq A^c = \text{scl}(A^c)$. Thus A^c is $f\psi$ -closed in (X, τ) . Hence A is $f\psi$ -open in (X, τ) .

The converse of Proposition 3.23 need not be true as seen from the following example.

Example 3.23: Let $X = \{a, b\}$ and $\alpha : X \rightarrow [0,1]$ be defined by $\alpha(a) = \alpha(b) = 0.5$. Then (X, τ) is a fuzzy topological space with $\tau = \{0_x, \alpha, 1_x\}$. In (X, τ) , the fuzzy subset $\lambda = (0.4, 0.4)$ is $f\psi$ -open set but not fs-open set in (X, τ) .

Proposition 3.24: Every $f\omega$ -open set is $f\alpha g_s$ -open.

Proof: If A is a $f\omega$ -open subset of (X, τ) . Then A^c is a $f\omega$ -closed subset of (X, τ) and G is any fuzzy semi-open set such that $G \geq A^c$, then $G \geq cl(A^c) \geq \alpha cl(A^c)$. Thus A^c is $f\alpha g_s$ -closed set. Hence A is $f\alpha g_s$ -open set

The converse of Proposition 3.26 need not be true as seen from the following example.

Example 3.25: Let $X = \{a, b\}$ and $\alpha : X \rightarrow [0,1]$ be defined by $\alpha(a) = 1, \alpha(b) = 0$. Then (X, τ) is a fuzzy topological space with $\tau = \{0_x, \alpha, 1_x\}$. In (X, τ) , the fuzzy subset $\beta = (1, 0.5)$ is $f\alpha g_s$ -open but not $f\omega$ -open set in (X, τ) .

Proposition 3.26: Every $f\ddot{g}$ -open set is $f\alpha g_s$ -open.

Proof: If A is a $f\ddot{g}$ -open subset of (X, τ) . Then A^c is a $f\ddot{g}$ -closed subset of (X, τ) and G is any fuzzy semi-open set such that $G \geq A^c$, since every fuzzy semi-open set is fsg-open, we have $G \geq cl(A^c) \geq \alpha cl(A^c)$. Thus A^c is $f\alpha g_s$ -closed. Hence A is $f\alpha g_s$ -open in (X, τ) .

The converse of Proposition 3.28 need not be true as seen from the following example.

Example 3.27: Let $X = \{a, b\}$ and $\alpha : X \rightarrow [0,1]$ be defined by $\alpha(a) = 1, \alpha(b) = 0$. Then (X, τ) is a fuzzy topological space with $\tau = \{0_x, \alpha, 1_x\}$. In (X, τ) , the fuzzy subset $\beta = (1, 0.5)$ is $f\alpha g_s$ -open but not $f\ddot{g}$ -open set in (X, τ) .

Proposition 3.28: Every $f\alpha g_s$ -open set is $f\alpha g$ -open.

Proof: If A is a $f\alpha g_s$ -open sub set of (X, τ) . Then A^c is a $f\alpha g_s$ -closed sub set of (X, τ) .

Suppose that $A^c \leq G$ and G is fuzzy semi-open such that $G \geq A^c$. Since every fuzzy semi-open set is fsg-open, we have $G \geq cl(A^c) \geq \alpha cl(A^c)$. Thus A^c is $f\alpha g$ -closed in (X, τ) . Hence A is $f\alpha g$ -open in (X, τ) .

The converse of Proposition 3.30 need not be true as seen from the following example.

Example 3.29: Let $X = \{a, b\}$ and $\alpha : X \rightarrow [0,1]$ be defined by $\alpha(a) = \alpha(b) = 0.5$. Then (X, τ) is a fuzzy topological space with $\tau = \{0_x, \alpha, 1_x\}$. In (X, τ) , the fuzzy subset $\beta = (0.6, 0.6)$ is $f\alpha g$ -open set but not $f\alpha g_s$ -open set in (X, τ) .

Proposition 3.30: Every $f\omega$ -open set is fg -open.

Proof: If A is a $f\omega$ -open sub set of (X, τ) . Then A^c is a $f\omega$ -closed sub set of (X, τ) . Suppose that $A^c \leq G$ and G is fuzzy open set such that $G \geq A^c$. Since every fuzzy open set is fuzzy semi open set, we have $G \geq cl(A^c)$. Thus A^c is fg -closed. Hence A is fg -open set in (X, τ) .

The converse of Proposition 3.32 need not be true as seen from the following example.

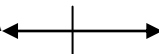
Example 3.31: Let $X = \{a, b\}$ and $\alpha : X \rightarrow [0,1]$ be defined by $\alpha(a) = 1, \alpha(b) = 0$. Then (X, τ) is a fuzzy topological space with $\tau = \{0_x, \alpha, 1_x\}$. In (X, τ) , the fuzzy subset $\beta = (0, 0.5)$ is fg -open but not $f\omega$ -open set in (X, τ) .

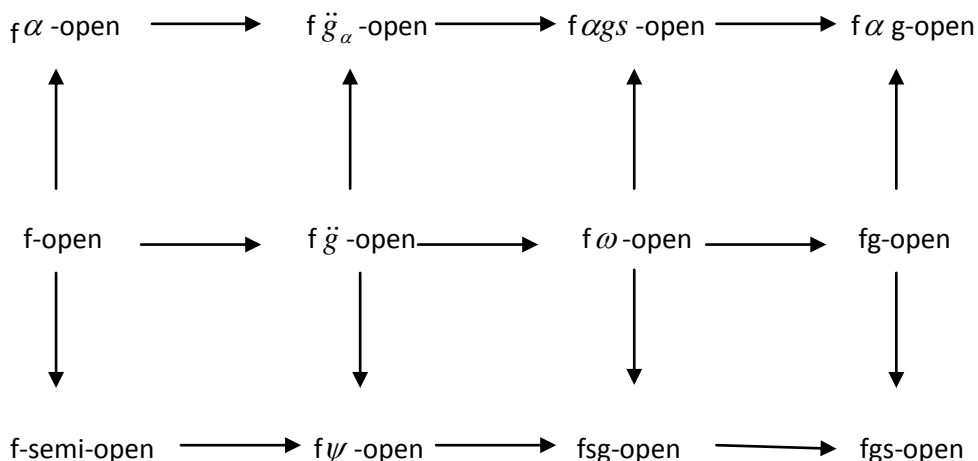
Remark 3.32: The following examples show that $f\ddot{g}$ -open sets are independent of $f\alpha$ -open sets and fuzzy semi-open sets.

Example 3.33: Let $X = \{a, b\}$ and $\alpha : X \rightarrow [0,1]$ be defined by $\alpha(a) = \alpha(b) = 0.5$ and $\beta(a) = \beta(b) = 0.6$. Then (X, τ) is a fuzzy topological space with $\tau = \{0_x, \alpha, \beta, 1_x\}$. In (X, τ) , the fuzzy subset $\lambda = (0.6, 0.6)$ is $f\ddot{g}$ -open but it is neither $f\alpha$ -open nor fuzzy semi-open in (X, τ) .

Example 3.34: Let $X = \{a, b\}$ and $\alpha : X \rightarrow [0,1]$ be defined by $\alpha(a) = 1, \alpha(b) = 0$. Then (X, τ) is a fuzzy topological space with $\tau = \{0_x, \alpha, 1_x\}$. In (X, τ) the fuzzy subset $\beta = (1, 0.5)$ is $f\alpha$ -open as well as fuzzy semi-open in (X, τ) but it is not $f\ddot{g}$ -open in (X, τ) .

Remark 3.35: we obtain the following diagram, where $A \rightarrow B$ (resp. $A \leftarrow B$) represents A implies B but not conversely (resp. A and B are independent of each other).





4.0 PROPERTIES OF $f\ddot{g}$ -OPEN SETS

In this section, we discuss some basic properties of $f\ddot{g}$ -open sets.

Theorem 4.1:If A and B are $f\ddot{g}$ -open sets in (X, τ) , then $A \wedge B$ is $f\ddot{g}$ -open in (X, τ) .

Proof:Considering the compliments in theorem 2.4, the proof follows.

Remark 4.2:The union of two $f\ddot{g}$ -open sets in (X, τ) is not necessarily $f\ddot{g}$ -open as seen from the following example.

Example 4.3:Let $X = \{a, b\}$ and $\alpha, \beta : X \rightarrow [0,1]$ be defined by $\alpha(a) = 0.6, \alpha(b) = 0$ and $\beta(a) = 0, \beta(b) = 0.3$. Then (X, τ) is a fts with $\tau = \{0_x, \alpha, \beta, \alpha \vee \beta, 1_x\}$. In (X, τ) , the fuzzy subset $\lambda_1 = (0.7, 0)$ and $\lambda_2 = (0, 0.3)$ are $f\ddot{g}$ -open, but $\lambda_1 \vee \lambda_2 = (0.7, 0.3)$ is not $f\ddot{g}$ -open.

Theorem 4.4:A fuzzy subset A of a fts (X, τ) is $f\ddot{g}$ -open iff $F \leq \text{int}(A)$ whenever $1-F$ is fsg-open and $F \leq A$.

Proof:Necessity: Assume that A is $f\ddot{g}$ -open in (X, τ) . Let $1-F$ be fsg-open such that $F \leq A$. Then $1-A \leq 1-F$ where $1-A$ is $f\ddot{g}$ -closed. Hence $\text{cl}(1-A) \leq 1-F$ and $F \leq 1-\text{cl}(1-A) = \text{int}(A)$.

Sufficiency: To prove that A is $f\ddot{g}$ -open under the given conditions, we prove $1-A$ is $f\ddot{g}$ -closed in (X, τ) . Let U be any fsg-open set such that $1-A \leq U$. Then $1-U \leq A$.

Taking $F = 1-U$, we have $F \leq A$ where $1-F$ is fsg-open. By assumption $F \leq \text{int}(A)$ which implies $1-U \leq \text{int}(A)$ and hence $1-\text{int}(A) \leq U$. Thus $\text{cl}(1-A) \leq U$ which proves that $1-A$ is $f\ddot{g}$ -closed and A is $f\ddot{g}$ -open.

Theorem 4.5:If A is $f\ddot{g}$ -open in (X, τ) and $\text{int}(A) \leq B \leq (A)$, then B is $f\ddot{g}$ -open in (X, τ) .

Proof:Let $\text{int}(A) \leq B \leq (A)$ implies $1-A \leq 1-B \leq 1-\text{int}(A) = \text{cl}(1-A)$ where $1-A$ is $f\ddot{g}$ -closed in (X, τ) . By Theorem 4.3, $1-B$ is $f\ddot{g}$ -closed and hence B is $f\ddot{g}$ -open in (X, τ) .

Proposition 4.6:If A is a fsg-open and $f\ddot{g}$ -closed in (X, τ) , then A is fuzzy closed in (X, τ) .

Proof:Since A is fsg-open and $f\ddot{g}$ -closed, $\text{cl}(A) \leq A$ and hence A is fuzzy closed in (X, τ) .

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