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# GENERALIZATIONS OF TOPOLOGICAL SPACES

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**ABSTRACT**

This is the fourth in a series of papers on **U**-spaces. Here several generalizations of topological spaces (I-spaces, CU-spaces, CUI-spaces, FU-spaces and FUI- spaces) have been introduced and many topological theorems have been generalized to I- spaces, as an extension of study of infratopological spaces. We have generalized some properties of topological spaces to the other spaces too.

**Key Words:** U-space, I-spaces, CU-spaces, CUI-spaces, FU-spaces and FUI-spaces, AMS Mathematics Subject Classification (2010): 54A40, 54D10, 54G12, 54D15



## INTRODUCTION

In a previous paper [1] we have introduced U- spaces and studied some of their properties. In this paper we use the terminology of [1]. Some study of these spaces was done previously in ([2],[3],[8],[12]) in less general form, and the spaces were called supratopological spaces. In this paper we have introduced the concepts of I-spaces, CU-spaces, CUI-spaces, FU-spaces and FUI-spaces as generalization of topological spaces. I-spaces have been called infratopological spaces by some authors [4], [9], [10]. The concepts of limit point of a set, Interior point of a set, closure of a set, three types of continuity, compactness, connectedness, and separation axioms in the topological spaces have been generalized to the case of I-spaces. The concepts can be defined similarly for CU-spaces, CUI-spaces, FU-spaces and FUI-spaces. We have constructed many examples and proved a number of theorems involving these concepts in case of I-spaces. For the other types of spaces some of these have been dealt with briefly.

## 2. I-SPACES

**Definition 2.1** Let X be a non- empty set. A collection I of subsets of X is called an **I- structure** on X if

(i) X, $\Phi \in$ I,

(ii) $G_1, G_2, G_3, G_4, G_5, \cdots\cdots, G_n \in I$, implies $G_1 \cap G_2 \cap G_3 \cap G_4 \cap G_5 \cap \cdots\cdots \cap G_n \in I$.

Then (X, I ) is called an I-space.

**Example** 2.1 For a non- empty X, {X, $\Phi$ } is an I-structure. In fact every topology is an I-structure on X, and so, every topological space is an I-space.

**Example** 2.2 Let X = Z, and I = {{m Z $|m \in$ N}$\cup\{\Phi\}$ }.

Then mZ $\cap$ $m'$ Z = l Z, where m, $m' \in$ N and l = l.c.m of m and $m'$.

Then (X, I ) is an I-space. However, X is not a U-space.

**Definition 2. 2** An I-space which is not a topological space is called a proper I-space.

**Example** 2.3 Let X ={a, b, c, d}, I = { X, $\Phi$ ,{a},{a, b},{a, c},{a, d},{a, b, d},{a, c, d}} is a proper I-structure which is not a **topology**, since {a, b}$\cup$ {a, c} = {a, b, c}$\notin$ I.

**Definition 2.3** Let X = R and I = {R, $\Phi$ , all finite intersection of sets of the form (a,b), a, b$\in$ R}. Then (X, I ) is an I-space and is called the usual I-space R of the first kind**.** Thus, I consists of R, $\Phi$ and the intervals (a, b).

**Definition 2.4** The usual I-space R of the second kind is the I-space (R, I ), where

I = The collection of the finite intersection of all rays (-$\infty$, b) and (a,$\infty$) together with R and $\Phi$. Thus, I consists of the sets of the form R, $\Phi$ , (-$\infty$,b), (a,$\infty$) and (a, b).

We may define the interior points and the interior of a set in an I-space as in the case of a topological space. The limit points and the closure of a subset in an I-space may be defined similarly. But in an I-space the interior and the closure of a subset may not have the properties of those in a topological space.

We consider below the following definitions in this situation. Let (X, I ) be an I-space. Let A$\subseteq$X. We have thus the following definitions.

**Definition 2.5** A point x$\in$X such that, for each I- open set G which contains x, G$\cap$A contains an element other than x, is called a limit point of A. The set of all limit points of A is called the derived set of A and is denoted by D(A).

**Definition 2.6** The closure of A written $\overline{A}$, is the subset of X consists of the elements x such that for each an I-open set G containing x, G$\cap$A$\neq$ $\Phi$. i.e., $\overline{A}$ = {x$\in$X$|$ for each G$\in$ I with x$\in$G, G$\cap$A$\neq$ $\Phi$ .}. Clearly, $\overline{A}$ = A$\cup$ D(A)

**Definition 2.7** A point x$\in$X is called **an interior point of A**, if there is an I-open set G such that x$\in$G and G$\subseteq$A.

**Definition 2.8** The set of all interior points of A is called the interior of A and is denoted by IntA. Thus, IntA = { x$\in$X$|\exists$ G$\in$ I such that x$\in$G$\subseteq$A }

**Comment 2.1**

For a subset A of a topological space X,

(i) $\overline{A}$ is an I-closed set and is the intersection of all I-closed supersets of A.

(ii) IntA is an I-open set and is the union of all I-open subsets of A.

But these properties may or may not hold for $\overline{A}$ and IntA in I-spaces. The truth of the comment follows from the following theorems and illustrations;

1. (i) Let X = The usual I-space R of the first or the second kind. Let A = Q. Then $\overline{A}$ = R, and R is I-closed and is the intersection of all I-closed supersets of Q.

(ii) Let X ={a, b, c, d}. Then I ={X, $\Phi$ ,{a},{a,b}, {a,c},{a, d},{a, b,d}, {a,c, d}} is proper I-structure on X. Then (X, I ) is a proper I-space. The I -closed sets are {c, d}, {b, d}, {b, c, d}, {c}, {b}, {b, c}, X, $\Phi$.

Let A ={b}.Then $\overline{A}$ ={b}. $\overline{A}$ is an I-closed and is the intersection of all I-closed supersets of A.

2. Let A = {d}. Then $\overline{A}$ = {d}. $\overline{A}$ is not an I-closed, but is the intersection of all I-closed supersets of A.

3.(i) Let X be the usual I- space R of the first or the second kind, and let A = N. then $\overline{A}$ = N, and N is neither I-closed nor is the intersection of all I-closed supersets of N.

(ii) Let X ={a, b, c, d} and let I = {X, $\Phi$ ,{b},{d}, {a, b},{b,d}}. Then (X, I ) is a proper I-space. The I-closed sets are {a, c, d}, {a, b, c},{c, d}, {a, c}, X, $\Phi$.

Let A = {b}. Then $\overline{A}$ = {a, b}. $\overline{A}$ is neither I-closed nor is the intersection of all I-closed supersets of A.

4.(i) Let X = The usual I- space R, A = Q. Then IntA = Int Q = $\Phi$ , and so IntA is an I - open and is the union of all I- open sets G $\subseteq$ A = Q.

(ii) Let X ={a, b, c, d} and I = {X, $\Phi$ ,{a},{a, c}, {a, d},{a, b, d}} is a proper I- structure. The (X, I ) is a proper I- space.

Let A = {a}. Then IntA = {a}, and so IntA is an I-open and is the union of all I- open sets G $\subseteq$ A.

5. (i) Let X = {a,b,c,d} and I = {X, $\Phi$ ,{a},{d},{a, c},{a, d},{b, d}} is a proper I-structure. The (X, I ) is a proper I-space.

Let A = {a, c, d}. Then IntA = {a, c, d}, and so IntA is not an I-open and is the union of all I-open sets G $\subseteq$ A.

(ii) Let X be the usual I-space R of the second kind, and let A = [a, b] $\cup$ [c, d], where a <b <c < d. I-open sets are of the form (- $\infty$ ,b),(a, $\infty$ ),(a, b), R, $\Phi$ .

IntA = (a, b) $\cup$ (c, d) is not an I-open set but is a union of I-open sets.

**3. I-CONTINUITY**

We define I-continuous, $\overline{I}$ -continuous and I*-continuous maps as we have done for U-continuous, $\overline{U}$ -continuous and U*-continuous maps.

**Definition 3.1** If X, Y are I-spaces (resp. X I-space, Y top-space; X top-space, Y I-space) a map f: X $\rightarrow$ Y is said to be **I-continuous** (resp. $\overline{I}$ -continuous,I*-continuous) if for each I-open set (resp. open, I-open) H in Y, $f^{-1}$(H) is an I-open (resp. I-open, open ) set in X .

**Example 3.1** Let X ={a, b, c, d }, I = {X, Φ, {a}, {b}, {d}, {a, b}, {a, d}, {b, c, d}}
Y = {p, q, r, s}, I ′ = {Y, Φ, {p}, {q}, {s}, {p, q},{p, s},{q, r, s}}. Let f: X $\rightarrow$ Y be defined by f(a) = p, f(b) = q, f(c) = r, f(d) = s.

Then f is I-continuous.

**Example 3.2** Let X= {a, b, c, d}, I = {X,Φ,{a},{b},{a, b},{a, c},{b, d}}.
Let Y = {p, q, r }, ***T*** = {Y, Φ, {p}, {q}, {p, r}}. Then (X, I ) is an I-space and (Y, I ) is a topological space. The function f: X $\rightarrow$ Y is defined by f(a) = p, f(b) = q, f(c) = r, f(d) = q. Then f is $\overline{I}$ -continuous.

**Example 3.3** Let X = {a, b, c, d}, ***T*** = {X, Φ,{b},{c},{b, c},{c, d},{b, c, d}}
Y = {p, q, r, s }, I = {Y,Φ, {p}, {q}, {p, q}, {p, r}, {q, s}}. Then (Y, I ) is an I- space. The function f: X $\rightarrow$ Y is defined by f(a) = p, f(b) = q, f(c) = q, f(d) = s.

Then f is I *-continuous.

**4. COMPACTNESS**

**Definition 4.1** Let (X,I ) be an I- space. An I-**open cover** of subset K is a collection {$G_\alpha$ } of I-open sets such that K $\subseteq \bigcup_\alpha G_\alpha$ .

**Definition 4.2** An I-space X is said to be compact if every I-open cover of X has a finite sub-cover.
A subset K of a I-space X is said to be compact if every I-open cover of K has a finite sub-cover.

**Example 4.1** Let X =N and let $A_{n_o}$ ={n $\in$ N$|$ n $\geq n_o$ }, I = {$\Phi$ ,{$A_{n_o}$ $|n_o \in$ N}}. Then ( X, I ) is an I-space. In this I-space, N is compact, because every I-open cover of N must contain $A_1$ = N.

**Comment 4. 1** We note however that

(i) For I-space (N, I ), where I = {N, $\Phi$ } $\cup$ {$n_o$ + 1, $n_o$ + 2, ....... $n_o$ + r$|$ $n_o$ , r $\in$ N}. N is not compact.

(ii) In the usual I-space R, of the first kind, (and also of the second kind), N is not compact.

For, {(n - $\frac{1}{2}$ , n + $\frac{1}{2}$ )$|$ n $\in$ N} is an I-open cover of N which does not have a finite subcover.

**Theorem 4.1** Every I-continuous image of a compact I-space is compact.
The proof is similar to that in topology.

The Heine-Borel Theorem of topology, 'A subset A of the usual space R is compact if and only if A is closed and bounded', has the following forms in the case of the usual I-space R of the first kind:

**Theorem 4.2**

(1) The compact subsets of R are precisely the finite subsets of R.

(2) No non-empty compact subset is I-closed.

(3) No non-empty I-closed subset is compact.

**Proof :**

**(1)** For, if A is an infinite subset of R, let A = $\{a_n\}_{n\in N}$ be a countable subset of R, and suppose $a_n < a_{n+1}$, for each n. Consider the intervals

$I_n = \left(a_n - \frac{\epsilon_n}{2}, a_n + \frac{\epsilon_n}{2}\right)$, where $\epsilon_n = \min\{a_{n+1} - a_n, a_n - a_{n-1}\}$. Then, $I_n \cap I_{n'} = \Phi$, if $n \neq n'$. If $\{I_n\}$ covers A, let C be this cover. Otherwise, let $\{J_k\}$ be a collection of I-open sets such that (i) $J_k \cap \left(\bigcup_n I_n\right) = \Phi$, for each k, and (ii) $\{I_n\} \cup \{J_k\}$ is a cover of A. In this case, let C denote this cover. In both the cases, C does not have a finite subcover. Thus, the compact subsets of R are finite.

**(2)** For, the definition of the I-structure on R shows that every non-empty I-closed set must contain subsets of the form $(-\infty, a]$ and $[b, \infty)$ both of which are infinite.Hence (2) follows from (1)

**(3)** The discussions in (1) and (2) prove (3).

For the usual I-space R of the second kind, the theorem corresponding to the Heine-Borel Theorem in topology is the following:

Theorem 4.3

(i) a compact subset need not be I-closed,

(ii) a compact subset need not be bounded,

(iii) every I-closed and bounded subset is compact.

**Proof :**

**(1)** Being finite, the subset {1, 2, 3,.......,n} of the usual I- space R of the second kind is compact. But it is not I-closed, since the non- trivial I-closed subsets of R are of the form $(-\infty, a]$, $[b, \infty)$ and$((-\infty, a] \cup [b, \infty))$, $(a < b)$. This proves (i).

**(2)** Any I-open cover C of N must contain either R, or an I-open subset of the form $(b, \infty)$, $b < 1$. Any one of these two sets covers N. Hence N is compact.

Clearly, N is unbounded.

**(3)** Let F be an I-closed and bounded subset of R. Then F = $\Phi$ or F = [a, b] or F = {c}, for some a, b, c$\in$ R, a < b. $\Phi$ and {c} are obviously compact. The proof that [a, b] is compact is exactly similar to the corresponding proof in topology.

**Definition 4.3** A subset A of an I- space (X, I ) is said to be **disconnected** if there exist I- open sets $I_1$and $I_2$ of X such that $A \cap I_1 \cap I_2 = \Phi$ and $I_1 \cup I_2 \supseteq A$.

A said to be **connected** if it is not disconnected.

**Example 4.2** Let X = {a, b, c, d} and I = {X, $\Phi$ , {c}, {a, c}, {c, d},{a, b, c},{a, c, d}}. Then (X, I ) is an I-space. Let A = {b, c, d} and B = {b, d}. Then A is connected and B is disconnected.

**Example 4.3** In the usual I-spaces R of the first and the second kinds, all intervals are connected subsets.

**Remark 4.**1 As in topological spaces, the closure of a connected subset of I-space is connected too.

**Remark 4.2** Although in the usual topological space R and the usual U-space R, N, Z, Q are disconnected, in the usual I-space R of the first kind, the above subsets of R are connected. However, these subsets are again disconnected in the usual I-space R of the second kind.

A Housdorff (resp. normal, regular, completely regular) I-space is defined as in topology. The usual I-spaces R of the first and the second kind are Hausforff.

**Remark 4.3.** A compact subset of a Hausdorff topological space is closed. But a compact subset of a Hausdorff I-space need not be I-closed.

Its truth follows from (2) of Theorem 4.2 as well as (1) of Theorem 4.3.

**Remark 4.4** Unlike the usual topological space R and the usual U-space R, the usual I-spaces R of the first kind and the second kind are normal but not regular.

**Proof:** Let X denote the usual I-space R of the first kind. The I-closed sets of X are R, $\Phi$ and sets of the form $(-\infty, a] \cup [b, \infty)$ with a < b. Let F = $(-\infty, a] \cup [b, \infty)$ and $x \notin$ F. Then $x \in (a, b)$. But the only I- open set containing F is R and it also contains x. Hence X is not regular.

The only pairs of disjoint I-closed sets of X are {R, $\Phi$ },{F, $\Phi$ } and { $\Phi$ , $\Phi$ }. Then the disjoint I-open sets R and $\Phi$ separate each of these pairs of disjoint I-closed sets. Thus, X is normal.

Now, let Y denote the usual I-space R of the second kind.

Then the I-closed sets of Y are R, $\Phi$ , and the sets of the form $(-\infty, a]$, $[b, \infty)$, $(-\infty, c] \cup [d, \infty)$ with c < d. As in the case of X, if F = $(-\infty, c] \cup [d, \infty)$ and $x \notin$ F, then $x \in (c, d)$. The only I-open set of Y which contains F is R which also contains x. Hence Y is not regular.

The only pairs of disjoint I-closed sets of Y are $P_1 = \{(-\infty, a], [b, \infty)\}$ (a < b), $P_2 = \{(-\infty, a] \cup [b, \infty), \Phi\}$, $P_3 = \{R, \Phi\}$. Then $P_1$ is separated by the each of disjoint I-open sets $\left(-\infty, \frac{a+b}{2}\right) and \left(\frac{a+b}{2}, \infty\right)$, while $P_2$ and $P_3$ is separated by the disjoint I-open sets R and $\Phi$ . Hence Y is normal.

**5. CU – SPACES**

**Definition 5.1** X be a non empty set and let CU a collection of subsets of X such that

(i) X, $\Phi \in$ C U ,

(ii) C U is closed under countable unions.

Then C U is called a CU-structure on X and (X, CU) is **called a CU-space. [**Clearly, every topology T (resp. every U-structure U ) on X is CU-structure on X and (X, T ) (resp. (X, U ) ) is a CU-space.] A CU-space, which is neither a topological space, nor a U-space will be called a proper CU-space**.**

**Example 5.1** Let X be an uncountable set and let CU consists of X, $\Phi$ and all countable unions of finite subsets of X. Then (X, C U ) is a proper CU-space.

**Example 5.2** The $\sigma$ algebra B of Borel sets on R is a proper CU-structure on R. Hence (R, B ) is a proper CU-space.

To see this, we first note that every singleton subset of R belongs to B. Let A be a proper uncountable subset of $Q^c$, the set of irrationals. Then A = $\bigcup_{x \in A}\{x\}$, A $\notin$ B. So, B is a proper CU-structure.

**Example 5.3** let X = R, CU = { R, $\Phi$ , all countable unions of all closed intervals [a, b]}. Then (X, CU) is a CU-space. C U properly contains the usual topology on R.

For,

(i) (a, b) = $\bigcup_{m=1}^{\infty}\bigcup_{n=1}^{\infty}\left[a+\frac{1}{m}, b-\frac{1}{n}\right] \in$ C U and every proper open set in the usual topology of R is a countable union of open intervals (a, b).

(ii) $[a, b] \in$ CU, but it does not belong to the usual topology of R.

**Definition 5.2** The usual U-space R is also a CU- space. It is called **the usual CU-space** R.

**Definition 5.3** The closure of A written $\overline{A}$, is the subset of X consisting of the elements x such that for each CU-open set G containing x, $G \cap A \neq \Phi$ . i.e, $\overline{A} = \{x \in X \mid$ for each $G \in$ CU with $x \in G$, $G \cap A \neq \Phi \}$.

**6. CUI- SPACES**

**Definition 6.1** Let X be a non- empty set. A collection CUI of subsets of X is called a **CUI-structure on X** if X, $\Phi \in$ CUI and CUI is closed under countable union, and finite intersection. Then (X,CUI ) is a **CUI-space**.

**Examples 5.1 and 5.2** of CU-spaces are examples of CUI-spaces too.

**Example 6.1** Let X = R and CU = {R, $\Phi$ , and the infinite countable subsets of R}.Then **(X, CU) is a CU-space. Let A = {n$\in$ Z $|-\infty<n<5$} and** B = { n$\in$ Z $|-7<n<\infty$}. Then A, B$\in$ CU. A$\cap$B = {- 6, - 5, - 4, - 3, - 2, - 1, 0, 1, 2, 3, 4}$\notin$ CU. (X, CU ) **is a proper CU-space** but not I-space.

**Example 6.2** Let X =RandC = {R, $\Phi$ , $\cup$ {(n, $\infty$ ) $|n \in$ Z}, $\cup$ {(- $\infty$ , n) $|n \in$Z}, $\cup\{[(m,\infty)\cup(-\infty,n),\ m,n \in$Z}}.

**Then** (R, C ) **is a U- space and so, a CU-space but not an I-space**.

**Example 6.3** Let X = N or, Z, and I = {X, $\Phi$ , all finite subsets of X}.

Then (X, I ) is an I-space but not a CU-space, and hence, not a U-space.

**Definition 6.2** The usual topological space R is defined to be the usual CUI- space R.

**7. FU- SPACES**

**Definition 7.1** Let X be a non-empty set and let FU be a collection of subsets of X such that

(i) X, $\Phi \in$ FU

(ii) FU is closed under finite unions.

Then FU is called an F**U-structure on X** and (X, FU ) is **called an FU-space.**

**Example 7.1** Topological spaces, U-spaces and CU-spaces are FU-spaces.

**Definition 7.2** A FU-space which is not a CU-space (and hence neither a U-space nor a topological space) is called a proper FU-space.

**Example 7.2** Let X be an infinite set and let FU be the collection of all finite subsets of X**.** Then (X, FU) is a proper FU-space.

**Example 7.3** Let X be R and FU the collection of all finite union of sets of the form (- $\infty$ , a) and (b, $\infty$ ). Then (X, FU) is FU-space.

**Definition 7.3** The usual FU-space R is R with the FU-structure consisting of R, $\Phi$ , and all finite unions of the sets of the form (- $\infty$ , a), (b, $\infty$ ) and (c, d).

**We thus note:**

**Remark 7.1** The FU-structure of the usual FU-space R consists precisely of the sets R, $\Phi$ and sets of the form (- $\infty$ , a), (b, $\infty$ ), (- $\infty$ , a)$\cup$(b, $\infty$ ) (a < b), ($a_1$, $b_1$)$\cup$($a_2$, $b_2$)$\cup\cdots\cup$($a_r$, $b_r$), for some positive integer r with $a_i < b_i$, $1\le i\le r$, and (- $\infty$ , a) $\cup$ (b, $\infty$ ) $\cup$ ($a_1$, $b_1$)$\cup$($a_2$, $b_2$)$\cup\cdots\cup$($a_s$, $b_s$), ($a < a_1< b_1< a_2 < b_2 <\cdots\cdots< a_s < b_s < b$ ) for some positive integer s.

**Definition 7.4** Let (X, FU) be an FU-space and let A be a subset of X. For x$\in$X, x is called an interior point of A if x$\in$G$\subseteq$A, for some FU-open set G in X.

**Definition 7.5** The set of all interior points of A is called **the interior of A**, and is denoted by IntA.

**Remark 7.**2 Unlike in topological spaces, IntA need not be FU-open in an FU-space.

To see this, let us consider the usual FU-space R. Let A = $\bigcup_{n=1}^{\infty}(2n,2n+1)$ .

Then, A = IntA . But A is not FU-open.

**Remark 7.3** However, for every FU-open set A in an FU-space, A = IntA.

The FU-closed sets of X are the complements of FU-open sets.

**Definition 7.6** The FU-closure $\overline{A}$ of a subset A of an FU-space X is defined by

$\overline{A}$ = {x$\in X | x \in G$ for each FU-open set G in X with G$\cap$A $\neq \Phi$ }.

**Theorem 7.1** Let X be an FU-space,

(i) For every FU-closed set F of X, $\overline{F}$ = F,

(ii) For a subset A of X, $\overline{A}$ need not be FU-closed.

**Proof: (i)** Let x$\in \overline{F}$. If x$\notin$F, then x$\in F^c$. Now x$\in \overline{F}$ and **s**ince $F^c$ is FU-open, and x$\in F^c$, $F^c \cap F \neq \Phi$, a contradiction. Hence x$\in$F.

(ii) Let X be the usual FU-space R and A = (1, 2)$\cup$(3, 4).

Then, $\overline{A}$ = [1, 2] $\cup$ [3, 4]. But this is not an FU-closed set in X, since the FU-closed subsets of X are precisely R, $\Phi$ and sets of the form [a, b], $[-\infty, a_1] \cup [a_2, b_1] \cup \cdots \cup [a_r, b_{r-1}] \cup [b_r, \infty]$$(a_1 < b_1 < a_2 < b_2 <$ --------$< a_r < b_r)$, $[a_1,b_1] \cup [a_2, b_2] \cup \cdots \cup [a_s, b_s]$, $a_1 < b_1 < a_2 < b_2 < \cdots\cdots < a_s < b_s$.

**Definition 7.7** A subset A of an FU-space X is called **compact** if every FU-open cover has a finite subcover.

**Example 7.4** In the usual FU-space R, N and the intervals [a, b] are compact subsets.

The proof that [a, b] is compact is similar to that in topology.

To see that N is compact, we note that every FU-open cover of N must contain a FU-open set of the form (a, $\infty$), a < 1. Then, at most [a] more FU-open sets of the cover are needed to cover A, where [a] is the largest positive integer less than or equal to a. Thus, N is compact.

**Theorem 7.2** Every FU-closed subsets of a compact FU-space is compact.

The proof is as in topology.

**Remark 7.4** The following is the FU-version of the Heine-Borel Theorem in topology: Let X be the usual FU-space R.

(i) Every FU-closed and bounded set in X is compact,

(ii) A compact set in X may be neither FU-closed nor bounded.

**Proof: (i)** It follows from the nature of the FU-closed sets in X that every non-empty FU-closed bounded set in X is of the form $[a_1,b_1] \cup [a_2, b_2] \cup \cdots \cup [a_r, b_r]$ which is obviously compact.

(ii) We have proved above (in Example 7.4) that N is compact.

However, N is neither FU-closed nor bounded.

**Definition 7.8** A non-empty subset A of an FU-space X is called disconnected if there exist FU-open sets $G_1$ and $G_2$, such that $A \cap G_1 \neq \Phi \neq A \cap G_2$, $A \cap G_1 \cap G_2 = \Phi$, $A \subseteq G_1 \cup G_2$. A is called connected if it is not disconnected.

**Example 7.5** In the usual FU-space R, the connected subsets are precisely R, $\Phi$ and sets of the form ($-\infty$, a), (b, $\infty$) and (c, d).

As in topology, we have every FU- continuous image of a connected set is connected.

**8. FUI- SPACES**

**Definition 8.1** Let X be a non-empty set. A collection **FUI** of subsets of X is called an **FUI-structure on X** if

(i) X, $\Phi \in$ FU I

(ii) UI is closed under finite unions and finite intersections.

Then FUI is called an F**UI-structure on X** and (X, FU I ) is called an F**UI-space**.

**Example 8.1** Every topological space and every CUI-space is an FUI-space.

**Example 8.2** Let X be an infinite set and FUI = {R, $\Phi$ ,all finite subsets of X}. Then, (X, FUI ) is an FUI-space which is neither a CUI-space nor a topological space.

**Example 8.3** Let X = R and FUI = The subsets of R obtained from the sets of the form ($-\infty$, a) and (b, $\infty$) under finite unions and intersections.

**Then,** (X, FUI ) is an FUI-space. It is called the usual FUI-space R. We note that here FUI consists of R, $\Phi$ and the sets of the form ($-\infty$, a), (b, $\infty$) and $(-\infty, a) \cup$ (b, $\infty$) (a < b), $(a_1, b_1) \cup (a_2, b_2) \cup \cdots \cup (a_r, b_r)$, and $(-\infty, a) \cup (b, \infty) \cup (a_1, b_1) \cup (a_2, b_2) \cup \cdots \cup$ $(a_s, b_s)$, $(a < a_1 < b_1 < a_2 < b_2 < \cdots\cdots < a_s < b_s < b)$ .Thus, the usual FUI-space is exactly the same as the usual FU-space R.

**Remark 8.1 Let X be a FUI-space. As in the case FU-spaces,**

(i) for each FUI-open subset A of X, A = IntA;

but (ii) InA need not always be FUI-open.

The first part is obvious and the second part follows the example in Remark 7.2.

**Remark 8.2 Example 7.**3 is an FU-space but not an FUI-space. Thus, the class of FU-spaces and the class of FUI-spaces are distinct.

**Theorem 8.1** Let X be an FUI-space,

(i) For every FUI-closed set F of X, $\overline{F}$ = F,

(ii) For a subset A of X, $\overline{A}$ need not be FUI-closed.

The proof is exactly similar to that of Theorem 7.1.

All the statements about the compact sets and the connected sets proved earlier for an FU-space, and in particular the statement corresponding to the Heine-Borel Theorem, hold for an FUI-space.

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